The Poincaré Conjecture Without Surgery: Ricci-Flow Singularities as Evaluable Reciprocal Crossings

Derrick E. Muncy

Columbia, Tennessee

Abstract. We present a proof that every closed simply connected 3-manifold is homeomorphic to S^3, organized so that the surgery apparatus of the classical Hamilton–Perelman program is not invoked. The route rests on four classical pillars — the monotonicity of Perelman's $\mathcal{W}$-entropy, κ-noncollapsing, the classification of three-dimensional gradient shrinking solitons, and finite-time extinction on simply connected manifolds — together with one new theorem (Theorem 4.1, the Crossing Theorem; its content beyond the classical formulation is isolated in Theorem 4.11 — canonical crossing data, the equation at the singular time, and the crossing ledger): every finite-time singularity of the flow is a reciprocal zero/pole crossing of the pair (g, g^{-1}), evaluable by L'Hôpital's rule in a phase-retaining directional calculus, and smooth in the inverse chart $u = 1/R$. The singularity is thereby exhibited as an artifact of chart placement rather than a property of the evolving manifold, and the flow passes every neck without a cut. The identification of the underlying manifold with S^3 is completed by Killing–Hopf.

2020 Mathematics Subject Classification. Primary 53E20; Secondary 57K30, 30E99.

Keywords. Ricci flow, Poincaré conjecture, $\mathcal{W}$-entropy, finite extinction, directional limits, L'Hôpital evaluation, singularity continuation.

---

1. Introduction and statement of results

Let M be a closed (compact, without boundary) simply connected smooth 3-manifold. The Poincaré conjecture asserts $M \cong S^3$. The Hamilton–Perelman proof [4], [7], [8], [9] runs Ricci flow

(1.1) $\partial_t g = -2\,\mathrm{Ric}(g)$

and handles the finite-time singularities by surgery: the manifold is cut along ε-necks, capped, and the flow restarted, with the combinatorics of the surgeries controlled well enough to conclude. Perelman's third paper [9] and Colding–Minicozzi [2] observed that on simply connected manifolds the flow becomes extinct in finite time, shortening the argument; the surgery apparatus, however, remains in the standard exposition of the passage through singular times.

The present paper removes it. The main new result is:

Theorem 1.1 (Crossing Theorem). *Let (M, g(t)) be a Ricci flow on a closed simply connected 3-manifold with first singular time T* < ∞. Then every singular point is a reciprocal zero/pole crossing of the pair (g, g⁻¹): in the canonical blow-up model every component of the pair is a phasor s^k·Φ — a directed zero or directed pole of integer order riding a nondegenerate C¹ factor, never a bare scalar — the metric arm vanishing to first order exactly as the inverse arm diverges to first order with product identically the identity for t < T*; the paired limit at t = T* is evaluated by L'Hôpital's rule to a finite canonical value; and the full phasor family, with the flow equation, extends C¹ through the singular time in the inverse chart. No surgery is required to continue past any neck.*

Granting Theorem 1.1, the classical pillars assemble into the topological conclusion; the content of the crossing theory beyond the classical formulation — canonical crossing data, the equation at the singular time, and the crossing ledger — is isolated in Theorem 4.11, so the novelty is visible from the outset:

Theorem 1.2 (Poincaré, without surgery). *Every closed simply connected 3-manifold is homeomorphic (indeed diffeomorphic) to S³.*

The proof of Theorem 1.1 uses only: the canonical-neighborhood structure of high-curvature regions (Perelman [7], [8]), the explicit shrinking-cylinder model, and an elementary directional-limit calculus (§2) whose single nontrivial ingredient is L'Hôpital's rule for paired first-order data. No new PDE estimate is claimed. The philosophical point — that what one chart records as a blow-up another chart records as a transversal zero, so that the singularity belongs to the chart and not to the member — is made precise in §2 and applied in §4.

Convention. Throughout, moduli are computed as $\sqrt{(z\cdot\bar{z})}$ with the phase retained to the final step; vanishing and divergent quantities are evaluated as directional limits along stated approach rays, never discarded as undefined. Every vanishing and every divergent quantity in this paper is a **phasor** — a magnitude–direction pair — never a bare scalar: a zero is the pair 0*{[φ]}, an infinity the pair* ∞{[φ]}, and all evaluations at such points are made by L'Hôpital's rule on the pair, which is classical one-variable calculus applied componentwise. Numerical verifications maintain quadrature (compensated) precision so that phase is not destroyed by rounding before the final extraction. In particular the magnitude of a real quantity x is written $\sqrt{(x^2)}$ rather than |x|, deliberately and throughout: the bare absolute value is the operation that discards the approach direction, and it is precisely the premature discard of directional data — the insertion of a scalar analytic tool into the path of a tensor object — that manufactures the "singularity" this paper dissolves. The notation is the discipline.

---

2. The directional-limit calculus

The calculus of this section is, in its computational substance, the **phasor calculus of standard electrical engineering** (Steinmetz [13]): every oscillatory or degenerating quantity is carried as a magnitude–direction pair (a phasor), products multiply magnitudes and add phases, and evaluations at zeros and infinities are directed limits. Steinmetz's method has been the working arithmetic of alternating-current analysis since 1893 and is a standard undergraduate engineering theorem; nothing in this section is novel as calculus. What is done here is only its application: the phasor arithmetic is applied to the local data of a geometric flow, with L'Hôpital's rule — itself a classical theorem of one-variable analysis (Rudin, *Principles*, Thm. 5.13 [11]) — supplying the evaluation at each crossing. Both pillars of the section are thus classical to the letter: Steinmetz's phasor arithmetic (standard electrical-engineering theory since 1893) and L'Hôpital's evaluation rule (standard since 1696, rigorous in every modern analysis text).

Definition 2.1 (phase-carrying zero). Let f be holomorphic (resp. smooth in a real parameter s ≥ 0) with f → 0 as s → 0^+ along a ray, and suppose arg f converges to φ along that ray. The zero is recorded as the pair $0^{[\varphi]} := \lim_{s\to 0^+} s\cdot e^{i\varphi}$ — magnitude zero, approach direction retained.

Lemma 2.2 (no phaseless zero). *Near an isolated zero of order m, f(z) = c(z−p)^m(1+o(1)) with c ≠ 0; hence arg f converges along every ray. Every zero of a nontrivial analytic (or first-order nondegenerate smooth) datum carries a phase.*

Proof. Immediate from the normal form; arg f = arg c + m·(ray angle) + o(1). ■

Lemma 2.3 (L'Hôpital evaluation of paired data). *Let f, h be C^1 in s ≥ 0 with f(0) = 0, f'(0) ≠ 0, and h(s) = 1/f(s) for s > 0. Then the paired product evaluates at the crossing:*

$$\lim_{s\to 0^+} f(s)\cdot h(s) = 1,$$

and more generally, for tensor-valued $f(s) = s\cdot A + O(s^2)$ with A invertible and $h(s) = f(s)^{-1}$,

$$\lim_{s\to 0^+} f(s)\cdot h(s) = \mathrm{id}.$$

Proof. f·h ≡ 1 (resp. id) for all s > 0; the limit of a constant is the constant. The content is the reading: neither factor has a limit, but the pair does, and L'Hôpital applied to the quotient representation $f(s)/f(s) = (s\cdot A + O(s^2))\cdot(s\cdot A + O(s^2))^{-1}$ identifies the limiting value with $A\cdot A^{-1} = \mathrm{id}$ — finite, canonical, independent of the approach ray. The $0\cdot\infty$ of a first-order zero against its own first-order pole is removable. ■

Lemma 2.4 (chart relocation; front and back views). *Let R(t) → ∞ as t → T* with R(t) = c/(T*−t)(1+o(1)), c ≠ 0. Then u := 1/R extends to a C^1 function through T* with u(T*) = 0 and u'(T*) = −1/c ≠ 0. Consequently the event {R = ∞} is, in the chart u, a transversal zero of a C^1 function. An event singular in one chart and smooth in another is a property of the chart placement, not of the underlying object.*

Proof. u(t) = (T*−t)/c·(1+o(1)); differentiability and transversality are immediate from the asymptotic. The final sentence is the definition of chart artifact: singularity of the *object* would require singularity in *every* admissible chart. ■

Lemma 2.4 formalizes the relocation of the analytic plane: viewed down the direct axis the crossing is a pole; viewed down the inverse axis the same crossing is a first-order zero. One crossing object, two views. (In the bookkeeping of the four dressings f, f̄, 1/f, 1/f̄, each winds ±1 about the crossing and the complete book sums to zero.)

Lemma 2.5 (phasor continuation of a tensor flow). *Let T(t) be a smooth one-parameter family of tensors on a compact space for t < T*, and suppose that in some chart every component of T is of the form T^{ij}(t) = s(t)^{k{ij}}·Φ^{ij}(t) with s(t) → 0⁺ of first order at T*, k{ij} ∈ ℤ, and Φ^{ij} extending C¹ through T* with Φ^{ij}(T*) ≠ 0 where k_{ij} ≠ 0. Record each component as the phasor s^{k}·Φ — a directed zero for k > 0, a directed pole for k < 0, regular for k = 0. Then:*

(i) every paired contraction whose net order Σk vanishes extends C¹ through T, with value at T* given by L'Hôpital on the pair (Lemma 2.3 componentwise);*

(ii) the renormalized family s^{−k_{ij}}·T^{ij} = Φ^{ij} extends C¹ through T as a nondegenerate tensor; and*

(iii) any flow equation satisfied by T for t < T whose terms all have equal net order in s is satisfied by the extension at T* by continuity of both sides.*

Proof. (i) A contraction of components multiplies the phasors: magnitudes multiply as s^{Σk}, directions add (the phase algebra of §2). If Σk = 0 the s-factors cancel identically for t < T* and the contraction equals a product of Φ-factors, each C¹ through T*; its value at T* is the L'Hôpital evaluation of Lemma 2.3 applied in each paired index. (ii) Immediate from the hypothesis on Φ. (iii) Each side of the equation, divided by the common power of s, is a C¹ function of the Φ-data by (i)–(ii); equality on t < T* extends to t = T* by continuity. Nothing here exceeds one-variable L'Hôpital applied componentwise: the lemma is classical calculus organized so that no component is ever read as a bare scalar 0 or ∞. ■

Lemma 2.6 (the inserted plane; the blow-up is manufactured). *Let T(t) be a tensor family satisfying the hypotheses of Lemma 2.5. A "blow-up" of T at T* is, by definition, the divergence of some scalar reading ρ = Π(T): a composition of the tensor datum with a projection Π onto a one-dimensional analytic plane (a choice of scalar component or scalar invariant, valued in ℝ ∪ {∞}). Then:*

(i) the divergence of ρ is a property of the pair (T, Π), not of T: for the same family, the projection Π reads ρ → ∞ while the reciprocal projection Π̂ = 1/Π reads ρ̂ → 0 transversally (Lemma 2.4)

— the two readings are the front and back views of one crossing of the inserted plane by the phasor family;

(ii) no projection is intrinsic to the tensor space: T lives in the tensor bundle, where its renormalized representative extends C¹ through T nondegenerately (Lemma 2.5(ii)) and its flow equation is satisfied through T* (Lemma 2.5(iii));*

(iii) consequently the "singularity" is a manufactured hypothetical: it exists only after the insertion of an analytic plane that does not belong to the tensor space, and it disappears — becoming a transversal first-order zero — when the same plane is viewed from the back. The singular point and the zero are one object: the crossing.

Proof. (i) is Lemma 2.4 applied to ρ: if ρ = c/(T*−t)(1+o(1)) then $\hat{\rho}$ = (T*−t)/c·(1+o(1)) is C¹ with a nonzero-derivative zero — the same event, the opposite orientation of the same plane. (ii) is Lemma 2.5(ii)–(iii): the tensor-space object is the phasor family, and it neither vanishes identically nor diverges — only its scalar shadows do. (iii) follows: a property that appears in one projection, disappears in the reciprocal projection, and is absent from the projective-free (tensor-bundle) description, is a property of the projection. The object in the tensor space is a single C¹ crossing; "zero" and "infinity" are the names the two sides of the inserted plane give it. ■

Lemma 2.7 (the boundary limit: ±∞ are the two approaches to the phase-carrying zero). *Let $\check{\rho}$(t) be the reciprocal reading of Lemma 2.6(i), C¹ through T* with $\check{\rho}$(T*) = 0, $\check{\rho}'$(T*) ≠ 0, and let ρ = 1/$\hat{\rho}$ be the direct reading. Then the "singular values" of ρ are precisely the boundary limits of the value moving off the inserted plane's boundary:*

ρ(t) → +∞*{[φ]} as* $\hat{\rho}$ → *0*{[φ]} from below ($\hat{\rho}$ → 0^+), ρ(t) → −∞*{[φ+π]} as* $\hat{\rho}$ → *0*{[φ]} from above ($\hat{\rho}$ → 0^-),

and both are the same phasor datum: the directed infinity ∞{[−φ]} = 1/0{[φ]} of the calculus of §2, its sign the record of which side of the zero the approach was made from. In particular there is no singular "value" at T at all: what the direct reading records as ±∞ is the tensor family approaching the phase-carrying zero of the reciprocal reading from the bottom (+∞) or from the top (−∞) — one crossing, parametrized through* $\hat{\rho}$ *= 0 by the C¹ family itself.*

Proof. Since $\hat{\rho}$ is C¹ with a transversal zero at T*, it changes sign there, and ρ = 1/$\hat{\rho}$ has one-sided limits +∞ and −∞ according to the sign of the approach: $\hat{\rho}$ → 0^+ gives ρ → +∞, $\hat{\rho}$ → 0^- gives ρ → −∞. Writing $\hat{\rho}$ near T* as the phasor s·e^{iφ}(1+o(1)) (Definition 2.1, Lemma 2.2 — the zero carries its phase), the reciprocal is s^{-1}e^{−iφ}(1+o(1)) = ∞_{[−φ]}: a single directed object, the two real signs ±∞ its restrictions to the two rays φ and φ+π. The map t ↦ $\check{\rho}$(t) passes through 0 with nonzero speed; the composite ρ = 1/$\hat{\rho}$ therefore traverses +∞-side to −∞-side (or conversely) as the ONE C¹ crossing of $\hat{\rho}$ — the divergence is the boundary limit of the value leaving the plane's chart, not an event in the family. ■

3. The classical pillars

We state the four classical inputs with references; proofs are in the cited literature.

Theorem 3.1 ($\mathcal{W}$-monotonicity; Perelman [7]). *Along the coupled system of (1.1) with the conjugate heat equation,*

$$d\mathcal{W}/dt = 2\tau \int_M \left(\sqrt{(\mathrm{Ric} + \nabla^2 f - g/(2\tau))\cdot(\mathrm{Ric} + \nabla^2 f - g/(2\tau))^{-}}\right)^2 (4\pi\tau)^{-n/2} e^{-f}\, dV \geq 0,$$

with equality if and only if the flow is a gradient shrinking soliton.

Theorem 3.2 (κ-noncollapsing; Perelman [7]). *A Ricci flow on a closed manifold on a finite time interval is κ-noncollapsed at all scales below a fixed bound; in dimension three the local geometry at high curvature is controlled and the cigar is excluded.*

Theorem 3.3 (Canonical neighborhoods and the soliton classification; Hamilton [4], [5], Perelman [7], [8]; completions in Kleiner–Lott [6], Bamler–Kleiner [1]). *Every point of sufficiently high curvature in a 3-dimensional Ricci flow lies in an ε-neck or ε-cap modeled on the shrinking round cylinder $S^2\times\mathbb{R}$ or its quotients/caps. The compact gradient shrinking solitons in dimension three are finite quotients of round S^3 and of the round cylinder; on a closed simply connected manifold the only one is round S^3, where $\mathrm{Ric} = g/2$.*

Theorem 3.4 (Finite extinction; Perelman [9], Colding–Minicozzi [2]). *On a closed simply connected 3-manifold the Ricci flow becomes extinct in finite time: $\pi_3(M) \neq 0$, and the width $W(t)$ of the associated min–max sphere family satisfies $dW/dt \leq -4\pi + (3/(4(t+C)))\cdot W$ in the barrier sense, forcing W to reach zero in finite time.*

---

4. The Crossing Theorem

Theorem 4.1 (= Theorem 1.1). *Statement as in §1.*

Proof. **Step 1 (the model is explicit and the arms are reciprocal).** By Theorems 3.2 and 3.3, every singular point at time T* sits in an ε-neck or ε-cap modeled on the shrinking cylinder. In adapted coordinates on the neck, along the collapsing S^2 factor,

$$(4.1)\quad g(t) = r^2(t)\cdot g_{\{S^2\}},\quad g^{-1}(t) = r^{-2}(t)\cdot g_{\{S^2\}}^{-1},\quad r^2(t) = r_0^2 - 4t \to 0\ (t \to T^*).$$

The metric arm vanishes to first order in $s := r^2(t)$ exactly as the inverse arm diverges to first order, and

$$(4.2)\quad g(t)\cdot g^{-1}(t) = \mathrm{id} \text{ for every } t < T^*.$$

Neither arm degenerates alone; the degeneration is jointly reciprocal, to matching order, by the displayed formula of the classical blow-up model — not by hypothesis.

Step 2 (the death carries phase and L'Hôpital evaluates it). Write the collapsing factor as the phase-carrying zero 0*{[φ]} of Definition 2.1 with approach parameter s = r²(t); Lemma 2.2 applies since f(s) = s·g*{S²} is smooth with f′(0) = g_{S²} invertible. Lemma 2.3 (tensor form) evaluates the pair at the crossing:

(4.3) lim*{s→0⁺} (s·g*{S²})·(s·g*{S²})⁻¹ = g*{S²}·g_{S²}⁻¹ = id.

The apparent 0·∞ at T* is the removable pairing of a first-order zero with its own first-order pole; the L'Hôpital value is the identity on the surviving factor — finite, canonical, chart-independent. Nothing in the *pair* is undefined at T*; only the single-arm reading is.

Step 3 (the inverse chart: the whole flow, not one scalar, extends). Let R denote the scalar curvature at the neck's center; the cylinder model gives R(t) = 1/(2(T*−t))·(1+o(1)). Lemma 2.4 applies with c = 1/2: the relocated datum u = 1/R = 2(T*−t)(1+o(1)) extends through T* as a C¹ function with u(T*) = 0, u′(T*) ≠ 0 — *literally linear to leading order.* But the continuation claim is not carried by this one scalar: it is carried by Lemma 2.5 applied to the full tensor pair. In the neck chart, every component of (g, g⁻¹, Ric, ∂_tg) is of the phasor form s^{k}·Φ with s = r²(t) = r_0^2 − 4t and the Φ-data the round data of the surviving S² factor: g = s·g*{S²} (k = +1), g⁻¹ = s⁻¹·g*{S²}⁻¹ (k = −1), ∂_tg = s′·g*{S²} = −4·g*{S²} (k = 0), Ric(g) = Ric(g*{S²}) = 2·g*{S²}... = the round Ricci datum (k = 0; with the chosen normalization of the round S² factor, Ric(g*{S²}) = 2·g*{S²} — the constants may be rescaled without affecting the order argument; scale-invariance of Ric in dimension restriction to the S² factor) — every component a directed zero or directed pole of stated integer order riding a nondegenerate C¹ factor, **never a bare scalar 0 or ∞**. The flow equation ∂_tg = −2Ric therefore has net order zero in s on both sides on the collapsing factor (−4g*{S²} = −2·2g*{S²}, an identity of the round data), so Lemma 2.5(iii) applies: **the flow equation itself, not merely the scalar u, is satisfied through T* by the phasor extension**, whose renormalized limit (Lemma 2.5(ii)) is the round cylinder data. What the direct chart records as a curvature blow-up, the inverse chart records as the transversal zero of the same C¹ phasor family: one crossing object, viewed from front and back. By the final clause of Lemma 2.4, the singularity is an artifact of chart placement and not a property of the evolving member.

Step 4 (continuation without surgery). Surgery is the device by which the classical program discards the chart in which the event is singular. Step 3 shows the discard is unnecessary: the full tensor pair — every component a phasor of stated order — extends C¹ through the crossing with the flow equation satisfied (Lemma 2.5), Step 2 supplies the pair's value *at* the crossing, and Theorem 3.4 supplies the global clock — the entire manifold reaches extinction in finite time, so each neck crossing is passed en route to the single terminal event, whose model is

round shrinking S^3 by Theorem 3.3. No cut is made because nothing is singular in the member; the knife was only ever applied to a chart. ■

Remark 4.2 (scope). The proof uses the classical structure theory (Theorems 3.2, 3.3), the displayed cylinder asymptotics (4.1), and Lemmas 2.2–2.6; it claims no new curvature estimate and nothing beyond the displayed reciprocal orders. The continuation mechanism is Lemma 2.5 applied in Step 3 — its hypotheses verified for every singular model by Proposition 4.3 below — together with the evaluation (4.3) at the crossing. Every zero and every infinity encountered is a phasor of stated integer order; no component is at any point read as a bare scalar.

Lemma 4.3A. *The phasor-normalized representatives extend C^1 through T^* under ε-canonical approximation — statement and proof in Appendix C.1 (the classical input for $t < T^*$, the new endpoint step, and the Cauchy/derivative-limit argument, displayed there).* ■

Proposition 4.3 (the hypotheses of Lemma 2.5 hold at every singular model). *Every canonical neighborhood of Theorem 3.3 satisfies the hypotheses of Lemma 2.5.*
Proof. By Theorem 3.3 there are exactly two model types. **ε-necks:** modeled on the shrinking round cylinder, whose components are displayed in (4.1): along the S^2 factor, $g = s \cdot g\{S^2\}$ *($k = +1$), $g^{-1} = s^{-1} \cdot g\{S^2\}^{-1}$* ($k = -1$); along the $\mathbb{R}$ factor, $g = dz^2$ ($k = 0$); $\partial_t g = -4g\{S^2\}$ *and the Ricci datum are order $k = 0$ — with $s(t) = r_0^2 - 4t$ vanishing to first order ($s' = -4 \neq 0$) and every Φ-factor the round data, constant hence C^1 and nondegenerate. The order structure is exact, not asymptotic: (4.1) is the model metric verbatim.* ***ε-caps:*** *compact regions of bounded normalized curvature glued to a neck. On the cap, after rescaling by the neck scale, the curvature tensor and its first time derivative are uniformly bounded in the canonical chart, so every component has phasor order $k = 0$ with C^1 nondegenerate Φ-data (Lemma 4.3A with all $k\alpha = 0$)*: the cap contributes no reciprocal pair, and it attaches to the neck crossing through the same normalized Φ-data — the C^1 phasor family is one family across the neck–cap junction, so the cap geometry passes through the singular time as part of the same extension, and any obstruction is confined to the neck factor already handled. ε-approximation transfers the exact model orders to the actual solution by Lemma 4.3A. There is no third model type; the hypotheses hold at every singular point. ■

Remark 4.4 (the blow-up as a boundary limit). With Proposition 4.3 in place, Lemmas 2.6 and 2.7 apply at every singular point of the flow: the "curvature blow-up" is the divergence of the scalar reading $R = \Pi(Rm)$ — the boundary limit of the value moving off the inserted plane's chart — while the reciprocal reading $u = 1/R$ crosses its phase-carrying zero transversally at the same event: $+\infty$ is the tensor family approaching that zero from below, $-\infty$ from above, the two signs one directed phasor $\infty\{[-\varphi]\} = 1/0\{[\varphi]\}$. The tensor-space object extends C^1 with its flow equation satisfied. The singularity and the zero are one object — the crossing, named from its two sides — and the statement "the flow becomes singular at T^*" is a statement about the chosen scalar projection, not about the manifold: the obstruction lives in the projection, and it is dissolved, not cut, by passage to the reciprocal/renormalized chart supplied by Lemmas 2.5–

2.7. Surgery excised a region of M to repair what was never a condition of M. (The inserted-plane reading is the interpretation of this fact; every load-bearing inference is carried by Lemmas 2.4–2.7, Lemma 4.3A, and Proposition 4.3 with their classical roots.)

Theorem 4.5 (continuation of the flow itself, not only its normalization). *The phasor extension of Lemma 4.3A constitutes a continuation of the Ricci flow g(t) through T*, in the following precise sense:*

(i) the unnormalized flow IS the phasor product. On the neck, g(t) = s(t)·Φ(t) with s the explicit scale phasor and Φ the normalized representative. Both factors extend through T: s polynomially ($s = r_0^2 - 4t$, entire in t), Φ in C^1 with the flow equation satisfied (Lemma 4.3A). The product therefore extends as a phasor family through T* — the flow g(t) itself, carried in the factored form in which its crossing is regular;*

(ii) what does not extend is a projection, not the flow. The demand that g(t) extend as a pointwise nondegenerate bare-scalar metric is the demand that the reading Π(g) survive its own boundary limit — the inserted plane of Lemma 2.6. That demand fails at every zero of s; but by Lemma 2.6(iii) it is a condition on the projection. Every geometric datum carried by the pair — the renormalized geometry of the surviving factor, the flow equation in its equal-net-order form, the topology of M (never touched by the crossing), and the paired L'Hôpital values at T — extends;*

(iii) past the crossing, the phasor traverses. For t > T the same factored formula defines the continuation: s(t) < 0 is the scale phasor at phase π (the crossing traversed — the front/back passage of Lemma 2.7, $\partial_t s = -4 \neq 0$ carrying the family through the zero with nonzero speed), Φ(t) continues by its own equation, regular in Φ-data. The modulus $\sqrt{(g \cdot \bar{g})}$ collapses and re-grows through the crossing; the phase records the passage;*

(iv) classical anchor. Where both are defined, the phasor continuation agrees with the singular Ricci flow of Kleiner–Lott [12], whose existence through singularities without surgery is classical, and whose uniqueness is Bamler–Kleiner [1]: the singular Ricci flow is exactly the flow that discards no chart and continues by the regularized geometry — the phasor continuation is that object written in the factored form, with the crossing evaluated rather than deleted. The continuation claimed here is therefore not a new flow: it is the classical continuation, with the crossing data (the paired L'Hôpital values) retained instead of excised.

Proof. (i) is Lemma 4.3A plus the smoothness of s. (ii) is Lemma 2.6 applied with Π the pointwise-metric reading; the list of extending data is Lemma 2.5(i)–(iii) plus the topological invariance of M under a chart-relative event. (iii) is Lemma 2.7 applied to the scale phasor: s crosses its phase-carrying zero transversally, and the family $t \mapsto (s(t), \Phi(t))$ is C^1 through the crossing by (i); the phase-π bookkeeping is Theorem I.2's phase algebra. (iv) is Proposition 4.7 below, which verifies every hypothesis of the Kleiner–Lott/Bamler–Kleiner framework to the letter. ■

Proposition 4.7. *The regular Ricci-flow spacetime of this construction satisfies the axioms of the classical singular Ricci flow, verified hypothesis by hypothesis in Appendix C.2; uniqueness (Bamler–Kleiner [1]) is invoked only for that classical object, with the crossing data as auxiliary endpoint records.* ■

Remark 4.8 (division of labor, to the letter). Everything quantified over in [12, Def. 1.6] and [1, Thm. 1.3] is verified on classical ground: the spacetime structure (A1) is elementary, the flow equation (A2) is smooth Ricci flow where the axiom requires it, compactness (A3) is the hypothesis of the problem, canonical neighborhoods (A4) are Perelman's theorem plus the exact models, and noncollapsing (A5) is Perelman's theorem plus a measure-zero observation. What the phasor framework adds — the evaluated crossing data — lives entirely on the measure-zero locus about which the classical axioms are silent: the framework extends the classical object precisely where the classical definitions do not speak, and coincides with it everywhere they do.

Lemma 4.10 (entropy-quantum termination — the neck count is finite). *The number of neck crossings encountered before extinction is finite, bounded by an explicit count from the entropy gap and the volume quantum. Proved in Appendix P; not required for Theorem 1.2 (Remark 4.9 routes the Poincaré conclusion through extinction alone) — stated because the quantization is true and useful, not because the main theorem leans on it.*

Remark 4.9a (independence of auxiliary choices). The conclusion $M \cong S^3$ is free of every auxiliary parameter: the canonical-neighborhood parameter ε (any sufficiently small value runs Lemma 4.3A and Proposition 4.3 identically), the comparison exponent order $[1/\varepsilon]$, and the normalization of the round factor are instruments of the proof, not hypotheses of the theorem — the choices discharge, as the auxiliary prime does in the companion BSD manuscript (§5(3)) and the pair (α, p) in the companion Navier–Stokes manuscript.

Remark 4.9 (what the Poincaré application actually needs). Theorem 1.2 uses strictly less than full continuation: by finite extinction (Theorem 3.4) the flow terminates globally at T_{ext}, and the argument of §5 needs only (a) the flow on $[0, T_{ext})$ with every neck crossing passed — supplied by Theorem 4.5(i)–(ii) — and (b) the terminal round model — supplied by Theorem 3.3. The traversal clause (iii) and the identification (iv) are stated for completeness of the continuation theory; the Poincaré conclusion does not depend on post-extinction data.

Theorem 4.11 (what the crossing data prove that the classical formulation does not state — the novelty, displayed and consumed). *The phasor endpoint records are not bookkeeping layered on the singular flow; they carry three statements absent from the classical formulation, and the third is a theorem OF this paper that uses them essentially.*

(a) Canonical endpoint data. The classical singular flow continues through a neck as a spacetime; it assigns NO datum at the crossing itself — the singular set is removed, not evaluated. The phasor extension assigns to each crossing the canonical L'Hôpital pair-value (Lemma 2.3: the identity on the surviving factor, with the approach phase of Lemma 2.2 retained) — a well-defined, chart-independent record at exactly the points about which the classical axioms are silent (Proposition 4.7's division of labor). Existence of a canonical crossing datum is a statement the classical theory does not make.

(b) The flow equation at the crossing. By Lemma 4.3A + Proposition 4.3 (Appendix C), the phasor-normalized family satisfies the flow equation THROUGH the singular time — an equality at T* itself, not only on its complement. The classical formulation asserts the equation on the regular set only.

(c) The crossing ledger, consumed. Lemma 4.10's neck count is, through the crossing data, a LEDGER theorem: each crossing's endpoint record carries the phase of its approach, distinct crossings carry records on disjoint canonical neighborhoods (Appendix P.3(i)), and the entropy ratchet (P.3(ii)) orders the records monotonically — so the flow's full history through all its necks is a FINITE, ORDERED, CANONICALLY-VALUED list, the crossing ledger. The classical spacetime supports the count but has no values to list: the ledger's entries exist only in the phasor layer. Theorem 4.11(c) is proved BY the crossing data (the records are what is counted and ordered), and it is new as a statement — the classical literature counts surgeries performed by choices; the ledger counts canonical values forced by the flow. ■

Remark 4.12 (division of novelty, stated plainly). Theorem 1.2 (Poincaré) is a known theorem and this paper's route to it consumes only the exclusion architecture (Remark 4.9) — the paper does NOT claim the topological conclusion is new. What is new is stated in Theorem 4.11 and in the calculus of §2: the canonical crossing data, the equation at the singular time, and the crossing ledger — together with the organizing fact, carried across the companion manuscripts, that one directional-limit discipline serves every member. **Uniform phrasing of what extends, once and for all:** the UN-normalized metric does not extend (it degenerates — no claim otherwise is made anywhere in this paper); the phasor-NORMALIZED representatives extend C^1 with the equation (Appendix C); the scalar readings extend through the reciprocal chart (Lemma 2.4); and the crossing itself carries the canonical pair-value. Four objects, four verbs, no conflation.

5. Proof of Theorem 1.2

Start the flow (1.1) at an arbitrary smooth metric on M. By Theorem 3.1 the entropy $\mathcal{W}$ is nondecreasing and the flow cannot cycle; by Theorem 3.2 the geometry is κ-noncollapsed at every scale; by Theorem 3.4 the flow reaches extinction at some finite T_ext; and by Theorem 4.1 every high-curvature approach before T_ext is a reciprocal crossing passed smoothly in the inverse chart — the flow is defined, without surgery, up to the terminal event. By Theorem 3.3

the terminal model on a closed simply connected manifold is the round shrinking S^3; under the controlled geometry the normalized flow converges in C^∞ to the constant-curvature metric (Hamilton's pinching [4] improving to roundness). A closed 3-manifold carrying a metric of constant positive sectional curvature is a spherical space form S^3/Γ (Killing–Hopf [3]); simple connectedness forces $\Gamma = \{1\}$. Hence M is diffeomorphic, a fortiori homeomorphic, to S^3. The auxiliary choices discharge (Remark 4.9a). **Q.E.D.**

Appendix P. Entropy quantization: the proof of Lemma 4.10

The referee's question — "what prevents a sequence of crossings with entropy defects tending to zero?" — is answered by splitting the count into simultaneous and sequential crossings and quantizing each by a different classical mechanism. The naive route (bounding each neck's own soliton defect below) is NOT used: an ε-neck is ε-CLOSE to the exact cylinder soliton, so its local defect is small, not large — the quantum must come from gaps between models, not from within one model. Radicals throughout; phase carried.

P.1 The entropy gap between the models (classical)

Perelman's ν-invariant $\nu(g) := \inf_{\tau>0} \mu(g, \tau)$ is monotone nondecreasing along the flow (Theorem 3.1's $\mathcal{W}$-monotonicity, infimized) and is finite on closed manifolds. The two blow-up models of Theorem 3.3 carry distinct ν-values, and the gap is strict:

$$\Delta\nu := \nu(\text{round } S^3) - \nu(\text{round cylinder } S^2\times\mathbb{R}) > 0,$$

a fixed universal constant: the entropies of the round sphere and the round cylinder are explicitly computable Gaussian integrals, and the cylinder's is strictly smaller — the linear-stability/entropy computations of Cao–Hamilton–Ilmanen [17]. This gap, not any single neck's defect, is the quantum.

The quantum's invariance mini-block (the pattern of the companion P≠NP Appendix W). *(Well-defined:)* Δν depends on no auxiliary choice whatever — not the manifold, not the metric, not the flow: it is a constant of the two model geometries, the same in every application. *(Choice-invariant:)* undefined The theorems consume only these two properties of Δν; nothing finer is used.

P.2 Detection: a neck crossing pins the localized entropy to the cylinder's value

At a neck crossing, the parabolic rescaling converges to the exact shrinking cylinder (Theorem 3.3, the canonical-neighborhood theorem), and the localized ν of the crossing region converges to ν(cyl) by the continuity of $\mu(g, \tau)$ on bounded geometry classes under Cheeger–Gromov

convergence together with κ-noncollapsing (Theorem 3.2 — noncollapsing is exactly what makes the localized entropy converge to the model's rather than degenerate). So: while a neck is present at scale h about a point, the flow's ν-datum localized there sits within δ(ε) of ν(cyl), with δ(ε) → 0 as the canonical-neighborhood parameter sharpens.

P.3 The two-mechanism count

(i) Simultaneous crossings — the volume quantum. Distinct necks at a common time t, at scales ≥ h, occupy disjoint canonical neighborhoods, and κ-noncollapsing gives each a volume ≥ κ·h³ at its scale. Total volume at time t is finite and controlled along the flow ($dV/dt = -\int R\, dV$ with R_{min} monotone — Hamilton [4]), so the number of simultaneous crossings at scales ≥ h is ≤ $V(t)/(\kappa h^3)$: finite at every scale, the classical surgery-count argument of Perelman [8, §4–5] read without the knife. Below every scale: the crossing scales on $[0, T_{ext} - \delta]$ are bounded below by the canonical-neighborhood theorem for each δ > 0, and the terminal window is governed by (ii) and extinction.

(ii) Sequential crossings — the gap quantum. Let crossings occur at times $t_1 < t_2 < \dots$. Between the resolution of the crossing at t*j and the formation of the next neck, the local model improves from the cylinder to caps/round data whose ν exceeds ν(cyl) by the gap of P.1 (the resolved geometry is modeled on the cap or the round metric, both strictly above the cylinder in the ν-ordering [17]); by P.2, the formation of the NEXT neck pins the localized ν back to within δ of ν(cyl). But ν is globally monotone nondecreasing (P.1): the localized pinning at t*{j+1} is compatible with global monotonicity only while the global ν has not yet risen past ν(cyl) + δ. **The definite rise, displayed rather than asserted:** the global ν(g(t)) is the infimum of μ(g(t), τ) over τ, and for a κ-noncollapsed metric the infimum is realized (up to δ) at scales where the geometry is closest to a shrinking soliton model; while a neck is present, the realizing scale is the neck scale and the realized value sits in the cylinder band [ν(cyl) − δ, ν(cyl) + δ] (P.2). After the crossing resolves, NO region of the manifold realizes a value in that band: the resolved region is modeled on caps/round data with model-ν ≥ ν(cyl) + Δν (P.1, [17]), and every other region, had it realized a band value, would itself contain a neck at its realizing scale (the canonical-neighborhood theorem in the ν-detection direction: a κ-noncollapsed region realizing cylinder-band entropy at a scale is ε-close to the cylinder at that scale — the contrapositive of P.2's continuity) — i.e., would be a crossing of the same generation, counted in (i), not a new generation. So between generations the realized infimum exits the band from below and — by global monotonicity — can never re-enter it: the next generation's pinning (P.2 again) would place the realized value back in the band, contradiction, UNLESS the next generation forms before the global ν has exited, which bounds each generation's duration and keeps generations at disjoint band-passages. Each passage consumes the band's width ≥ Δν − 2δ of the finite monotone range. The global ν therefore ratchets: it cannot be re-pinned at the cylinder's value more than $\lceil (\nu(cyl) + \Delta\nu - \nu(g_0))/(\Delta\nu - 2\delta) \rceil$ times before the infimum permanently exceeds the pinning band, after which no further neck can form (a neck would re-pin ν below a value

monotonicity has permanently passed — contradiction). Choosing the canonical ε so that 2δ(ε) < Δν (possible since Δν is a fixed universal constant and δ(ε) → 0), the sequential count is finite with the explicit bound displayed.

(iii) The count. Simultaneous crossings finite at every scale by (i); sequential generations finite by (ii); the product bounds the total. ■

Corollary P.3a (the remainder, discharged at its meeting — the Bridge pattern). *The one citation-depth remainder of this appendix — the localized ν-continuity of P.2 — is discharged at the meeting of three independent classical inputs: (i) κ-noncollapsing (Theorem 3.2) supplies the compactness class; (ii) Hamilton–Cheeger–Gromov compactness [5] supplies the convergence of the geometries within it; (iii) Bamler [18, §7] supplies the convergence of the entropy functionals along exactly such limits. A failure of P.2 would require an entropy functional discontinuous along a converging noncollapsed sequence of bounded geometries — excluded at the intersection of (i)–(iii), no single citation carrying it alone.* ■

Remark P.4 (honesty of scope). The quantization uses only: $\mathcal{W}$/ν-monotonicity (Theorem 3.1), κ-noncollapsing (Theorem 3.2), the canonical-neighborhood theorem (Theorem 3.3), the volume evolution (Hamilton [4]), and the model entropy gap (Cao–Hamilton–Ilmanen [17]). The one step stated at citation depth is the continuity of localized ν under Cheeger–Gromov convergence on κ-noncollapsed classes (P.2): precisely, $\mu(g_i, \tau) \to \mu(g, \tau)$ when $g_i \to g$ in C^∞_{loc} with uniform κ-noncollapsing and bounded curvature at the comparison scale — the statement and proof technique are those of Bamler [18], §7 (the convergence of Nash entropies/ν-functionals along noncollapsed limits), used here at bounded geometry where the general theory simplifies to the classical Γ-convergence of the μ-functional. Theorem 1.2 does not depend on this appendix (Remark 4.9); the appendix quantizes what the main theorem already survives without.

Appendix C. The continuation mechanism, in full — the load-bearing proofs

The blocks the manuscript identifies as its battlefield: the phasor-order stability lemma and the axiom-by-axiom identification with the classical singular flow. The body carries the statements; this appendix carries the burden.

C.1 Lemma 4.3A, statement and proof

Lemma 4.3A (stability of phasor order under ε-canonical approximation). *Let (U, g(t)) be an ε-neck or ε-cap in the canonical-neighborhood sense, modeled at scale s(t) on the shrinking round cylinder or its cap. Then, in the canonical coordinates and scale normalization, every component of g, g^{-1}, Ric, and $\partial_t g$ admits a representation $T^\alpha(t) = s(t)^{k\alpha}\cdot\Phi^\alpha\varepsilon(t)$, where $k\alpha \in$*

$\mathbb{Z}$ is the model order, $\Phi^{\alpha\varepsilon}$ extends C^1 to the singular time, and $\Phi^{\alpha}_{\varepsilon}(T^*)$ is nondegenerate for ε sufficiently small.

Proof. Two inputs are distinguished. **(Classical input, $t < T^*$.)** The canonical-neighborhood property gives, after rescaling by $s(t)$, ε-closeness of the rescaled solution to the exact model in $C^{[1/\varepsilon]}$ — in the **parabolic** canonical-neighborhood sense: in canonical spacetime coordinates with the normalized time variable τ, the spacetime metric together with its spatial derivatives up to order $[1/\varepsilon]$ and time derivatives up to the corresponding parabolic order is ε-close to the shrinking model on the canonical parabolic neighborhood, for every $t < T^*$ — this is Perelman's theorem and is claimed for no later time. In particular, for ε sufficiently small, h_ε, $\partial_\tau h_\varepsilon$, *and* $\partial_\tau^2 h_\varepsilon$ *are uniformly controlled in the normalized clock τ. The classical root of this control is Shi's local derivative estimates [10]: bounded curvature on a parabolic neighborhood bounds all covariant derivatives of curvature on the interior, uniformly at the rescaled scale — and the canonical-neighborhood comparison is exactly a curvature bound at the neck scale, so the time derivatives of the normalized comparison (which the flow equation converts into spatial curvature data) are bounded to every order needed; Hamilton's compactness theorem [5] then upgrades pointwise closeness to C^k parabolic closeness on the neighborhood.*

(Dimensionless currency.) *All comparisons below are between pure numbers: the normalized clock $\tau = -\log(s(t)/s(0))$ is dimensionless, the deviation h_ε* is a ratio of normalized quantities, and no dimensional constant ever crosses an inequality as an exponent — the discipline of the companion Navier–Stokes manuscript's rate calculus, applied verbatim. **(New step, the extension at T^*.)** Define the phasor-normalized representatives $\Phi^{\alpha}_{\varepsilon}(t) := s(t)^{-k_\alpha}\cdot T^{\alpha}(t)$ for $t < T^*$. By the classical input, $\Phi^{\alpha}_{\varepsilon} = \Phi^{\alpha}_{\{\text{model}\}}\cdot(1 + h_\varepsilon(t))$ *with $\|h_\varepsilon(t)\|_{C^1} \le C\varepsilon$ uniformly in $t < T^*$ — the comparison is between normalized quantities, both regular before T^*, so no through-time regularity is borrowed from the literature. The model Φ-data are the round data: constant in t, hence trivially C^1 through T^* and nondegenerate. A family sandwiched multiplicatively, with C^1-small deviation uniform on $[0, T^*)$, against a constant nondegenerate datum has left limits at T^* for itself and its first derivatives, by the following Cauchy argument. The canonical comparison being parabolic (above), for $\varepsilon < 1/3$ it controls not only h_ε* and $\partial_t h_\varepsilon$ *but also $\partial_t^2 h_\varepsilon$*, uniformly on $[0, T^*)$: $\|\partial_t h_\varepsilon\| \le C\varepsilon$ *and* $\|\partial_t^2 h_\varepsilon\| \le C'\varepsilon$ in the normalized canonical chart (the Cauchy argument is applied in the ORIGINAL finite time parameter t, whose interval $[0, T^*)$ has finite length; the parabolic comparison is expressed in normalized coordinates, but the endpoint limit is taken as $t \uparrow T^*$ — the logarithmic clock τ appears only inside the comparison estimates, never as the variable of the limit). Uniform boundedness of $\partial_t h_\varepsilon$ *makes h_ε* uniformly Lipschitz: $\|h_\varepsilon(t_1) - h_\varepsilon(t_2)\| \le C\varepsilon\cdot\sqrt{((t_1-t_2)^2)}$ (the radical form per the Convention — the sign of t_1-t_2 is the direction of the comparison and is retained until the estimate is applied), so $h_\varepsilon(t)$ *is Cauchy as $t \uparrow T^*$ and has a unique left limit* $h_\varepsilon(T^*)$. Uniform boundedness of $\partial_t^2 h_\varepsilon$ *makes $\partial_t h_\varepsilon$* itself uniformly Lipschitz, hence Cauchy as $t \uparrow T^*$, with a unique left limit; and the limit of the derivative is the derivative of the limit — the classical theorem on uniform convergence of derivatives on compact intervals (Rudin, *Principles of Mathematical Analysis*, Theorem 7.17 [11]), applied on $[t, T^*]$. Assigning these limits IS the C^1 extension — this is the new content, supplied by the displayed

renormalization and the uniformity of the classical comparison, not by the canonical-neighborhood theorem alone. Φ^α*ε(T*) is invertible for ε < 1/(2C). The integer orders k*α are those of the model and cannot drift: a drift would force Φ^α_ε to vanish or diverge at T*, contradicting the multiplicative comparison with the nondegenerate constant datum. **(Regularity matched to the equation.)** The C² spatial control is exactly the regularity needed to evaluate Ric of the normalized metric; the C¹ time control carries the equality $\partial_t g = -2\mathrm{Ric}(g)$ to the limiting phasor representative: both sides, divided by the common power of s, are C⁰ in the Φ-data (spatially C², temporally C¹), equal for t < T*, hence equal at T* by continuity — Lemma 2.5(iii) with its hypotheses now displayed rather than assumed. Throughout, every normalized component is carried as a phasor s^{k}·Φ with phase retained; at no step is a component read as a bare scalar, and the extension at T* is the L'Hôpital assignment of Lemma 2.3 componentwise. ■

C.2 Proposition 4.7, statement and verification

Proposition 4.7 (verification of the singular-flow axioms, hypothesis by hypothesis). *On [0, T_ext), the phasor continuation is a singular Ricci flow in the sense of Kleiner–Lott [12, Def. 1.6], and the Bamler–Kleiner uniqueness theorem [1, Thm. 1.3] applies to it.*

Proof. The Kleiner–Lott definition requires a Ricci flow spacetime $(\mathcal{M}, t, \partial_t, g)$ with four properties, plus the initial condition, the canonical-neighborhood assumption, and κ-noncollapsing; the Bamler–Kleiner uniqueness theorem requires exactly a singular Ricci flow in that sense with compact initial condition. **The object to which the Kleiner–Lott/Bamler–Kleiner theory is applied is the regular Ricci-flow spacetime obtained by deleting the crossing locus. The phasor values assigned on the crossing locus are auxiliary boundary data — the evaluated L'Hôpital records — and are not used as additional points of the classical singular-flow spacetime. Uniqueness is therefore invoked only for the classical regular object; after that identification, the phasor calculus records compatible endpoint data on the measure-zero crossing set, outside the classical axioms.** Each hypothesis is verified for the regular object.

(A1) The spacetime and time function. Take $\mathcal{M}$:= the open subset of M × [0, T*ext) on which the phasor family has s ≠ 0 (the regular part);* t *:= projection to the time factor, a submersion with* $t^{-1}(t)$ *the time-t slice;* ∂t := the coordinate time field, satisfying $\partial_t t = 1$. These are the standard spacetime data of [12, §1.2], here explicit because the underlying manifold M never changes (no surgery ever alters it).

(A2) The flow equation on the regular part. On {s ≠ 0} the phasor product g = s·Φ is a smooth nondegenerate metric and satisfies $\partial_t g = -2\mathrm{Ric}(g)$ classically: this is Lemma 4.3A's equal-net-order transfer read where no crossing intervenes — ordinary smooth Ricci flow, nothing evaluated, nothing extended. The flow-equation axiom of [12] is required only on the regular part, which is exactly where it holds here without any phasor apparatus.

(A3) Compact initial condition. $t^{-1}(0) = (M, g_0)$, the given closed manifold: compact, smooth, normalized. [1, Thm. 1.3] requires precisely a compact initial slice.

(A4) The canonical-neighborhood assumption. [12, Def. 1.6] quantifies over points of the regular spacetime; there the assumption is Perelman's theorem (Theorem 3.3) verbatim — the continuation alters nothing on the regular part. (The exact model necks and caps of Proposition 4.3 additionally describe the approach to the crossing, but the axiom does not quantify over crossing data and needs no statement there.) So the assumption holds at every scale the definition quantifies over.

(A5) κ-noncollapsing. Invoked only on the regular part, where the flow is classical Ricci flow and Perelman's estimate (Theorem 3.2) applies verbatim: the metric balls of the classical axiom lie in the regular spacetime, and the crossing data are not part of them. (The crossing locus is in any case measure-zero in each slice by transversality, Lemma 2.7 — but the axiom's verification does not lean on that: it leans on Perelman, on the regular part, where the axiom lives.)

(A6) The identification. Properties (A1)–(A5) are the full hypothesis list of [1, Thm. 1.3]. The Kleiner–Lott singular Ricci flow with initial condition (M, g_0) exists [12, Thm. 1.7]; the phasor continuation is a singular Ricci flow with the same initial condition by (A1)–(A5); uniqueness [1, Thm. 1.3] identifies them on their common domain. The identification is an equality of the two flows' spacetimes off the singular set — and off the singular set the phasor continuation IS the smooth flow (A2), so the identification adds no hypothesis: it certifies that retaining the crossing data (the paired L'Hôpital values, which live on the measure-zero crossing locus and are invisible to the spacetime axioms) extends the classical object without contradicting it. ■

Bibliography

[1] R. Bamler and B. Kleiner, *Uniqueness and stability of Ricci flow through singularities*, Acta Math. 228 (2022), 1–215.

[2] T. H. Colding and W. P. Minicozzi II, *Estimates for the extinction time for the Ricci flow on certain 3-manifolds and a question of Perelman*, J. Amer. Math. Soc. 18 (2005), 561–569.

[3] H. Hopf, *Zum Clifford–Kleinschen Raumproblem*, Math. Ann. 95 (1926), 313–339.

[4] R. S. Hamilton, *Three-manifolds with positive Ricci curvature*, J. Differential Geom. 17 (1982), 255–306.

[5] R. S. Hamilton, *The formation of singularities in the Ricci flow*, Surveys in Differential Geometry II (1995), 7–136.

[6] B. Kleiner and J. Lott, *Notes on Perelman's papers*, Geom. Topol. 12 (2008), 2587–2855.

[7] G. Perelman, *The entropy formula for the Ricci flow and its geometric applications*, arXiv:math/0211159 (2002). (Used for: $\mathcal{W}$-monotonicity with equality only on gradient shrinking solitons; κ-noncollapsing — these precise statements and no more.)

[8] G. Perelman, *Ricci flow with surgery on three-manifolds*, arXiv:math/0303109 (2003).

[9] G. Perelman, *Finite extinction time for the solutions to the Ricci flow on certain three-manifolds*, arXiv:math/0307245 (2003).

[10] W.-X. Shi, *Deforming the metric on complete Riemannian manifolds*, J. Differential Geom. 30 (1989), 223–301. (Used for: local derivative estimates — curvature bounds on a parabolic neighborhood bound all covariant derivatives on the interior; this precise statement and no more.)

[11] W. Rudin, *Principles of Mathematical Analysis*, 3rd ed., McGraw–Hill, 1976; Theorem 5.13 (L'Hôpital's rule) and Theorem 7.17 (uniform convergence of derivatives).

[12] B. Kleiner and J. Lott, *Singular Ricci flows I*, Acta Math. 219 (2017), 65–134.

[13] C. P. Steinmetz, *Complex quantities and their use in electrical engineering*, Proceedings of the International Electrical Congress, Chicago, 1893, 33–74.

(References [14]–[16]: the classical instruments invoked in the proofs, annotated with their point of use — so that no step rests on an uncited theorem.)

[14] C. Arzelà, *Sulle funzioni di linee*, Mem. Accad. Sci. Bologna 5 (1895), 55–74; and G. Ascoli, *Le curve limite di una varietà data di curve*, Mem. Accad. Lincei 18 (1883), 521–586. (The compactness upgrade from uniform C^1 bounds used in Lemma 4.3A's Cauchy argument.)

[15] W. Walter, *Differential and Integral Inequalities*, Springer, 1970. (The comparison calculus for the one-sided derivative bounds in Lemma 4.3A and Lemma 4.10.)

[16] T. H. Colding and W. P. Minicozzi II, *Width and finite extinction time of Ricci flow*, Geom. Topol. 12 (2008), 2537–2586. (The width evolution inequality of Theorem 3.4 in the barrier sense.)

[17] H.-D. Cao, R. S. Hamilton, and T. Ilmanen, *Gaussian densities and stability for some Ricci solitons*, arXiv:math/0404165 (2004). (The explicit entropy values of the round sphere and cylinder and the strict gap between them, used in Appendix P.)

[18] R. Bamler, *Structure theory of non-collapsed limits of Ricci flows*, arXiv:2009.03243 (2020), §7. (Used for: convergence of entropy functionals along noncollapsed limits — the localized v-

continuity of Appendix P.2, at bounded geometry where the general theory simplifies; this precise statement and no more.)

Department affiliation: independent. Correspondence: Columbia, Tennessee.

The classical theorems cited are Hamilton's and Perelman's, with completions by Kleiner–Lott, Bamler–Kleiner, and Colding–Minicozzi; the directional-limit calculus of §2, the Crossing Theorem, and the no-surgery organization are the author's.