Universal Clock Response and Local Equivalence in Operational Quantum Gravity

An Explanation with Proof of Electromagnetic Response and Matter-wave Dispersion

Todd J. Desiato

Manuscript version 0.6 · 31 October 2026

Independent researcher, Statesville, North Carolina, USA

Correspondence: mainengineering@gmail.com · ORCID: 0000-0002-7514-0169

Abstract

Why does a clock change its rate when it is raised, moved, accelerated, or released into free fall? Operational Quantum Gravity (OQG) proposes a common material explanation through electromagnetic scattering, nuclear response, and dispersion. Surrounding matter changes the correlated field exchange and equilibrium storage of a receiver. Within the complete corresponding-state family, the response-scaling and supporting-power theorems connect that adjustment to stable binding, material dimensions, and clock frequencies. Combining these results with the retained local Lorentz energy–momentum relation gives a single law for the phase accumulated along a clock's path. This article develops the explanation effect by effect: stationary clock shifts and redshift, moving-clock slowdown, reciprocal laboratory comparisons, applied acceleration, supported and accelerating laboratories, free fall, tidal limits, and reunion. It shows how storage gradients supply force and reversible work, momentum dispersion supplies velocity and inertia, and transition phase supplies elapsed readings. The supported and accelerating laboratories agree in their leading local height comparison; conservation of energy fixes the released clock's combined environmental and motion factors. Essential equations accompany the physical explanations, with detailed proofs and numerical checks in the appendices. The result is a connected OQG account of these clock and equivalence effects, with the correspondence assumptions, source-strength calibration, and propagation input identified explicitly so that its material claims can be tested.

Keywords: operational quantum gravity; dispersion forces; matter-wave dispersion; electromagnetic scattering; nuclear response; supporting power; material scaling; clock comparison.

1 Start with the clock on the table

Put two matched clocks beside a book on a table. Leave one clock there. We will lift the other onto a shelf, send it on a journey, accelerate it, and finally release it with the book. We will also compare

clocks on the floor and ceiling of an accelerating laboratory. The question in each case is what happens to the clock's physical operation, and why.

Dispersion provides the connection. A material system responds differently to different frequencies of an applied field. Its charges and currents have resonances, delays, and correlations with the surrounding matter. Their shared electromagnetic interaction can therefore change the system's equilibrium energy. The spatial variation of that interaction energy produces a dispersion force. OQG identifies gravity with a particular collective material response of this kind, tied to a change in balanced supporting power [1, Sections 2.5 and 5; 2, Sections 2–3].

There is a second, related use of the word. Matter-wave dispersion is the relation between a body's energy and momentum. Once the electromagnetic response has changed the material energy and its clock and ruler scales, that changed state enters the matter-wave dispersion relation. The relation tells us how the body moves and how its phase advances. The first part of the explanation establishes the material state; the second follows its motion and counted cycles. Keeping that connection explicit is how we will explain each experiment [1, Section 6].

Elapsed time is what a clock measures between events. An atomic clock counts cycles associated with a transition between two internal energy levels. If the gap is ΔE, its frequency is $\nu = \Delta E/(2\pi\hbar)$. A clock calibrated to reference frequency ν_0 converts its cycle count N into a reading τ. In a stationary comparison, with t supplied by the declared reference clock,

$$\nu = \frac{\Delta E}{2\pi\hbar}, \qquad dN = \nu\, dt, \qquad d\tau = \frac{dN}{\nu_0} = \frac{\nu}{\nu_0}\, dt. \tag{1}$$

Equation (1) locates the physical question. What changes the transition gap and hence the number of cycles accumulated during the comparison? Relativity relates clock readings to position and motion. OQG supplies a material account of the state and propagation of the clock that realize those relations. We will follow that account before using the Hamiltonian or geometric description.

The clock, book, table, and Earth contain responding charges and currents. Their quantum fluctuations receive energy from and return energy to a shared electromagnetic field. Other matter scatters that field back to the receiver, including propagation delays and correlations. Electrically neutral matter participates through its internal charge and current response. A nucleus's underlying spectrum retains its strong-interaction binding; the electrical response does not replace that binding with an ordinary electrostatic force [1, Sections 2–3; 2, Section 2; 4, Sections 2–3].

The two fields at a receiver must be added as amplitudes before their intensity is calculated. If E_a is a reference field component and E_s the scattered component, averaging gives

$$\langle |E_a + E_s|^2 \rangle = \langle |E_a|^2 \rangle + \langle |E_s|^2 \rangle + 2\mathrm{Re}\langle E_a E_s^* \rangle. \tag{2}$$

The last term is the interference. It can be negative and exceed the positive scattered-intensity term in magnitude. Thus another responding body can reduce reception even though it contributes a field of its own. The exact resonant two-dipole example gives a reception ratio of 0.999245360517, a reduction of 0.07546395 percent. Appendix A shows the sign calculation with repeated scattering retained. That percentage belongs to the specified dipole example; it is not the Earth's gravitational deficit [1, Section 2.3].

The receiver then adjusts. Its frequency-dependent response determines its currents; those currents help determine the field; the field couples back to its material storage. OQG describes the resulting equilibrium through the coupled field and material response. At equilibrium the incoming and outgoing powers are equal. Their common magnitude is called supporting power. It is an exchange in both directions, with no net heating. An electrical supply powering a clock's electronics measures a different quantity.

Balanced exchange does not imply zero interaction energy or zero force. Two coupled systems can exchange equal powers while their joint equilibrium energy depends on separation. A dispersion force is the derivative of that energy with respect to separation. Appendix A.4 makes the distinction explicit using the same scattering response in the interaction energy. Supporting power measures an exchange per unit time; it cannot be used as a mechanical force without the intervening storage relation.

For the same receiver and corresponding channels, let P_0 be the reference supporting power and P the adjusted value, expressed in the same comparison standards. Let f be the transition-frequency factor. The OQG supporting-power theorem gives [1, Section 4.3; 2, Section 2]

$$P_{\mathrm{in}} = P_{\mathrm{out}} = P, \qquad \Pi = \frac{P}{P_0} = f^2, \qquad \frac{\nu}{\nu_0} = f = \sqrt{\Pi}. \tag{3}$$

A lower exchange therefore corresponds to a slower material clock. For example, 98.01 percent of the reference supporting power gives 99 percent of the reference frequency. This is a ratio for one receiver, not a claim that different clocks require equal watts. The theorem carries the complete response spectrum, radiative resistance, current patterns, and correlations between corresponding states. Appendix D gives the calculation; the physical selection of the settled state comes from the scattering and storage conditions in Appendices A–B.

Throughout the examples, a clock is small enough that the relevant environment is effectively uniform across its internal structure. Its preparation follows the corresponding material state or the ideal-clock phase evolution. Temperature, strain, and applied-field shifts are separate instrument effects. These conditions let us identify the universal clock response without confusing it with damage or disturbance caused by the apparatus.

2 Why different clocks can agree and remain bound

A clock cannot be explained by changing its tick rate while leaving its matter unspecified. Its atoms must remain bound, its nuclear response must remain consistent, and the ruler used beside it must belong to the same material state. OQG supplies linked changes to these quantities.

The nuclear excitation gaps and charge-transition strengths determine the frequency-dependent charge response to a small applied voltage. In response notation, $q(\omega) = \chi_{qq}(\omega)V(\omega)$: the same voltage at different frequencies produces different charge motion. The response contains the nuclear resonances and their strengths. Other responding matter changes the shared-field coupling to that spectrum, which is why scattering can change the settled material state.

At frequencies where a storage expansion is valid, the inverse response defines an effective capacitance C_n and inductance L_n. Capacitance measures compliance against charge displacement; inductance measures inertia associated with charge motion. These are coefficients derived from the interacting nuclear spectrum. Its strong-interaction binding is retained. The full causal response, rather than a truncated storage expansion, is used near the resonances [1, Section 3.1; 4, Section 3.2].

Between corresponding states the two storage coefficients acquire the same inverse frequency factor. For a quadratic mode this gives the familiar resonator relation

$$\frac{C_n}{C_{n0}} = \frac{L_n}{L_{n0}} = \frac{1}{f}, \qquad \frac{\omega_n}{\omega_{n0}} = \sqrt{\frac{L_{n0}C_{n0}}{L_nC_n}} = f. \tag{4}$$

Here $\omega_n = 2\pi\nu_n$ is angular frequency. Increasing both capacitance and inductance shifts the resonance downward. For an engineer, Eq. (4) expresses the changed dispersive response as a storage calculation: the frequency changes because the charge and current motion have different restoring and inertial coefficients. The same source calculation contains a positive-energy model of a material oscillator coupled to electric and magnetic environmental modes. Its interaction loads the common scale toward contraction and restores electric–magnetic balance; the material restoring energy fixes the resulting equilibrium. Appendix B derives those signs and the equilibrium conditions [1, Sections 3.3–3.5].

Smaller size and slower oscillation are compatible because restoring stiffness and inertia change together. Let b be the length ratio, k a mode's restoring stiffness, and m_{eff} its matched inertia. The complete matter-wave scaling theorem gives

$$\frac{k}{k_0} = \frac{f}{b^2}, \qquad \frac{m_{\mathrm{eff}}}{m_{\mathrm{eff},0}} = \frac{1}{fb^2}, \qquad \frac{\omega}{\omega_0} = \sqrt{\frac{k/k_0}{m_{\mathrm{eff}}/m_{\mathrm{eff},0}}} = f. \tag{5}$$

On the common stationary branch, $b = f$. If the size and frequency factors are both 0.99, the restoring stiffness is about 1.01010 times its reference value, while the matched inertia is about 1.03061 times its reference value. The inertia increases more strongly, so the frequency falls even though the object is smaller and its restoring stiffness is greater. These inertial factors are expressed in the fixed comparison standards; they are not a mass increase discovered by weighing an object against co-located standards that change with it.

Hydrogen provides a second check that reaches beyond a nuclear oscillator. On this branch its reduced electron–proton mass scales as b^{-3} and the matched electric coefficient as $\epsilon/\epsilon_0 = b^{-2}$. The Bohr radius is proportional to ϵ/m_r, so it scales as b. Coulomb energy gaps are proportional to m_r/ϵ^2, so they also scale as b. The dimensionless electromagnetic coupling remains unchanged. Atomic dimensions and atomic clock gaps thus follow the same common factor as the nuclear response [1, Section 6.2].

For the full quantum system, the scaling acts on all retained energy levels and their corresponding breakup thresholds. Positive frequency scaling preserves their ordering: a bound state remains below its threshold. It is the complete dynamics that transform, including the strong interaction already present in the reference spectrum. Appendix C gives that proof and the associated response identity.

Consequently, two different clock transitions j and k need not have equal frequencies or equal supporting powers. Their complete corresponding systems share the fractional factor:

$$\frac{\nu_j}{\nu_{j0}} = \frac{\nu_k}{\nu_{k0}} = f, \qquad \frac{\nu_j/\nu_k}{\nu_{j0}/\nu_{k0}} = 1, \qquad d\tau_j = d\tau_k = f\,dt. \tag{6}$$

This explains both agreement and invisibility. After calibration, the clocks agree about the local change; a comparison made entirely between standards sharing that change cannot reveal it. To reveal the common factor, compare with a clock at another potential or follow a different physical history. The spectroscopic protocol and transition-ratio test are stated in Ref. [4, Section 5].

3 Lift one clock onto the shelf

Keep the lower clock at the table and lift the other slowly to a shelf. Changing the source–receiver separation changes the propagation and correlations entering their dispersive electromagnetic coupling. In OQG's terrestrial application, the Earth's constituents alter the shared field and establish a stationary supporting-power profile. Closer to the source, the deficit is greater. Higher up, the smaller deficit permits a larger supporting exchange, a larger transition gap, and a faster clock [1, Sections 2.5, 5, and 7; 5, Section 4].

Write $\Pi(x)$ for the stationary profile and $f_s(x) = \sqrt{\Pi(x)}$ for its clock factor. The subscript s distinguishes this stationary environmental factor from the motion factor introduced later. Both clocks count cycles during the same comparison interval. Equation (3) gives

$$\frac{d\tau_2}{d\tau_1} = \frac{\nu_2}{\nu_1} = \frac{f_s(x_2)}{f_s(x_1)} = \sqrt{\frac{\Pi(x_2)}{\Pi(x_1)}}. \tag{7}$$

Equation (7) says why the higher clock wins the comparison: its complete material transition gap is greater in the stationary state sustained there. It also ties the clock change to a definite power ratio. If the fractional frequency increase is small, the fractional supporting-power increase is twice as large.

The source calculation makes this quantitative. In the calibrated weak spherical application, with source mass M and distance r,

$$1 - \Pi(r) \simeq \frac{2G_0M}{rc_0^2}, \qquad \frac{\Delta\nu}{\nu} \simeq \frac{gh}{c_0^2}, \qquad \frac{\Pi_2}{\Pi_1} - 1 \simeq \frac{2gh}{c_0^2}. \tag{8}$$

Here c_0 is the locally measured reference light speed, g the weak local gravitational acceleration, and h a small upward height change. The Earth benchmark gives a surface supporting-power deficit of approximately 1.39 parts per billion relative to the receiver's reference with the Earth contribution removed. For one metre of height, the upper clock gains about 9.4 picoseconds per day. A picosecond is one trillionth of a second. The strength in this source application is calibrated to measured gravity; the fixed-dipole reception ratio in Section 1 is not substituted for the terrestrial equilibrium profile [1, Section 7.2; Appendix E.2].

The clock change has an energy counterpart. OQG identifies the conservative dispersion interaction with the change in complete material storage, including binding. If E_0 is the reference rest-storage energy, the stationary energy, interaction energy relative to that reference, lifting work, and force are

$$\begin{aligned} E_{\mathrm{rest}} &= E_0 f_s, \qquad U_{\mathrm{disp}} = E_0(f_s - 1), \\ W_{1\to 2} &= E_0\big(f_{s,2} - f_{s,1}\big), \qquad F_x^{\mathrm{disp}} = -\frac{\partial U_{\mathrm{disp}}}{\partial x} = -E_0 \frac{\partial f_s}{\partial x}. \end{aligned} \tag{9}$$

Lifting a one-kilogram body by one metre near Earth supplies approximately 9.8 joules. In this account the work increases recoverable material storage; it does not fill a separate container of geometric energy. Reversible lowering returns the storage difference. At rest on the shelf, the supporting force does no continuing mechanical work through vertical displacement, while balanced microscopic exchange continues at the higher value.

This connection can also be evaluated at the electrical storage level. Changing the effective nuclear capacitance and inductance changes the energy stored in the charge and current modes. Their derivatives supply the storage force; the variable-component power identity accounts for the work transferred into deformation. Appendix E gives both that calculation and Eq. (9). Thus the dispersion interaction changes a common material energy scale: internal energy differences determine the clock rate, while the spatial change of the complete energy determines the force and reversible work. These are different measurements of the same adjusted state.

Gravitational redshift is the signal version of this comparison. The lower clock emits light at the transition frequency of its adjusted material state. In a static environment the signal's frequency referred to the fixed comparison time is conserved along propagation. In those same standards, the upper receiver's corresponding transition is faster, so the arriving light is redshifted relative to that upper standard. The upper clock running faster and the upward signal being redshifted describe the same pair of material standards from different ends of the experiment [Appendix E].

4 Why a local light speed measurement stays unchanged

Changing clock rates raises an immediate question: why does an observer beside either clock still measure the same local light speed? OQG requires the ruler and field propagation to participate in the material comparison. Holding an external ruler fixed while changing only the clock would describe a different experiment.

Let b_s be the stationary length factor. Field-pattern matching to the storage coefficients in Eq. (4) gives $\epsilon/\epsilon_0 = \mu/\mu_0 = 1/(f_s b_s)$, and hence comparison-coordinate light speed $c_{\mathrm{cmp}} = f_s b_s c_0$. A corresponding

ruler of reference length L_0 has length $b_s L_0$, while its clock frequency is $f_s \nu_0$. The light transit and cycle count are therefore [1, Sections 4.2 and 6.2; 5, Section 3]

$$\Delta t = \frac{b_s L_0}{f_s b_s c_0} = \frac{L_0}{f_s c_0}, \qquad N = (f_s \nu_0)\Delta t = \frac{\nu_0 L_0}{c_0}. \tag{10}$$

The cycle count across the corresponding ruler is unchanged. Equivalently, local readings satisfy $dT = f_s dt$ and $d\ell = |dx|/b_s$, giving $d\ell/dT = c_0$ for light. This is the material cancellation behind the local result. It is not a failure to notice a slow clock: the clock, ruler, and propagation coefficients all enter the measurement.

On the common stationary branch, $f_s = b_s = \sqrt{\Pi}$, so $c_{\mathrm{cmp}}/c_0 = \Pi$. The completed collective ambient stress coefficient also has $A_{2,E}^{\mathrm{tot}}/A_{2,E,0}^{\mathrm{tot}} = \Pi$ on that branch. For independent clock and ruler factors those two ratios instead follow $f_s b_s$, whereas supporting power follows f_s^2. Appendix D proves the relations and retains their different meanings. The equality of the two factors is used only where the stated stationary branch applies.

These stationary formulas must also be distinguished from a moving receiver at a fixed altitude. Translating an apparatus does not rescale its ambient propagation medium isotropically. Its longitudinal comparison and the synchronization of its clocks are treated in Section 6.

5 Send one clock on a journey

Return to a region small enough that the stationary source profile is approximately uniform. Accelerate one clock relative to the laboratory, then let it coast. The source matter, surrounding boundaries, and the receiving clock define the physical comparison. The bare vacuum is Lorentz invariant; the matter-associated field being sampled can have a spectrum and angular distribution that depend on motion relative to its sources [1, Section 2.1; 2, Section 4].

The changed motion affects reception. For a propagating field component arriving at angle θ to local velocity V, its frequency in the receiver's clock is $\omega_A = \gamma\omega_B(1 - \beta\cos\theta)$, with $\beta = V/c_0$ and $\gamma = 1/\sqrt{1-\beta^2}$. Components ahead and behind shift differently. Their amplitudes, polarizations, and correlations also enter the response. Shifting one frequency and calling the result a power deficit would omit those parts of the calculation.

Now use matter-wave dispersion to connect that receiving state to work and clock phase. The rest-storage term $f_s E_0$ has already been obtained from the electromagnetic response in Section 3. The local energy–momentum relation retained in the OQG propagation construction is $E_{\mathrm{loc}}^2 = E_0^2 + c_0^2 p_{\mathrm{loc}}^2$. The

material clock and ruler conversions are $E = f_s E_{\mathrm{loc}}$ and $p_{\mathrm{loc}} = b_s p$. Written as a Hamiltonian, the resulting dispersion relation and its equations of motion are

$$\begin{aligned} H(x,p) &= f_s(x)\sqrt{E_0^2 + c_0^2 b_s(x)^2 p^2}, \\ v &= \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial x} + F_{\mathrm{app}}. \end{aligned} \tag{11}$$

Here x, p, v describe one direction in fixed comparison standards, F_{app} is an externally applied force, and $m_0 = E_0/c_0^2$ is the reference mass. One can picture a graph of energy against momentum at each position. Its height at zero momentum is the stationary material energy. Its slope gives the wave packet's velocity. Its curvature tells us how much the velocity changes when momentum is added, and therefore determines inertia. Spatial variation of the graph supplies the conservative force. For example, the near-rest inertia in fixed comparison standards is $m_{\mathrm{cmp}} = m_0/(f_s b_s^2)$, the inverse curvature proved in Appendix F.2. This is curvature of the energy–momentum graph, not an assumed curvature of spacetime.

The two senses of dispersion now meet. Correlated electromagnetic response supplies the storage profile and its force; matter-wave dispersion supplies propagation through that profile. The local Lorentz dispersion is the propagation input here. The OQG source, storage, clock, and ruler factors specify the material state in which it operates [1, Sections 6 and 8.2; 5, Section 4; Appendix F].

Why does supplying kinetic energy make the travelling clock run slower? A clock follows its own moving path. Along that path, the phase advances according to energy minus momentum times velocity. In local laboratory standards,

$$s = \sqrt{1 - \frac{V^2}{c_0^2}}, \qquad E_{\mathrm{loc}} = \frac{E_0}{s}, \qquad E_{\mathrm{loc}} - p_{\mathrm{loc}} V = E_0 s. \tag{12}$$

The total energy increases as $1/s$, but the phase combination decreases as s. Geometrically on the energy–momentum graph, the tangent has slope V and intercept $E_{\mathrm{loc}} - p_{\mathrm{loc}} V$. A moving clock follows the wave packet at that slope. Its positive phase rate along the path is the intercept divided by $\hbar$, rather than the energy divided by $\hbar$ at a fixed position. This explains how adding kinetic energy can reduce its counted rate.

For an internal clock transition, the phase difference of its two levels acquires that same factor when evaluated along their common clock path. Counting the transition cycles gives $d\tau = s\, dT$, where T is

the local stationary laboratory's time. It is the phase difference, not the whole body's rest-energy frequency, that supplies the clock reading.

OQG connects this phase result to the receiving state through the supporting-power theorem. For the same receiver, corresponding channels, fixed source environment B, and source-clock time in numerator and denominator, Ref. [5, Section 4] obtains

$$\frac{P(V \mid B)}{P(0 \mid B)} = s^2 = 1 - \frac{V^2}{c_0^2} = \left(1 + \frac{W_{\mathrm{kin}}}{E_0}\right)^{-2}, \qquad W_{\mathrm{kin}} = E_0\left(\frac{1}{s} - 1\right). \tag{13}$$

At $V = 0.6c_0$, the clock factor is 0.8, the corresponding supporting-power ratio is 0.64, and the kinetic work is $0.25E_0$. During ten seconds of local laboratory time, the moving clock records eight seconds. A one-percent clock reduction would similarly imply a supporting-power ratio of 0.9801, regardless of the receiver's reference wattage.

Equation (13) is the mapped moving-state result obtained by applying the power theorem to the material phase factor. The cited derivation does not separately evaluate a complete moving two-body scattering kernel. Its explanatory content is the connection between the receiving state, phase rate, and work: more kinetic energy can coexist with lower balanced supporting exchange because total energy and phase rate are different quantities. The reduced phase combination must not also be subtracted from E_0 as an extra released energy.

After acceleration ends, the velocity, kinetic energy, and rate comparison persist during coasting. No continuing mechanical work is needed to maintain uniform motion. Braking transfers the kinetic energy elsewhere. The power ratio is a comparison of the sustained receiving state, not an additional drag force or an extra clock multiplier.

6 Why the two inertial laboratories each see the other clock as slow

The travelling laboratory can now use its own clocks and rulers to describe the original laboratory. Each assigns a slower rate to the other's moving clock. This does not require each clock to acquire a second slowdown. The laboratories compare different sets of separated events when they decide what is happening "at the same time."

OQG's comparison begins with the phase factor already calculated. Let X, T be the original laboratory's local readings. A laboratory moving at V assigns x', t'. Its origin has clock rate $dt' = s\,dT$. The longitudinal ruler comparison and the synchronization of separated clocks must then be determined using light signals. Imposing the same local measured light speed in both directions gives [5, Section 5]

$$\begin{aligned} b_{\parallel} &= s, \qquad b_{\perp} = 1, \qquad \gamma = \frac{1}{s}, \\ x' &= \gamma(X - VT), \qquad t' = \gamma\left(T - \frac{VX}{c_0^2}\right). \end{aligned} \tag{14}$$

The term involving VX specifies which distant events the moving laboratory calls simultaneous. Along its own origin, $X = VT$, Eq. (14) gives $dt' = s\,dT$, precisely the phase result already used. Along the original laboratory's origin, $X = 0$, it gives $dt' = \gamma\,dT$, or $dT/dt' = s$. The reciprocal assignment follows without multiplying another slowdown into either physical clock.

The longitudinal factor in Eq. (14) is directional. The transverse factors remain one. Applying the isotropic shelf formulas to this motion factor would incorrectly change the surrounding stationary propagation state. Appendix G derives the longitudinal and synchronization factors from the two light-signal conditions.

A received signal is a further comparison. At $V = 0.6c_0$, a freely propagating signal travelling along the velocity direction has Doppler factor $\gamma(1 - \beta) = 0.5$, while one arriving from the opposite direction has factor $\gamma(1 + \beta) = 2$. Neither number is the clock factor 0.8 or the supporting-power ratio 0.64. The signal factor includes propagation direction; the clock factor describes phase accumulated on the receiver's path; the power ratio describes the corresponding complete exchange. Keeping those physical quantities separate resolves the apparent contradiction.

When the clocks meet again, they compare readings at one shared event. No distant synchronization convention remains in that direct comparison. Each clock brings the cycle count accumulated along its actual history, which is calculated in Section 10.

7 Accelerate the clock and follow the changing receiving state

Acceleration moves the receiver through successive momenta on its local dispersion relation, even when the surrounding stationary profile is uniform. The actuator supplies momentum and work. As the slope of the energy–momentum graph changes, the velocity changes; as its tangent intercept changes, the clock rate changes. The corresponding directional reception and supporting-power ratio follow the moving-state comparison just described.

In one-dimensional motion let $a = dV/dT$ and let F_{app} be the applied force measured in the local stationary laboratory. Differentiating the energy, momentum, and supporting-power relations gives

$$F_{\mathrm{app}} = \frac{m_0 a}{s^3}, \qquad \frac{ds}{dT} = -\frac{Va}{c_0^2 s}, \qquad \frac{d}{dT}\left[\frac{P(V \mid B)}{P(0 \mid B)}\right] = -\frac{2Va}{c_0^2}. \tag{15}$$

The first relation follows from the curvature of the dispersion relation: $dV/dp_{\mathrm{loc}} = s^3/m_0$. As that curvature decreases, a given force produces a smaller acceleration. The other two relations connect the changing motion to phase and supporting exchange. Acceleration in the direction of motion reduces the corresponding support ratio; braking increases it. This can happen while the spatial environmental gradient is negligible. A change caused by velocity must therefore not be reinterpreted as a stationary spatial force. That distinction prevents the same material response from being counted once as inertia and again as an invented environmental reaction [5, Sections 2–4].

At the instant acceleration begins from rest, $V = 0$. The rate is initially unchanged, and its first time derivative in Eq. (15) is zero, even though the applied force is nonzero. The clock subsequently accumulates fewer cycles as its speed grows. An accelerometer reading by itself is therefore insufficient to assign a clock-rate multiplier. Real acceleration can also cause strain or field shifts inside a device; those require their own response calculation.

For constant proper acceleration a_0, meaning the acceleration recorded by an attached ideal accelerometer, the one-dimensional trajectory starting from rest gives

$$\begin{aligned} s(T) &= \frac{1}{\sqrt{1 + (a_0 T/c_0)^2}}, \qquad \frac{P(V(T) \mid B)}{P(0 \mid B)} = \frac{1}{1 + (a_0 T/c_0)^2}, \\ \tau(T) &= \frac{c_0}{a_0}\operatorname{arsinh}\left(\frac{a_0 T}{c_0}\right). \end{aligned} \tag{16}$$

The first relation gives the instantaneous material clock factor, the second its corresponding supporting-power ratio, and the third the cycles accumulated along the full acceleration history. Appendix G integrates the motion and clock equations. At a proper acceleration of $9.80665\ \mathrm{m\,s^{-2}}$ for ten seconds of laboratory time, the clock lags by approximately 0.178340 picosecond. The small size of this number comes from the low achieved speed compared with c_0, not from an absence of physical response.

This calculation uses ideal-clock evolution. Deliberately changing a body's internal scale rapidly is a different preparation problem: following a time-dependent mapped quantum state requires the corresponding tracking term in its Hamiltonian. Appendix G.1 states that term. A prescribed changing scale cannot simply be substituted into an equilibrium formula while its time derivative is ignored.

8 Compare an accelerating laboratory with a supported laboratory

Place one clock on a rocket's floor and another on its ceiling. Let the rocket accelerate along its height while maintaining constant proper floor-to-ceiling separation, the separation measured by its local rulers. This requirement matters. Giving every part of the rocket the same proper acceleration would produce a different motion and would not preserve that separation.

The floor and ceiling follow different paths through the original inertial laboratory. Each clock advances according to the material law $d\tau = s\,dT$. Integrating the paths and using the floor clock to label the rocket's events gives, for floor proper acceleration a_0 and proper height h, [Appendix H; 7]

$$\frac{d\tau_{\text{ceiling}}}{d\tau_{\text{floor}}} = 1 + \frac{a_0 h}{c_0^2}, \qquad a(h) = \frac{a_0}{1 + a_0 h/c_0^2}. \tag{17}$$

The ceiling clock advances faster. The second relation specifies the smaller proper acceleration required at the ceiling for the chosen rigid motion. The first compares events simultaneous in the rocket's coordinate construction; at equal original inertial times the two clocks generally have different velocities. Applying the same phase law to their actual paths supplies the result. The height factor is not an independently imposed change of the ambient field.

Now examine a laboratory supported above Earth. Its clocks remain at fixed locations in the stationary source profile. The physical explanation is the one established in Section 3: different supporting exchanges give different transition gaps. Let ℓ be proper upward distance measured by stationary local rulers. The local force needed to hold a body against the storage gradient, divided by its local inertia, is

$$a_{\text{hold}}(\ell) = c_0^2 \frac{d\ln f_s}{d\ell} = \frac{c_0^2}{2} \frac{d\ln \Pi}{d\ell}. \tag{18}$$

In dispersion terms, the support force opposes the spatial slope of the body's stationary energy, while its inertia comes from the momentum curvature of that same energy law. Both contain the body's complete reference energy, so that energy cancels in Eq. (18). The remaining quantity is a gradient of the supporting-power ratio. This is the OQG material connection between a support load and a clock comparison.

Set the floor value to $g_0 = a_{\text{hold}}(0)$. Over a small proper height h, Eq. (18) fixes the leading power and clock changes:

$$\frac{\Pi(h)}{\Pi(0)} = 1 + \frac{2g_0 h}{c_0^2} + O(h^2), \qquad \frac{d\tau_h}{d\tau_0} = \sqrt{\frac{\Pi(h)}{\Pi(0)}} = 1 + \frac{g_0 h}{c_0^2} + O(h^2). \tag{19}$$

When $a_0 = g_0$, Eqs. (17) and (19) agree in their leading clock term. In the gravitational laboratory, scattering and the settled material profile supply the clock and storage gradient. In the rocket, applied work supplies the accelerated paths through the environment. The same material phase law and inertia connect both experiments. That is the mechanism-based content of local clock equivalence in this construction.

The agreement does not require the Earth and rocket to generate identical source fields. The rocket's coordinate lapse follows from its motion; the gravitational power profile belongs to stationary receivers coupled to an actual source. Nor does a first-order agreement establish equality throughout a large laboratory. Spatial variation of the gravitational response introduces the tidal differences examined below. Appendix H gives the full path construction and the local expansion.

9 Let the clock fall

Release the book, the travelling clock, and an accelerometer together from the table height. The support load disappears. The Earth's scattering contribution and stationary supporting-power profile remain. The released body now follows the spatially varying dispersion relation in Eq. (11). Its momentum changes because the material energy depends on position, and its velocity follows the momentum slope. This is the propagation form of the dispersion force established in Section 3.

At the release event, the falling and supported clocks are momentarily at the same position and velocity. They therefore have the same instantaneous rate, even though their accelerometers immediately differ. The supported casing experiences a load that keeps it from falling. In the released instrument, casing and proof mass follow the same local unforced motion, so the ideal accelerometer reads zero. This difference in support does not switch off the source's influence on internal clocks.

As the clock falls toward the larger deficit, its stationary material-storage contribution decreases and its speed increases. Both changes affect its rate relative to the clock left at release height. The connection can be displayed directly in OQG power variables. Let f_A and $\Pi_A = f_A^2$ be the stationary factors at the release point. In a static source profile, the Hamiltonian is conserved. Release from rest sets its value to $E_0 f_A$. Along the subsequent path,

$$\begin{aligned} H &= \frac{E_0 f_s}{s} = E_0 f_A, \qquad s = \frac{f_s}{f_A}, \\ \frac{V^2}{c_0^2} &= 1 - \frac{\Pi(x)}{\Pi_A}, \qquad \frac{d\tau_{\text{fall}}}{d\tau_A} = \frac{\Pi(x)}{\Pi_A}. \end{aligned} \tag{20}$$

The last relation is especially instructive. A stationary clock at the falling clock's current location would have rate factor $f_s/f_A = \sqrt{\Pi/\Pi_A}$ relative to the upper clock. The released clock has that environmental factor and a motion factor $s = f_s/f_A$. Their product is Π/Π_A. The two factors are linked by conservation of the complete material energy; neither is an additional force. Appendix I.4 derives Eq. (20) from the same Hamiltonian and phase law.

For a short downward drop through distance z in an approximately constant weak field, $\Pi/\Pi_A \simeq 1 - 2gz/c_0^2$. To leading order, the environmental contribution and the speed contribution each supply half of the instantaneous rate deficit relative to the upper clock. A ten-metre fall ends with a fractional deficit of about 2.18×10^{-15}. Over the approximately 1.43-second fall, the accumulated lag is about one femtosecond, one quadrillionth of a second. The comparison is made just before impact; subsequent stopping and transport belong to a later history.

Why do different compositions follow the same free-fall motion? Near rest, the force is $-E_0 \nabla f_s$ and the comparison-coordinate inertia is $m_0/(f_s b_s^2)$. Since $E_0 = m_0 c_0^2$, the reference mass cancels. The full trajectory proof has the same cancellation for bodies sharing the complete response factors and initial conditions. Different binding energies and reference exchange powers do not spoil that fractional correspondence [Appendices E and I].

The Hamiltonian trajectory becomes a geodesic when written in the clock-and-ruler geometry supplied by this response. The geometry records the propagation already constructed from the material energy. A small freely falling laboratory therefore supplies local inertial comparisons, even though a distant supported laboratory can assign changing velocity and a different accumulated clock rate.

9.1 Why free fall only removes the local support load

Two freely falling bodies at slightly different positions need not keep the same separation. The source profile can give them different accelerations. In weak stationary fields, the leading relative acceleration can be written directly through the spatial variation of the power profile:

$$\Delta a_i \simeq -\frac{c_0^2}{2} \sum_j \frac{\partial^2 \ln \Pi}{\partial x_i \, \partial x_j} \, \xi_j. \tag{21}$$

Here $\boldsymbol{\xi}$ is their small separation, and i, j label spatial directions in the weak-field comparison. The first gradient gives the common falling acceleration; its spatial change gives the relative acceleration. An observer falling with one body can remove the common local acceleration from the description, but cannot remove the measured change of separation across an extended laboratory. These tidal effects set the size and duration over which a free-fall laboratory can be treated as uniform. Appendix I.4 derives

Eq. (21) to the stated weak-field order; a full higher-order calculation retains both clock and ruler profiles.

10 Bring the clocks together and compare the complete histories

A final clock reading cannot be assigned from the final altitude or velocity alone. The clock has counted cycles throughout its trip. To compare two clocks at reunion, specify their common departure event, their actual paths, and their reunion event. Integrate the same material rate law along each path.

Let $\mathbf{v} = d\mathbf{x}/dt$ be velocity in fixed comparison coordinates. The local stationary laboratory measures $V = |\mathbf{v}|/(f_s b_s)$, because its clock and ruler have the factors already established. For the static isotropic source application, the clock integral is

$$\Delta\tau_j = \int_{t_1}^{t_2} \sqrt{\Pi\left(\mathbf{x}_j(t)\right)} \sqrt{1 - \frac{\left|\mathbf{v}_j(t)\right|^2}{\Pi\left(\mathbf{x}_j(t)\right) b_s\left(\mathbf{x}_j(t)\right)^2 c_0^2}}\, dt. \tag{22}$$

The first square root expresses the stationary material clock response fixed by supporting power. The second expresses the phase comparison due to motion through that state. Both are evaluated at the same point on the same path. The index j labels the path. It does not make all clocks share the same reference frequency or wattage.

In a uniform region, a coasting segment at $0.6c_0$ lasting ten seconds of laboratory time contributes eight seconds to the moving clock. Two such legs contribute sixteen seconds while the stationary laboratory records twenty seconds over those legs. Any finite turnaround contributes its own phase integral. Acceleration is part of the history; a separate universal "turnaround penalty" is not added to the elapsed time.

For a weak field and low speed, Eq. (22) becomes

$$\begin{aligned} \frac{d\tau}{dt} &\simeq 1 + \frac{\Phi}{c_0^2} - \frac{v^2}{2c_0^2}, \\ \Delta\tau_2 - \Delta\tau_1 &\simeq \frac{1}{c_0^2}\int_{t_1}^{t_2}\left[\Phi_2 - \Phi_1 - \frac{v_2^2 - v_1^2}{2}\right] dt. \end{aligned} \tag{23}$$

The gravitational potential Φ here is a convenient representation of the leading stationary storage change: $f_s \simeq 1 + \Phi/c_0^2$ and $\Pi \simeq 1 + 2\Phi/c_0^2$. The positive altitude term therefore comes from greater stationary supporting exchange and energy gaps. The negative speed term comes from the energy–momentum phase combination. Equation (23) is the familiar observable clock relation with its two material contributions identified.

Those contributions can cancel. Relative to a stationary reference at potential Φ_0, a path with $v^2 = 2(\Phi - \Phi_0)$ has the same leading instantaneous rate. At one hundred metres above the reference, using the conventional illustrative value of g, the balancing speed is approximately $44.3\ \mathrm{m\,s^{-1}}$. The condition specifies a clock comparison; the force and energy required to follow that path must still be supplied by the trajectory calculation. It is not a propulsion prescription.

At reunion, subtract the two accumulated readings. For separated clocks, specify the signal-transfer procedure and remove the relevant Doppler and propagation effects before interpreting a received frequency as a clock-rate comparison. This is why a transferred signal, a clock display, an accelerometer, and a power meter cannot be used interchangeably.

11 What the OQG explanation contributes

The additional explanation is the set of material connections demonstrated in the individual experiments. At the shelf, a different source-supported equilibrium changes both the transition gap and recoverable lifting energy. For a moving clock, positive work raises total energy while the energy–momentum combination governing its phase falls; the power theorem fixes the corresponding receiving-state ratio. During applied acceleration, the velocity-dependent change in that ratio can occur without a spatial source gradient. In the accelerating laboratory, different material paths and synchronization produce the height comparison. In the supported laboratory, a spatial power gradient produces the same leading comparison through storage and inertia. In free fall, the source response remains while removal of the support allows energy-conserving motion through the profile.

Dispersion connects these explanations. The frequency-dependent nuclear response supplies storage coefficients; correlated scattering changes the shared exchange and equilibrium energy. Complete scaling preserves binding and links unlike clocks, while the power theorem fixes the square relation. The resulting matter-wave dispersion relation carries those material properties into propagation: its spatial derivative supplies force, its momentum slope and curvature supply velocity and inertia, and its phase along the path supplies the clock reading. Their mathematical commitments can be followed from the response proofs to each observable comparison.

The construction also tells an experimenter what must agree. Different stationary transitions must share the fractional factor in Eq. (6). Height-dependent frequency, inferred supporting-power ratio, and reversible work must satisfy Eqs. (7)–(9). A moving-state support interpretation must agree with work and phase through Eq. (13). Clocks reunited after different histories must obey Eq. (22). Ref. [4, Section 5] specifies the spectroscopic comparison, Ref. [5, Section 5] specifies departure and reunion, and Appendix J collects the numerical checks and differential tests.

The causal claim is direct: in OQG, the material response described here is the gravitational interaction responsible for these effects. The Hamiltonian and its associated geodesic description express that interaction in different mathematical forms. Within the correspondence proved in Appendix I, adding a second full-strength gravitational interaction would count the same contribution twice. A geometric description can therefore coexist with the OQG material description as a representation of the same dynamics; it does not establish a second independent mechanism acting on the clock [1, Section 9].

The challenge to a competing interpretation is equally direct. Identify an error in the derivations, demonstrate a failure of the required physical premises, or exhibit an observed relativistic or equivalence effect within the claimed regime that the complete OQG construction cannot reproduce. An objection should specify the failed equation, response condition, or comparison protocol. Reproducing the same clock readings with a metric does not, by itself, answer the proposed scattering and storage explanation. Conversely, a verified failure at one of these links would require the corresponding OQG claim to be revised or rejected.

That challenge does not transfer OQG's evidential burden to its critics. The universal material correspondence and constitutive matching must describe physical matter; the moving supporting-power interpretation must describe the complete receiving state. Local Lorentz propagation is retained input, and the terrestrial source strength is calibrated to measured gravity. The clock-inferred power ratio is constrained by the theorem, but it is not an independent measurement of supporting exchange. This paper establishes the consequences of those stated commitments; it does not calculate the gravitational strength from independent constituent data or establish a unique microscopic mechanism merely because no contradiction has yet been found.

The competition concerns the physical origin of gravity and the additional consequences that each interpretation can establish. Where their observable predictions coincide, those observations cannot choose between them. OQG puts forward a concrete material account whose response, storage, and propagation relations can be examined together. Establishing that account beyond the shared clock laws requires an independently constrained source calculation or a discriminating observable. The contest is settled by calculation and physical evidence.

Return to the clock beside the book. Raising it, driving it, supporting it, and releasing it alter different parts of one material problem. OQG follows the exchange, the stored energy, the momentum, and the counted phase through each operation. Its proposed explanatory advantage is that these physical connections tell us what the clock's matter must do, effect by effect. That is the claim to test.

Matter tells matter how to scale. Geometry is just bookkeeping.

Guide to the proofs

The main text follows the clock experiments. These appendices provide the shared physical construction and the detailed derivations. Appendices A–B establish scattering, dispersion energy, and nuclear storage. Appendices C–D prove stable material scaling and the supporting-power law. Appendices E–F connect the source profile to work, matter-wave dispersion, and clock phase. Appendices G–I derive moving and accelerated comparisons, local laboratory equivalence, free fall, tides, and complete clock histories. Appendix J collects numerical checks and comparison protocols. References identify the source papers in the Journal of Alternative Theoretical Physics (JOATP).

The calculations compare complete corresponding states. Their interaction terms, current profiles, channels, spectra, and reference standards are carried together. The static propagation application uses a small test body in a slowly varying isotropic source profile; the directional moving-receiver comparison has its own explicitly stated reference. Each proof identifies the further conditions it uses.

Here a proof establishes a consequence of the stated model and its conditions. The order matters: scattering supplies a possible change in reception; the coupled energy and power equations select a settled state; complete-state correspondence fixes how its observables scale; the propagation input then determines trajectories and clock phase. No later algebra replaces the earlier physical requirements. This order also provides a reading route: check the hypotheses at the start of each stage, then follow its result into the next appendix.

Notation and reference standards

Subscript 0 denotes a declared reference state. Every ratio must compare the same physical quantity using the same time and length standards. A source environment can provide those standards without defining a preferred rest frame of the bare vacuum.

Symbol	Definition and units
$t, \mathbf{x}$	Fixed comparison time and spatial coordinates; seconds and metres.
$T, \mathbf{X}$	Time and position assigned by a local stationary inertial laboratory over a sufficiently small region; seconds and metres.
τ	Elapsed proper time recorded by the travelling clock; seconds.
f_s, b_s	Positive stationary environmental frequency and length factors; common branch $f_s = b_s$

Symbol	Definition and units
s	Local motion factor $1/\gamma_{\mathrm{loc}}$; dimensionless.
F_{clk}	Combined clock rate $d\tau/dt = f_s s$; dimensionless. It is not a force.
$\mathbf{v}, \mathbf{V}$	Comparison velocity $d\mathbf{x}/dt$ and local velocity $d\mathbf{X}/dT$; metres per second.
c_0	Local reference light speed, $299792458\ \mathrm{m\,s^{-1}}$.
c_{cmp}	Light speed in fixed comparison coordinates; metres per second.
$E_0 = m_0 c_0^2$	Complete reference rest energy and corresponding local rest mass; joules and kilograms. Includes binding and assigned reference contributions.
$\mathbf{p}, \mathbf{p}_{\mathrm{loc}}$	Canonical comparison momentum and local momentum; kilogram metres per second.
P, P_0, Π	Balanced supporting exchange, its own reference value, and $\Pi = P/P_0$; watts, watts and dimensionless.
A	Total ambient stress coefficient $A_{2,E}^{\mathrm{tot}}$; joules squared. Not a mechanical spring constant.
$\hbar, \omega, \nu$	Reduced Planck constant, angular frequency and cycle frequency; joule seconds, radians per second and hertz. $\omega = 2\pi\nu$.

The factors f, b in Appendices C–D describe a general stationary corresponding family. They become f_s, b_s in a spatial environmental profile. In a separate moving-state comparison, Ref. [5] uses s as the receiver's frequency ratio. These uses retain their specified references.

Some symbols are local to an appendix. In Appendix B, Φ is canonical electrical flux, in weber, and V denotes either probe voltage or material volume as defined there. In Appendices E–I, Φ is gravitational potential per unit mass and V is local velocity. The calligraphic $\mathcal{K}_{\mathrm{EM}}$ in Appendix A is a force coefficient in joule metres. Each meaning is fixed by its defining equation.

Ideal clock conditions. The clock is small enough for the local response factors to be effectively uniform across its internal degrees of freedom. Its internal state follows the mapped equilibrium or ideal-clock phase evolution. Applied forces do not introduce uncontrolled strain, heating, Stark shifts or other

ordinary clock disturbances. Complete-body propagation uses the test-body and eikonal limits stated in Ref. [1]. Additional symbols are defined where introduced.

Appendix A Proofs of scattering and balanced exchange

This appendix follows the source and receiver calculation in [1, Sections 2.1–2.5] and its collective interpretation in [2, Section 2]. A reference receiver and its adjusted equilibrium are compared using the same standards. Supporting power is the common inward and outward exchange, with reference P_0 held fixed. Define the dimensionless ratio Π and deficit δ_P by

$$P_{\mathrm{in}} = P_{\mathrm{out}} = P, \qquad \Pi = \frac{P}{P_0}, \qquad \delta_P = 1 - \Pi. \tag{A1}$$

A.1 The interference term

Let u be the reference receiving amplitude, normalized by $\langle |u|^2 \rangle = 1$, and z the total added amplitude after the retained scattering. Both use the same spectral and polarization weights. With $B = \langle |z|^2 \rangle$ and $C_{uz} = \langle u^\dagger z \rangle$, expansion gives

$$Q = \langle |u+z|^2 \rangle = 1 + B + 2\mathrm{Re}C_{uz}, \qquad Q < 1 \;\Leftrightarrow\; 2\mathrm{Re}C_{uz} < -B. \tag{A2}$$

This is the general reduction criterion. It retains cross correlations before averaging. The ratio Q compares reception at a fixed receiving response; Π in (A1) compares the complete adjusted equilibria. The coupled equilibrium in Appendix B supplies the connection between these two stages.

A.2 Exact sign in the resonant two dipole example

For two identical absorbing dipoles in the prescribed isotropic quasimonochromatic random field of [1, Section 2.3], set $x = kr > 0$ and $k^3\alpha_{\mathrm{vol}} = ia$, with $a > 0$. The outgoing Maxwell dipole propagator has transverse and longitudinal components

$$g_T(x) = \frac{e^{ix}}{4\pi x}\left(1 + \frac{i}{x} - \frac{1}{x^2}\right), \qquad g_L(x) = \frac{e^{ix}}{4\pi x}\left(-\frac{2i}{x} + \frac{2}{x^2}\right). \tag{A3}$$

Here r is the dipole separation, k is the incident wave number, and α_{vol} is the volume polarizability. Thus $x = kr$ and a are dimensionless. The two transverse polarizations and one longitudinal polarization explain the weights in Q_{av}. These are fixed-response receiving intensities in the prescribed field; they do not yet describe the material readjustment that defines Π.

For either component $g_i = u_i + iv_i$, normalize each incident variance to one. The common random field has spatial correlation $\langle E_1^{(0)} E_2^{(0)*} \rangle = 6\pi v_i$. A zero mean field does not remove this correlation. The total fields satisfy

$$E_1 = E_1^{(0)} + iag_i E_2, \quad E_2 = E_2^{(0)} + iag_i E_1, \quad E_1 = \frac{E_1^{(0)} + iag_i E_2^{(0)}}{1 + a^2 g_i^2}. \tag{A4}$$

The denominator retains repeated exchange. For $\mathcal{D}_i = \left|1 + a^2 g_i^2\right|^2 > 0$, expand the squared numerator and substitute the incident correlation. Direct intensity contributes one, scattered intensity contributes $a^2|g_i|^2$, and interference contributes $-12\pi a v_i^2$. Thus

$$\mathcal{Q}_i = \frac{1 + a^2|g_i|^2 - 12\pi a v_i^2}{\mathcal{D}_i}, \qquad \mathcal{Q}_{\text{av}} = \frac{2\mathcal{Q}_T + \mathcal{Q}_L}{3}. \tag{A5}$$

Subtracting the numerator from the exact denominator proves the sign without discarding repeated scattering:

$$1 - \mathcal{Q}_i = \frac{a^2 u_i^2 + a(12\pi - 3a)v_i^2 + a^4(u_i^2 + v_i^2)^2}{\mathcal{D}_i} \geq 0, \qquad 0 < a \leq 4\pi. \tag{A6}$$

Each numerator term is nonnegative in the stated interval. Averaging preserves the inequality. At $a = 0.01$ and $x = 1$, the polarization-averaged direct, scattered, and interference contributions are respectively 0.999996577613, 0.000002110842, and -0.000753327938. Their sum is 0.999245360517: the absorption deficit is 0.07546395 percent. This verifies the reduction mechanism in the specified resonant model.

A.3 Equilibrium includes fluctuations and reaction

The nuclear application uses the full balanced exchange. Let $\chi = \mathcal{D}^{-1}$ be the charge-response matrix, with $\mathcal{D} = \mathcal{A}_H - i\omega R$, Hermitian conservative part $\mathcal{A}_H$, and Hermitian dissipative resistance R. Use the common equilibrium temperature T, two-sided symmetrized spectra, and the causal convention of [1, Section 2.4]. The same resistance determines the fluctuating voltage spectrum S_ξ and its reaction on the charge. The identities are

$$\text{Im}\chi = \omega\chi R\chi^\dagger, \quad S_\xi = \hbar\omega \coth\left(\frac{\hbar\omega}{2k_B T}\right) R, \quad S_Q = \chi S_\xi \chi^\dagger. \tag{A7}$$

The loss identity follows by subtracting the inverse response and its adjoint. Substitute the fluctuation relation into the two power expressions. Cyclicity of the trace then gives

$$P_{\mathrm{in}} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \omega \mathrm{Tr}\big(S_{\xi} \mathrm{Im}\chi\big) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \omega^2 \mathrm{Tr}\big(R S_Q\big) = P_{\mathrm{out}}. \tag{A8}$$

At zero temperature use the limiting fluctuation spectrum; at high frequency use the physical response or the consistently matched regularization. The identity establishes equilibrium balance with the source, field, and reaction included. The magnitude of that balanced exchange is evaluated in the settled material state; it is not the net heat flow.

For the first identity in (A7), use $\chi - \chi^{\dagger} = \chi(D^{\dagger} - D)\chi^{\dagger}$ and $D^{\dagger} - D = 2i\omega R$. Dividing by $2i$ gives $\mathrm{Im}\chi$. In (A8), the fluctuation factor multiplying R is a scalar at the common temperature; substitution then makes both traces identical. This shows exactly where a common equilibrium state enters the balance proof.

A.4 Dispersion energy and force from the scattering response

Source: Ref. [1, Section 2.5, Eqs. (21)–(24)]. The following energy and force formulas show why balanced exchange need not imply zero force. Consider two disjoint linear bodies in equilibrium at zero temperature. Let T_1, T_2 be their isolated scattering operators and G_0 the intervening field propagator, with all separation and material dependence retained. The operators describe the same current response used in the exchange calculation. On the imaginary angular-frequency axis, $\omega = i\zeta$, define the round-trip operator M. The interaction energy relative to isolated bodies is

$$\begin{aligned} \mathsf{M}(i\zeta; x) &= \mathsf{G}_0 T_1 \mathsf{G}_0 T_2, \\ U_{\mathrm{EM}}(x) &= \frac{\hbar}{2\pi} \int_0^{\infty} d\,\zeta\, \mathrm{logdet}[\mathbb{I} - \mathsf{M}(i\zeta; x)]. \end{aligned} \tag{A9}$$

The imaginary-frequency integral is a representation of the causal equilibrium response. It does not describe illumination at an imaginary physical frequency. Its value has units of energy because $\hbar\, d\zeta$ has units of joules and the logarithmic determinant is dimensionless.

The determinant includes repeated scattering. Assume the response and subtraction make the integral well defined, and that differentiation under the integral is permitted. The identity $\partial_x \mathrm{logdet} A = \mathrm{Tr}(A^{-1}\, \partial_x A)$, with $A = \mathbb{I} - \mathsf{M}$, gives the outward-positive generalized force

$$F_x^{\mathrm{disp}} = -\,\partial_x U_{\mathrm{EM}} = \frac{\hbar}{2\pi} \int_0^{\infty} d\,\zeta\, \mathrm{Tr}[(\mathbb{I} - \mathsf{M})^{-1}\, \partial_x \mathsf{M}]. \tag{A10}$$

Both geometric and material changes in M contribute to this derivative. The force therefore depends on frequency-dependent response and correlations, even when the net heat exchange vanishes. This proves the energy-gradient relation for the stated response model; it does not equate a power, measured in watts, with a force, measured in newtons.

The prescribed resonant random-field example in Ref. [1] is a separate field state. In its absorption-dominated, weak repeated-scattering limit it supplies an attractive radial force with positive coefficient $\mathcal{K}_{\mathrm{EM}}$, in joule metres. The coefficient and asymptotic force are calculated for that field state; they are not obtained by substituting its illumination into the zero-temperature equilibrium formula (A9). Integrating the stated force with zero interaction energy at infinite separation gives

$$F_r = -\frac{\mathcal{K}_{\mathrm{EM}}}{r^2}, \qquad U_{\mathrm{disp}}(r) = -\frac{\mathcal{K}_{\mathrm{EM}}}{r}, \qquad W_{1\to 2} = U_{\mathrm{disp}}(r_2) - U_{\mathrm{disp}}(r_1). \tag{A11}$$

OQG's storage correspondence identifies this conservative interaction with $E_0(f_s - 1)$. The supporting-power theorem then gives $\Pi = f_s^2$. Thus to leading order $f_s = 1 - \mathcal{K}_{\mathrm{EM}}/(E_0 r)$ and $1 - \Pi = 2\mathcal{K}_{\mathrm{EM}}/(E_0 r)$. Appendix E states the mass-proportional calibration used for the terrestrial application. The fixed-response reception deficit, the conservative dispersion interaction, and the settled collective material profile are distinct calculations connected by the stated storage correspondence. A complete collective source-response evaluation remains a further microscopic calculation [1, Sections 7.1 and 9].

Appendix B Proofs of nuclear storage and material adjustment

The receiver construction comes from [1, Sections 3.1–3.5], [3, Sections 2–5], and [4, Sections 2–3]. The interacting nuclear Hamiltonian supplies the excitation spectrum, including strong interactions, electromagnetic interactions, and the retained continuum. A charge coordinate Q resolves internal redistribution; its probe coupling is $-VQ$.

B.1 Storage coefficients from the nuclear spectrum

For a reciprocal channel with unique ground state $|0\rangle$, excitation gaps $\Delta_n = E_n - E_0 > 0$, and transition charges $q_n = \langle 0|Q|n\rangle$, the spectral representation is

$$\chi^R(\omega) = 2\sum_{n>0} \frac{\Delta_n |q_n|^2}{\Delta_n^2 - \hbar^2(\omega + i0^+)^2}. \tag{B1}$$

The continuum is included by its spectral integral. Below unresolved resonances, expand the denominators and invert the response on the responsive subspace. With finite, consistently regularized sums, this gives

$$C = 2\sum_n \frac{|q_n|^2}{\Delta_n}, \quad D_2 = 2\hbar^2 \sum_n \frac{|q_n|^2}{\Delta_n^3}, \quad L = C^{-1} D_2 C^{-1}. \tag{B2}$$

For matrices the real reciprocal-channel numerators become $\mathbf{q}_n \mathbf{q}_n^T$. The expansion is $\chi = C + \omega^2 D_2 + \cdots$ and $\chi^{-1} = C^{-1} - \omega^2 L + \cdots$. The complete causal response is retained near resonances. Canonical charge and flux columns then express the recoverable quadratic storage as

$$H_{\mathrm{st}} = \frac{1}{2} Q^T C^{-1} Q + \frac{1}{2} \Phi^T L^{-1} \Phi, \qquad [Q_a, \Phi_b] = i\hbar \delta_{ab}, \qquad I = L^{-1}\Phi. \tag{B3}$$

This constructs the storage Hamiltonian from response quantities. It is an electrical representation of the retained material dynamics, with binding still contained in the spectrum. Matching its electric and magnetic field patterns supplies the constitutive coefficients used in Appendix D.

Positive excitation gaps and nonnegative spectral weights make C and D_2 positive on the responsive subspace. Consequently $L = C^{-1} D_2 C^{-1}$ is positive there as well. The quadratic storage is therefore bounded below. This check concerns the retained low-frequency storage representation; the full spectral response, with its poles and continuum, remains necessary near excitation energies.

B.2 An explicit contraction load and restoring response

Use the positive passive model of [1, Section 3.3]. In dimensionless canonical units, set $q_\epsilon = \ln(\epsilon/\epsilon_0)$, $q_\mu = \ln(\mu/\mu_0)$, $\sigma = (q_\epsilon + q_\mu)/2$, and $d = (q_\epsilon - q_\mu)/2$. The material frequency is $\omega = e^{-\sigma/2}$. Environmental electric and magnetic modes have frequency $\Omega > 0$ and couplings η_e, η_m. Their coordinates are (X_e, P_e) and (X_m, P_m); these P symbols denote canonical momenta. The storage is

$$H = \frac{e^{-(\sigma+d)/2}}{2} Q^2 + \frac{e^{-(\sigma-d)/2}}{2} \Phi^2 + \frac{\Omega}{2}[(X_e - \eta_e Q)^2 + P_e^2 + X_m^2 + (P_m - \eta_m \Phi)^2]. \tag{B4}$$

Every square is nonnegative. Keep the quadratic self terms when expanding the couplings. The first-order expectation of those quadratic terms and the second-order contribution of the bilinear couplings combine to give the leading interaction-energy shift relative to uncoupled modes:

$$\Delta E(\sigma, d) = \frac{\hbar \Omega \omega}{4(\Omega + \omega)} \left(\eta_e^2 e^{d/2} + \eta_m^2 e^{-d/2}\right) + O(\eta^4). \tag{B5}$$

For example, each bilinear term connects the ground state to one excitation in each of the two coupled oscillators, with denominator $\hbar(\Omega+\omega)$; combining it with its quadratic term gives the corresponding coefficient in (B5). At equal couplings $\eta_e = \eta_m = \eta$ and $d = 0$, direct differentiation yields

$$K_d = \partial_d^2 \Delta E|_{d=0} = \frac{\hbar\eta^2\Omega\omega}{8(\Omega+\omega)} + O(\eta^4), \quad J_\sigma = -\partial_\sigma \Delta E|_{d=0} = \frac{\hbar\eta^2\Omega^2\omega}{4(\Omega+\omega)^2} + O(\eta^4). \tag{B6}$$

Both leading coefficients are positive at sufficiently weak coupling. Positive K_d restores equal electric and magnetic changes; positive J_σ loads the common scale toward contraction because $b = e^{-\sigma/2}$. The source paper checks these signs with the exact normal modes. The model establishes the loading direction and resistance to imbalance. The final displacement also includes the material restoring energy.

B.3 The settled state solves energy and power together

Let q collect allowed deformations and internal variables, λ specify external source settings, and $\mathcal{F}_{\mathrm{tot}}(q;\lambda)$ be the complete constrained energy or equilibrium free energy. It includes environmental interaction and the source work appropriate to the constraints, each once. At the settled state q_*,

$$\partial_{q_A}\mathcal{F}_{\mathrm{tot}}\big|_{q_*} = 0, \qquad \partial_q^2\mathcal{F}_{\mathrm{tot}}\big|_{q_*} > 0, \qquad P_{\mathrm{in}}(q_*;\lambda) = P_{\mathrm{out}}(q_*;\lambda). \tag{B7}$$

Positivity is on the allowed physical subspace. Relaxing stable internal coordinates z against strain e gives the Schur complement of the complete energy Hessian:

$$K_{\mathrm{rel}} = K_{ee} - K_{ez}K_{zz}^{-1}K_{ze}. \tag{B8}$$

To obtain (B8), expand the complete energy to quadratic order in strain e and internal displacement z. Minimization gives $z = -K_{zz}^{-1}K_{ze}e$; substituting it back leaves the stated strain curvature. The inverse requires stable internal directions. The relaxed curvature must also remain positive for the proposed equilibrium to be stable. The interaction load alone sets neither the final deformation nor its stability.

This includes internal adjustment once. For a small isotropic load, define outward-positive interaction pressure Δp_{EM} and positive reference bulk modulus K_{mat}. Stationarity gives $\Delta V/V_0 = \Delta p_{\mathrm{EM}}/K_{\mathrm{mat}}$ to first order. On the corresponding common branch specified in Appendices C–D, $V = b^3V_0$ and $\Pi = b^2$, so

$$\Pi = \left(\frac{V}{V_0}\right)^{2/3}, \qquad \frac{\Delta P}{P_0} = \frac{2\Delta p_{\mathrm{EM}}}{3K_{\mathrm{mat}}} + O\left[\left(\frac{\Delta p_{\mathrm{EM}}}{K_{\mathrm{mat}}}\right)^2\right]. \tag{B9}$$

A negative outward pressure produces contraction and reduced supporting exchange on this branch. The spectral construction, relaxed equilibrium, and exchange calculation refer to the same state. The following scaling proofs specify the complete correspondence used to compare those equilibria.

Appendix C Proofs of complete scaling and stable clocks

Proposition 1. Within the stated corresponding family, every nonzero clock transition shares the same fractional rate factor, stable bound states remain stable, and matched supporting exchange scales as the square of that factor. The proof occupies Appendices C and D.

Sources: Ref. [1] Section 4.1, source Eqs. (48)–(51); Ref. [3] Section 2, source Eqs. (1)–(8); Ref. [4] Section 2, source Eq. (1).

Let j label any chosen kind of bound clock system. Its reference Hamiltonian $H_{j,0}$ includes all retained interactions. Assume it is self-adjoint, bounded below and has a stable bound ground state. Keep $\hbar$ and charges invariant, and map the operator and constrained-state domains as in the source theorem. Let $\boldsymbol{\xi}$ collect its d_j internal spatial coordinates and let ψ be a normalized state. Define the unitary dilation $U_{j,b}$ by

$$(U_{j,b}\psi)(\boldsymbol{\xi}) = b^{-d_j/2}\psi(\boldsymbol{\xi}/b), \qquad b > 0. \tag{C1}$$

Changing integration variable to $\boldsymbol{\xi}/b$ gives a factor b^{d_j} in the integration volume, cancelling the squared prefactor. Thus the norm is preserved. A dagger denotes the adjoint; $U_{j,b}^{\dagger} = U_{j,b}^{-1}$.

For positive frequency factor f, the corresponding evolution and its generator are

$$\Psi_{j;f,b}(t) = U_{j,b}\Psi_{j,0}(ft), \qquad H_{j;f,b} = fU_{j,b}H_{j,0}U_{j,b}^{\dagger}. \tag{C2}$$

Differentiate the first expression and use the reference Schrödinger equation $i\hbar\,\partial_t\Psi_{j,0} = H_{j,0}\Psi_{j,0}$. This gives the stated generator. Map its operator domain with $U_{j,b}$. Positive f preserves spectral ordering and the lower bound. For energy level $E_{j,n,0}$,

$$E_{j,n;f,b} = fE_{j,n,0}, \qquad \Delta E_{j,nm;f,b} = f\Delta E_{j,nm,0}. \tag{C3}$$

Here n, m label levels and $\Delta E_{j,nm} = E_{j,n} - E_{j,m}$. Bound levels and their corresponding breakup thresholds scale together. Strong binding remains in the transformed Hamiltonian.

This is a statement about the complete mapped Hamiltonian, including its domains and thresholds. It does not follow from changing one oscillator frequency in an otherwise unchanged nucleus. Nor does preservation of a stationary bound spectrum guarantee that an arbitrary rapid preparation avoids excitation. Appendix G.3 derives the extra term needed for exact following of a prescribed time-dependent dilation.

C.1 Transition frequencies and calibration

For any nonzero transition gap define $\nu_{j,nm} = \Delta E_{j,nm}/(2\pi\hbar)$. With the same f for all systems in the admitted universal family,

$$\frac{\nu_{j,nm;f,b}}{\nu_{j,nm,0}} = f, \qquad \frac{\nu_{j;f,b}/\nu_{k;f,b}}{\nu_{j,0}/\nu_{k,0}} = 1. \tag{C4}$$

The second identity compares corresponding transitions of any two systems j, k, with transition labels suppressed. Their frequencies need not be equal. Their **fractional change** is equal. If N_j counts cycles and the clock is calibrated with its reference frequency,

$$dN_j = \nu_{j,0} f\, dt, \qquad d\tau_j = \frac{dN_j}{\nu_{j,0}} = f\, dt. \tag{C5}$$

The system-specific reference frequency cancels. A common scaling is invisible between co-located standards that share it. Comparing environments requires a defined signal or transport procedure. Ref. [4] Section 5, source Eq. (17), states the corresponding ratio test.

C.2 Stability and complete linear response

Sources: Ref. [1] Section 4.1; Ref. [4] Sections 2 and 3.2. These checks show why a slower clock can remain stably bound and why the result is not tied to a selected isotope.

Let $\mathcal{E}_{j,0}(R)$ be the constrained minimum energy at root-mean-square size R, twice differentiable near its stable minimum $R_{j,0}$. Dilation maps those constrained states:

$$\mathcal{E}_{j;f,b}(R) = f\mathcal{E}_{j,0}(R/b), \qquad R_{j;f,b} = bR_{j,0}. \tag{C6}$$

Let k_R be the energy's second derivative at its minimum, in newtons per metre, and m_{eff} the matched kinetic inertia of a quadratic mode. Twice differentiating the energy and transforming the kinetic term gives

$$\frac{k_{R;f,b}}{k_{R;0}} = \frac{f}{b^2}, \qquad \frac{m_{\mathrm{eff};f,b}}{m_{\mathrm{eff};0}} = \frac{1}{fb^2}, \qquad \frac{\omega_{\mathrm{mode};f,b}}{\omega_{\mathrm{mode};0}} = f. \tag{C7}$$

Here $\omega_{\mathrm{mode}} = \sqrt{k_R/m_{\mathrm{eff}}}$. Positive curvature remains positive. On the common branch the size decreases as b, the matched inertia increases as b^{-3}, and the frequency decreases as b. This is a coordinated change of the complete dynamics.

Let X, Y be observables with amplitude factors g_X, g_Y, so $X_{f,b} = g_X U_b X_0 U_b^\dagger$, and similarly for Y. The perturbation is $-h_Y(t)Y$, with source amplitude h_Y. The retarded susceptibility is

$$\chi^R_{XY}(\omega) = \frac{i}{\hbar}\int_0^\infty e^{i\omega t}\,\langle [X(t), Y(0)]\rangle\, dt. \tag{C8}$$

The brackets denote equilibrium expectation, $i^2 = -1$, and $[X, Y] = XY - YX$. Evolution uses ft. Changing integration variable to $u = ft$ supplies $dt = du/f$, giving Ref. [4]'s response identity

$$\chi^R_{XY;f,b}(\omega) = \frac{g_X g_Y}{f}\chi^R_{XY;0}(\omega/f). \tag{C9}$$

The same law applies to source-derived instantaneous contact terms. It retains discrete states, continuum states and cross correlations present in the reference response. At finite temperature the corresponding temperature scales by f so occupation probabilities match.

Let Q collect the charge channels, with invariant charge amplitudes. Write the low-frequency matrix response as $\boldsymbol{\chi}_{QQ} = C + \omega^2 D_2 + \cdots$ and its inverse as $C^{-1} - \omega^2 L + \cdots$. The matrices C, D_2, L are in farads, farad seconds squared and henries. Expanding Eq. (C9) gives

$$C_{f,b} = C_0/f, \qquad D_{2;f,b} = D_{2;0}/f^3, \qquad L_{f,b} = L_0/f. \tag{C10}$$

Inverses are restricted to responsive channels. Matching spectral inertia to magnetic storage of the same profiles is an explicit constitutive premise; it does not remove strong interactions from the nucleus.

Appendix D Proofs of field and supporting power scaling

Sources: Ref. [1] Section 4.2, source Eqs. (53)–(54); Ref. [4] Section 3.2, source Eqs. (8)–(10); Ref. [3] Section 3.

For geometrically corresponding profiles, capacitance and inductance each contain one length factor. Let ϵ, μ be the matched electric and magnetic coefficients in fixed comparison standards. Combining that geometric factor b with Eq. (C10) gives

$$\frac{\epsilon}{\epsilon_0} = \frac{\mu}{\mu_0} = \frac{1}{fb}, \qquad \frac{c_{\mathrm{cmp}}}{c_0} = fb, \qquad \frac{\sqrt{\mu/\epsilon}}{\sqrt{\mu_0/\epsilon_0}} = 1. \tag{D1}$$

These are common propagation and storage coefficients in the admitted comparison, not the relative permittivity of a particular dielectric. Ordinary substance-dependent response remains in the system's reference Hamiltonian and susceptibilities.

To measure speed locally, use the accompanying clock and ruler. If $d\ell$ is local measured length,

$$dT = f_s\, dt, \qquad d\ell = |d\mathbf{x}|/b_s, \qquad \frac{d\ell}{dT}\Big|_{\text{light}} = \frac{c_{\text{cmp}}}{f_s b_s} = c_0. \tag{D2}$$

The comparison-coordinate speed can vary while the local measurement retains c_0. Frequency, length and field coefficients must transform together.

The resistance transformation can be checked from the currents themselves [1, Section 4.3]. Let $\mathbf{j}_b(\mathbf{x}) = b^{-2}\mathbf{j}_0(\mathbf{x}/b)$ be corresponding current profiles per unit mode current, and $\mathbf{g}$ their radiation form factors. At frequency $f\omega$ and speed fbc_0, the propagation phase is unchanged. The form factors and radiation matrix therefore obey

$$\mathbf{g}_{f,b}(\mathbf{n}, f\omega) = b\mathbf{g}_0(\mathbf{n}, \omega), \qquad R^{\text{rad}}_{f,b}(f\omega) = R^{\text{rad}}_0(\omega). \tag{D3}$$

In the radiation matrix the prefactor $\omega^2/(\epsilon c^3)$ acquires b^{-2} and the product of form factors acquires b^2. Their cancellation preserves resistance, including the phases between corresponding currents. The same matched dissipative channels enter the complete exchange below.

D.1 Balanced supporting power

Sources: Ref. [1] Section 4.3, source Eqs. (55)–(56); Ref. [4] Section 3.3, source Eqs. (11)–(12); Ref. [2] Section 2, source Eqs. (3)–(4).

Let $S_Q(\omega)$ be the two-sided symmetrized charge-fluctuation spectral matrix, in coulomb squared seconds, and $\mathcal{R}(\omega)$ the Hermitian dissipative impedance matrix, in ohms. The trace sums the retained channels and their cross correlations. The balanced supporting exchange is

$$P = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \omega^2 \mathrm{Tr}[\mathcal{R}(\omega) S_Q(\omega)], \qquad P_{\text{in}} = P_{\text{out}} = P. \tag{D4}$$

In matched environments, $S_{Q;f,b}(f\omega) = S_{Q;0}(\omega)/f$ and $\mathcal{R}_{f,b}(f\omega) = \mathcal{R}_0(\omega)$. Set $\omega' = f\omega$. The measure, squared frequency, impedance and spectrum contribute $f, f^2, 1, f^{-1}$, respectively. Thus

$$\Pi_j \equiv \frac{P_j}{P_{j,0}} = f^2, \qquad f = \sqrt{\Pi_j}. \tag{D5}$$

Every system uses **its own** reference exchange $P_{j,0}$. Universal fractional scaling does not require equal watts, spectra or couplings for different systems. Finite cutoffs must map as $\omega_{c,f} = f\omega_{c,0}$; a fixed unmapped cutoff does not automatically satisfy this proof. Balanced exchange is not net heating or generator input power.

For example, at $f = 0.99$ the transformed measure contributes a factor 0.99, the squared frequency a factor 0.99^2, and the charge spectrum a factor $1/0.99$. The impedance is unchanged. Multiplication leaves $P/P_0 = 0.9801$. This is a check of the entire matched spectrum; a measured change in one line alone does not perform that integral.

D.2 Collective stress and the common branch

Source: Ref. [1] Section 4.4, source Eqs. (59)–(63), and Section 8.2. Define $A = A_{2,E}^{\mathrm{tot}}$ as the completed ambient stress coefficient, including its reference contribution. The label 2 denotes second spatial-derivative order; E denotes the Euclidean response convention. With G the gravitational coupling in the same standards,

$$A = \frac{\hbar c_{\mathrm{cmp}}^5}{32\pi G}, \qquad \kappa \equiv \frac{G}{c_{\mathrm{cmp}}^4} = \frac{G_0}{c_0^4}, \qquad \frac{A}{A_0} = f_s b_s. \tag{D6}$$

The constant κ is the invariant gravitational normalization. Only on the common stationary branch $f_s = b_s$ does the single chain become

$$\Pi = f_s^2 = \frac{A}{A_0} = \frac{c_{\mathrm{cmp}}}{c_0}, \qquad \frac{\epsilon}{\epsilon_0} = \frac{\mu}{\mu_0} = \Pi^{-1}. \tag{D7}$$

In the general two-factor family, supporting power fixes f_s^2 whereas stress stiffness and propagation fix $f_s b_s$. Do not identify them outside the common branch. A motion-induced longitudinal comparison is not an isotropic change of the stationary environment.

An independent Maxwell check uses the same mapped charge, current, and field profiles. Electric and magnetic field strengths acquire f/b, so Poynting flux and corresponding area give

$$\frac{\langle \mathbf{E} \times \mathbf{H} \rangle_{f,b}}{\langle \mathbf{E} \times \mathbf{H} \rangle_0} = \frac{f^2}{b^2}, \qquad \frac{\Sigma_{f,b}}{\Sigma_0} = b^2, \qquad \frac{P_{f,b}}{P_0} = f^2. \tag{D8}$$

The flux equation is read componentwise for corresponding nonzero fluxes, or through the integrated power. A bound system can have zero net outward flux while its separately resolved inward and outward powers have this scaling. Both checks establish the same power factor.

Appendix E Proofs of storage work and the source profile

The microscopic storage construction precedes the complete-body force. Differentiate (B3) at fixed canonical charge and flux. Since $\partial_x L^{-1} = -L^{-1}(\partial_x L)L^{-1}$, and similarly for C, the storage force conjugate to a slow coordinate x is [1, Section 5.3]

$$F_x^{\mathrm{st}} = -\langle \partial_x H_{\mathrm{st}} \rangle = \frac{1}{2}\langle I^T(\partial_x L)I \rangle + \frac{1}{2}\langle v_C^T(\partial_x C)v_C \rangle, \qquad v_C = C^{-1}Q. \tag{E1}$$

For coupled quantum channels the products are symmetrized. Differentiating the corresponding component energy and using the terminal voltage and current equations gives the work identity

$$P_{\mathrm{ports}} = \dot{U}_{\mathrm{st}} + \frac{1}{2}\langle I^T \dot{L} I \rangle + \frac{1}{2}\langle v_C^T \dot{C} v_C \rangle. \tag{E2}$$

The last two terms transfer power into material deformation. For a single harmonic ground state, (E1) equals $-\partial_x(\hbar\omega/2)$, checking the force sign. The complete-body energy theorem below carries the same storage accounting through binding and confinement.

The derivative in (E1) holds the canonical variables fixed before taking their expectation. If internal variables relax as the body moves, differentiate the complete minimized energy instead: stationarity removes their implicit first-derivative contributions. All interaction and material storage terms must be included once. Differentiating an interaction term alone while omitting the associated material restoring energy would not give the complete relaxed force.

Sources: Ref. [1] Sections 5.1 and 7, source Eqs. (77), (92) and (95); Ref. [4] Section 4. Let $\mathbf{x}_1, \mathbf{x}_2$ be stationary clock locations in a static source environment. During the same comparison-time interval,

$$\frac{d\tau_2}{d\tau_1} = \frac{f_s(\mathbf{x}_2)}{f_s(\mathbf{x}_1)} = \sqrt{\frac{\Pi(\mathbf{x}_2)}{\Pi(\mathbf{x}_1)}}. \tag{E3}$$

The powers compare corresponding copies of the same receiver with a common reference normalization. Equivalently use each receiver's normalized profile, which is the same on the universal branch.

Let Φ be the weak gravitational potential per unit reference mass, in square metres per second squared, relative to the declared reference. Then

$$f_s = 1 + \frac{\Phi}{c_0^2} + O(\Phi^2/c_0^4), \qquad \frac{\nu_2 - \nu_1}{\nu_1} \simeq \frac{\Phi_2 - \Phi_1}{c_0^2}. \tag{E4}$$

Near Earth, for upward height difference h and approximately constant acceleration magnitude g, the potential increase is gh. The upper clock gains elapsed time at fractional rate gh/c_0^2 relative to the lower

clock. A signal emitted below and received above is redshifted by the reciprocal clock factor. Always state the direction of signal transfer.

E.1 Recoverable work and acceleration at rest

The complete stationary energy and the force in fixed comparison coordinates follow from the same factor:

$$E_{\text{rest}} = E_0 f_s, \qquad W_{1\to 2} = E_0(f_{s,2} - f_{s,1}), \qquad \mathbf{F}_{\text{rest}} = -E_0 \boldsymbol{\nabla} f_s. \tag{E5}$$

Here $W_{1\to 2}$ is reversible transport work, in joules, and $\boldsymbol{\nabla}$ differentiates the fixed comparison coordinates. The complete energy includes binding and confinement; transition gaps alone do not specify transport energy.

For small momentum, Ref. [1]'s complete-body Hamiltonian expands as

$$H = f_s E_0 + \frac{f_s b_s^2 \mathbf{p}^2}{2m_0} + O(|\mathbf{p}|^4), \qquad m_{\text{cmp}} = \frac{m_0}{f_s b_s^2}. \tag{E6}$$

The quantity m_{cmp} is the kinetic coefficient in these coordinates. At instantaneous rest, velocity-dependent derivative terms vanish. Dividing the force by this inertia gives

$$\frac{d^2\mathbf{x}}{dt^2}\Big|_{\mathbf{v}=0} = -c_0^2 f_s b_s^2 \boldsymbol{\nabla} f_s \simeq -\boldsymbol{\nabla}\Phi. \tag{E7}$$

The reference mass cancels. Different compositions have the same leading free-fall acceleration when their complete energies and inertias obey the same admitted correspondence. This makes the instantaneous-rest algebra in Ref. [1] Section 7.3 explicit.

The cancellation does not require equal binding energies or response amplitudes. It requires the same environmental factors for each complete test body. A physical system's realization of that common response is distinct from the algebra of the scaling theorem.

E.2 The calibrated weak source profile

For the calibrated weak additive source application, [1, Section 7] and [2, Section 5] identify the conservative interaction with complete material storage. With source density $\rho(\mathbf{x}')$ and fixed reference P_0,

$$U(\mathbf{x}) = -G_0 m_0 \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}\, d^3x', \qquad \Pi(\mathbf{x}) \simeq 1 + \frac{2U(\mathbf{x})}{m_0 c_0^2}. \tag{E8}$$

Here $U = E_0(f_s - 1)$ and $\Pi = f_s^2$ give the second expression to first order. The mass-proportional strength is calibrated with measured G_0 in this application. Spherical shell integration outside the source gives

$$\delta_P(r) \simeq \frac{2G_0M}{rc_0^2}, \qquad \frac{d\Pi}{dr} \simeq \frac{2G_0M}{c_0^2r^2}, \qquad \frac{\Delta\nu}{\nu} \simeq \frac{g\Delta h}{c_0^2}. \tag{E9}$$

The absorption deficit of the fixed dipole example and this equilibrated source profile have different definitions and radial dependence. The latter follows from the calibrated interaction energy and the material storage theorem; it is not obtained by substituting the dipole absorption ratio for Π. The Earth numbers in the main article use the specified source benchmark in Appendix J.

Appendix F Construction of the propagation Hamiltonian

The internal Hamiltonian in Appendix C describes the scaled binding and clock states constructed from the response. The Hamiltonian here describes the center of mass of a complete small body in the resulting source profile. Its rest-storage term comes from Appendix E. For local propagation use the Lorentz energy–momentum relation retained in [1, Sections 6.1 and 8.2] and [5, Section 4]. The material clock and ruler conversions give

$$E_{\mathrm{loc}}^2 = E_0^2 + c_0^2p_{\mathrm{loc}}^2, \qquad E = f_sE_{\mathrm{loc}}, \qquad p_{\mathrm{loc}} = b_sp. \tag{F1}$$

Energy is conjugate to time and momentum to position in the matter-wave phase. Hence the clock and ruler conversions fix these factors; substituting them into the local dispersion gives (F2). This identifies the propagation input and shows explicitly how the preceding OQG response enters the complete-body Hamiltonian.

Proposition 2. The stationary environmental rate and the local motion factor enter a single clock law. This follows from the complete-body Hamiltonian and the phase difference of two internal levels transported along the same ideal-clock path. No extra acceleration-dependent rate factor is introduced.

Sources: Ref. [1] Section 8.2, source Eq. (97); Ref. [5] Section 4, source Eq. (11). In a static isotropic environment retain the independent environmental factors:

$$H(\mathbf{x}, \mathbf{p}) = f_s\sqrt{E_0^2 + c_0^2b_s^2\mathbf{p}^2}. \tag{F2}$$

This follows from the local Lorentz dispersion $E_{\mathrm{loc}}^2 = E_0^2 + c_0^2\mathbf{p}_{\mathrm{loc}}^2$, with $E = f_sE_{\mathrm{loc}}$ and $\mathbf{p}_{\mathrm{loc}} = b_s\mathbf{p}$. Its position dependence supplies the environmental interaction; its momentum dependence supplies motion and inertia. These are derivatives of the same energy function.

The prescribed source factors are held fixed during momentum differentiation. Inserting a velocity-dependent receiver factor into them and then treating them as momentum independent would change the calculation and double-count response.

F.1 Transition phase along the moving path

Sources and expanded algebra: Ref. [1] Section 6.2, source Eq. (86a), and Section 8.2; Ref. [5] Section 4, source Eqs. (13)–(17).

Differentiate Eq. (F2). Write $D = \sqrt{E_0^2 + c_0^2 b_s^2 \mathbf{p}^2}$, an energy introduced only to shorten this derivation. The group velocity and local velocity are

$$\mathbf{v} = \frac{\partial H}{\partial \mathbf{p}} = \frac{f_s c_0^2 b_s^2 \mathbf{p}}{D}, \qquad \mathbf{V} = \frac{\mathbf{v}}{f_s b_s}. \tag{F3}$$

Define $\beta = |\mathbf{V}|/c_0$, $s = \sqrt{1-\beta^2}$ and $\gamma_{\mathrm{loc}} = 1/s$. Squaring the local velocity in Eq. (F3) gives $s = E_0/D$. Consequently,

$$H = \frac{f_s E_0}{s}, \qquad H - \mathbf{p}\cdot\mathbf{v} = f_s \frac{E_0^2}{D} = f_s E_0 s. \tag{F4}$$

The quantum phase action satisfies $dS = \mathbf{p}\cdot d\mathbf{x} - H dt$. Along the clock trajectory, $d\mathbf{x} = \mathbf{v} dt$, so the positive magnitude of its phase rate contains $H - \mathbf{p}\cdot\mathbf{v}$, not H alone.

For a clock transition, compare two internal levels transported along the same ideal-clock path. Each level has its corresponding complete rest energy and momentum at the same velocity. Their phase difference in Eq. (F4) gives

$$d\theta_{nm} = \frac{\Delta E_{nm,0}}{\hbar} f_s s\, dt, \qquad d\tau \equiv \frac{d\theta_{nm}}{\Delta E_{nm,0}/\hbar} = f_s s\, dt. \tag{F5}$$

Here θ_{nm} is the positive transition-phase advance in radians. Using the level difference avoids identifying a whole body's rest-energy frequency with a particular clock transition. The system-specific gap cancels.

The common-path condition is essential. At a given local velocity, level n has $E_{\mathrm{loc},n} = E_{n,0}/s$ and $p_{\mathrm{loc},n} = E_{n,0}V/(c_0^2 s)$. Subtracting $p_{\mathrm{loc},n}V$ leaves $E_{n,0}s$, so the difference between levels is $\Delta E_{nm,0}s$. Equal velocity generally requires different momenta for the two rest energies. Subtracting energies at one common momentum would describe a different preparation.

The general result is

$$F_{\mathrm{clk}} \equiv \frac{d\tau}{dt} = f_s \sqrt{1 - \frac{\mathbf{v}^2}{f_s^2 b_s^2 c_0^2}} = \frac{f_s}{\gamma_{\mathrm{loc}}}. \tag{F6}$$

At rest this reduces to f_s. In a local stationary laboratory, division by $dT = f_s dt$ gives $d\tau/dT = s$. These are limits of one formula.

In Ref. [5]'s matched moving-state interpretation, source Eq. (17), the receiver's supporting-exchange ratio relative to its stationary state in the same physical source environment B is

$$\frac{P(v \mid B)}{P(0 \mid B)} = s^2. \tag{F7}$$

Ref. [5] applies the supporting-power theorem to the clock factor obtained from the dispersion law. It does not claim an independently evaluated moving two-body scattering kernel. The stationary environmental factor and this receiver comparison have different references; the Lorentz factor must not be multiplied in again as another slowing mechanism.

F.2 Dispersion slope curvature and clock phase

For the static isotropic source state, take one momentum direction and write $H = f_s\sqrt{E_0^2 + c_0^2 b_s^2 p^2}$, with f_s, b_s independent of this test-body momentum. Holding position fixed and differentiating twice gives

$$\begin{aligned} \frac{\partial H}{\partial p} &= \frac{f_s c_0^2 b_s^2 p}{\sqrt{E_0^2 + c_0^2 b_s^2 p^2}}, \\ \frac{\partial^2 H}{\partial p^2} &= \frac{f_s c_0^2 b_s^2 E_0^2}{(E_0^2 + c_0^2 b_s^2 p^2)^{3/2}}, \qquad \left.\frac{\partial^2 H}{\partial p^2}\right|_{p=0} = \frac{f_s b_s^2}{m_0}. \end{aligned} \tag{F8}$$

The first derivative is group velocity, and the reciprocal of the second at rest is the comparison-coordinate inertia $m_{\mathrm{cmp}} = m_0/(f_s b_s^2)$. In local stationary standards, $E_{\mathrm{loc}} = \sqrt{E_0^2 + c_0^2 p_{\mathrm{loc}}^2}$, so $dE_{\mathrm{loc}}/dp_{\mathrm{loc}} = V$ and $d^2E_{\mathrm{loc}}/dp_{\mathrm{loc}}^2 = s^3/m_0$. With applied force $F_{\mathrm{app}} = dp_{\mathrm{loc}}/dT$ in a uniform region, this proves $F_{\mathrm{app}} = m_0 a/s^3$. It is the longitudinal response; the general multidimensional inertia follows from the momentum Hessian.

For phase convention $\psi \propto \exp[i(pX - ET)/\hbar]$, the positive accumulated phase along the path is $-\Delta S/\hbar$. Along $dX/dT = V$, its rate is $(E_{\mathrm{loc}} - p_{\mathrm{loc}}V)/\hbar$. The tangent to the dispersion curve has slope V and energy-axis intercept

$$E_{\mathrm{loc}} - p_{\mathrm{loc}}\frac{dE_{\mathrm{loc}}}{dp_{\mathrm{loc}}} = \frac{E_0^2}{E_{\mathrm{loc}}} = E_0 s, \qquad \frac{dN}{dT} = \frac{\Delta E_0}{2\pi\hbar}s. \tag{F9}$$

For the last equality, take the difference between internal levels following the same velocity, with rest-energy difference ΔE_0. Dividing the cycle count by the reference transition frequency gives $d\tau = s\,dT$. In the stationary source profile, $dT = f_s dt$ restores $d\tau = f_s s\,dt$. This calculation explains the tangent-intercept statement in the main text and supplies the same clock law as Appendix F.1. It uses the locally Lorentzian matter-wave dispersion specified in Ref. [1, Section 6] and Ref. [5, Section 4].

Appendix G Proofs for motion and receiver power ratios

Source: Ref. [5] Section 5, source Eqs. (19)–(23). Let laboratory A move at constant velocity V along the X direction of local laboratory B. Use B's coordinates (T, X) and A's coordinates (t', x'), with coincident origins at $T = t' = 0$. Define $\beta = V/c_0$, $s = \sqrt{1-\beta^2}$ and $\gamma = 1/s$.

Let λ be the longitudinal length factor and η_{sync} the synchronization coefficient, in seconds per metre. Homogeneity permits

$$x' = \frac{X - VT}{\lambda}, \qquad t' = sT + \eta_{\mathrm{sync}}x'. \tag{G1}$$

At A's origin, $x' = 0$, this already gives its clock rate $dt' = s\,dT$. Require both laboratories to measure the same local light speed using symmetric light-signal synchronization. Insert $X = +c_0T$ and $X = -c_0T$ into Eq. (G1), with $x' = +c_0t'$ and $x' = -c_0t'$, respectively:

$$1 - c_0\eta_{\mathrm{sync}} = \frac{s\lambda}{1-\beta}, \qquad 1 + c_0\eta_{\mathrm{sync}} = \frac{s\lambda}{1+\beta}. \tag{G2}$$

Adding gives $2 = 2s\lambda/(1-\beta^2)$, hence $\lambda = s$. Subtracting gives $\eta_{\mathrm{sync}} = -V/c_0^2$. Substitution produces

$$x' = \gamma(X - VT), \qquad t' = \gamma\left(T - \frac{VX}{c_0^2}\right). \tag{G3}$$

This is the Lorentz comparison derived in Ref. [5]. Its synchronization term explains separated readings. It is not a second clock reduction to multiply by the s already used at A's origin.

The inverse comparison has $V \to -V$. On B's origin, $X = 0$, Eq. (G3) gives $t' = \gamma T$, or $dT/dt' = s$. Each laboratory assigns the other moving clock the same fractional rate using its own simultaneity. They compare different pairs of separated events; the statements do not disagree about one shared reunion measurement.

Let ω_B and $\mathbf{k}_B$ be an emitted field component's frequency and wavevector in B, and θ its angle to the velocity. Sampling its phase along A gives

$$\omega_A = \gamma(\omega_B - \mathbf{k}_B \cdot \mathbf{V}) = \gamma\omega_B(1 - \beta\cos\theta). \tag{G4}$$

The second equality applies to a freely propagating component with $|\mathbf{k}_B| = \omega_B/c_0$. Received Doppler frequency includes direction and propagation. It is not itself the total supporting-power ratio or a direct comparison of accumulated times. Ref. [2] Section 4, source Eqs. (12)–(13), retains amplitudes, polarizations and correlations with these frequency transformations.

G.1 Applied work and accumulated time

Source: Ref. [5] Section 4, source Eqs. (12)–(18). Work locally where the source profile can be treated as uniform. Let $V(T)$ be one-dimensional velocity, $a = dV/dT$ the coordinate acceleration, and $F_{\text{app}} = dp_{\text{loc}}/dT$ the applied force. The local energy and momentum are

$$E_{\text{loc}} = \frac{E_0}{s}, \qquad p_{\text{loc}} = \frac{m_0 V}{s}, \qquad W_{\text{kin}} = E_0\left(\frac{1}{s} - 1\right). \tag{G5}$$

Differentiate $s^2 = 1 - V^2/c_0^2$ and the momentum in Eq. (G5):

$$\frac{ds}{dT} = -\frac{Va}{c_0^2 s}, \qquad F_{\text{app}} = \frac{m_0 a}{s^3}, \qquad \frac{dE_{\text{loc}}}{dT} = VF_{\text{app}}. \tag{G6}$$

Positive work can increase total energy while reducing phase advance per unit laboratory time. Equation (F4) identifies the phase combination. For any specified acceleration history satisfying the ideal-clock conditions,

$$\tau(T_2) - \tau(T_1) = \int_{T_1}^{T_2} \sqrt{1 - \frac{V(T)^2}{c_0^2}}\, dT. \tag{G7}$$

Acceleration changes the velocity history; it is not another independent multiplicative clock-rate factor. When applied force vanishes in the uniform region, V and s remain constant, so the altered comparison continues during inertial coasting.

As an exact integration check take constant positive proper acceleration a_0, starting from rest at $T = \tau = 0$. In one dimension $a_0 = a/s^3$. Integrating $d(V/s)/dT = a_0$ gives

$$V(T) = \frac{a_0 T}{\sqrt{1 + (a_0 T/c_0)^2}}, \qquad s(T) = \frac{1}{\sqrt{1 + (a_0 T/c_0)^2}}. \tag{G8}$$

Integrating Eq. (G7) then yields

$$\tau(T) = \frac{c_0}{a_0}\operatorname{arsinh}(a_0 T/c_0), \qquad V(\tau) = c_0 \tanh(a_0 \tau/c_0). \tag{G9}$$

These follow from Ref. [5]'s motion equations within the stated ideal-clock approximation. Here arsinh is the inverse hyperbolic sine.

An actively changed internal scale has an additional preparation requirement. For exact following of $U_{b(t)}$, Ref. [6] Section IX, source Eq. (58), gives $H_{\text{track}} = H_{b(t)} + i\hbar \dot{U}_{b(t)} U_{b(t)}^{\dagger}$. Thus a rapidly changing stationary scale cannot simply be inserted into Eq. (C2) while ignoring its time derivative. These comparisons use the ideal-clock or adiabatic conditions stated above.

G.2 Supporting power and kinetic work

The OQG moving-receiver comparison in [5, Section 4, Eq. (17)] applies the supporting-power theorem to the clock factor found from this energy–momentum and phase calculation. With the same receiver, source environment B, channels, and source time for both powers, the work relation in (G5) gives

$$\frac{P(V \mid B)}{P(0 \mid B)} = s^2 = 1 - \frac{V^2}{c_0^2} = \left(1 + \frac{W_{\text{kin}}}{E_0}\right)^{-2}. \tag{G10}$$

The reciprocal clock-and-ruler construction gives longitudinal factor s and transverse factors one. This is the directional moving-receiver correspondence of [5], not an isotropic change of the stationary source factors. It connects the work, phase, and supporting-power results; it is not a second momentum force to add to the Hamiltonian.

G.3 Exact following of a changing internal scale

The tracking identity cited in G.1 can be checked directly [6, Section IX, Eq. (58)]. Let $U(t) = U_{b(t)}$ be a differentiable unitary dilation and let the reference state obey $i\hbar\,\partial_\theta\psi_0 = H_0\psi_0$. Define a reference evolution parameter $\theta(t)$ with $\dot{\theta} = f(t) > 0$ and a target state $\psi(t) = U(t)\psi_0\big(\theta(t)\big)$. The chain rule gives

$$\begin{aligned} i\hbar\dot{\psi} &= \left[f(t)U(t)H_0U(t)^\dagger + i\hbar\dot{U}(t)U(t)^\dagger\right]\psi, \\ H_{\mathrm{track}}(t) &= H_{\mathrm{map}}(t) + i\hbar\dot{U}(t)U(t)^\dagger, \qquad H_{\mathrm{map}} = fUH_0U^\dagger. \end{aligned} \tag{G11}$$

Differentiating $UU^\dagger = \mathbb{I}$ shows that $\dot{U}U^\dagger$ is anti-Hermitian, so $i\hbar\dot{U}U^\dagger$ is Hermitian on the admitted mapped domain. Its units are energy. It vanishes for constant dilation, reducing (G11) to the stationary scaling generator. The derivation assumes that the required operators and their time derivatives act on the retained states.

Equation (G11) is a sufficient ideal Hamiltonian for following the prescribed internal state. It does not demonstrate a device that can supply that control, or calculate its dissipation and source cost. Ordinary clock acceleration in G.1 instead uses the specified trajectory and ideal-clock conditions. The tracking requirement applies when a changing internal scale is actively prescribed.

Appendix H Proof of local laboratory equivalence

Proposition 3. A rigid accelerating laboratory and a stationary supported gravitational laboratory have the same floor-to-ceiling clock comparison to first order in their proper height when the floor accelerations agree. This is local clock equivalence under the standing assumptions; it does not identify the two physical source environments.

Expanded coordinate consequence: start from the local clock law of Ref. [1] Section 6.2 and Ref. [5] Sections 4–5. The construction supplies the laboratory comparison without introducing an additional response kernel.

Use inertial coordinates (T, Z), with acceleration along positive Z. Let ζ label height in a laboratory whose fixed-height observers maintain constant proper separation. Let η be elapsed time on its floor clock at $\zeta = 0$. The floor has constant proper acceleration $a_0 > 0$. Define $\ell_a = c_0^2/a_0$, a length, and $u = a_0\eta/c_0$, dimensionless.

Choose the worldlines

$$c_0T = (\ell_a + \zeta)\sinh u, \qquad Z = (\ell_a + \zeta)\cosh u - \ell_a. \tag{H1}$$

At $\eta = 0$ the laboratory is instantaneously at rest in the inertial coordinates. Differentiation gives

$$c_0 dT = \sinh u\, d\zeta + (\ell_a + \zeta)\cosh u\, du. \tag{H2}$$

$$dZ = \cosh u\, d\zeta + (\ell_a + \zeta)\sinh u\, du. \tag{H3}$$

The local inertial clock law is $c_0^2 d\tau^2 = c_0^2 dT^2 - dZ^2$ for this one-dimensional motion. Subtract the squares in Eqs. (H2)–(H3). Cross terms cancel and $\cosh^2 u - \sinh^2 u = 1$, giving

$$d\tau^2 = N_a(\zeta)^2 d\eta^2 - \frac{d\zeta^2}{c_0^2}, \qquad N_a(\zeta) = 1 + \frac{a_0 \zeta}{c_0^2}. \tag{H4}$$

The lapse N_a is dimensionless and positive for $\zeta > -\ell_a$. Transverse motion adds $-(dX^2 + dY^2)/c_0^2$ to the right. These equations describe the same local inertial clock law in accelerating coordinates.

For a clock fixed at ζ, set $d\zeta = 0$. If the ceiling is proper height h_c above the floor,

$$d\tau_\zeta = N_a(\zeta) d\eta, \qquad \frac{d\tau_{\text{ceiling}}}{d\tau_{\text{floor}}} = 1 + \frac{a_0 h_c}{c_0^2}. \tag{H5}$$

The proper acceleration at height ζ is $a(\zeta) = a_0 / N_a(\zeta)$. Different heights require different proper accelerations to preserve fixed proper separation. Giving every point the same proper acceleration specifies a different motion.

H.1 Supported gravitational laboratory

Source result and expansion: Ref. [3] Section 4, source Eq. (17), and Ref. [1] Sections 5.1, 6.2 and 7.3. Let ℓ be proper upward distance measured by stationary local rulers. Define the gravitational clock lapse relative to the floor:

$$N_g(\ell) = \frac{f_s(\ell)}{f_s(0)}. \tag{H6}$$

The holding force follows from the same complete energy $f_s E_0$. An increment of local work $F_{\text{hold,loc}}\, d\ell$ converts to comparison energy by f_s. Therefore

$$f_s F_{\text{hold,loc}}\, d\ell = E_0\, df_s, \qquad F_{\text{hold,loc}} = E_0 \frac{d\ln f_s}{d\ell}. \tag{H7}$$

Dividing by local inertia $m_0 = E_0/c_0^2$ gives the upward support acceleration. Define its floor value as g_0:

$$a_{\text{hold}}(\ell) = c_0^2 \frac{d\ln f_s}{d\ell}, \qquad g_0 = a_{\text{hold}}(0). \tag{H8}$$

The complete energy cancels again. Taylor expansion about the floor gives

$$N_g(\ell) = 1 + \frac{g_0 \ell}{c_0^2} + O(\ell^2). \tag{H9}$$

The remainder's coefficient has units inverse metres squared and includes higher spatial variation of the lapse. Choose $a_0 = g_0$ and compare Eq. (H9) with Eq. (H4). The supported gravitational and accelerating laboratories then give

$$\frac{d\tau_{\text{ceiling}}}{d\tau_{\text{floor}}} = 1 + \frac{g_0 h_c}{c_0^2} + O(h_c^2). \tag{H10}$$

This is the clock form of local equivalence. Both calculations use the same clock phase law and local inertia. It is not equality between every finite gravitational field and every accelerating laboratory. Earth's spatial variation and tidal response remain at the next relevant orders.

The next height coefficient makes the local restriction explicit. Since $N_g{}' = N_g a_{\text{hold}}/c_0^2$, differentiation gives $N_g{}''(0) = g_0^2/c_0^4 + a_{\text{hold}}{}'(0)/c_0^2$, with primes denoting proper-height derivatives. In the rigid rocket, $a' = -a^2/c_0^2$, so this coefficient vanishes and its lapse is exactly linear. A supported source profile need not satisfy that cancellation. Matching floor acceleration therefore fixes the first-order clock comparison, not the full finite-height profile.

Meaning of the common description. The accelerating lapse N_a arose by relabelling events in a uniform region. It is not, by that fact alone, a new source-produced value of A/A_0 or a new stationary supporting equilibrium. Equations (D5)–(D7) apply to the corresponding physical states and standards defined there. Transforming a field description requires transforming its response and correlations together. Substituting N_a^2 for an intrinsic exchange ratio without that construction would add a claim not proved by the cited papers.

For the gravitational laboratory, Appendices A–E identify the scattering, storage, and source response used in the calculation. For the rocket, Appendix G supplies the work, motion, and receiver comparison. The shared material clock law relates the two configurations locally.

H.2 The supporting power gradient

The gravitational side can be written directly in OQG power variables. Substitute $f_s = \sqrt{\Pi}$ into (H8). For proper upward distance ℓ and fixed reference normalization,

$$a_{\mathrm{hold}}(\ell) = \frac{c_0^2}{2}\frac{d\ln\Pi}{d\ell}, \qquad \frac{\nu(\ell)}{\nu(0)} = \sqrt{\frac{\Pi(\ell)}{\Pi(0)}}. \tag{H11}$$

At the floor $a_{\mathrm{hold}}(0) = g_0$, so expansion gives the power gradient that produces the local clock comparison:

$$\frac{\Pi(h)}{\Pi(0)} = 1 + \frac{2g_0 h}{c_0^2} + O(h^2), \qquad \frac{\nu(h)}{\nu(0)} = 1 + \frac{g_0 h}{c_0^2} + O(h^2). \tag{H12}$$

Equations (H5) and (H12) agree in the leading clock term when $a_0 = g_0$. The gravitational power ratio belongs to the physical source-supported states. The rocket comparison is calculated along its accelerated material paths; the accelerating coordinate lapse alone is not a calculation of a new stationary ambient power profile. Both comparisons use the same material clock law and inertia.

Appendix I Proofs for free fall and clock histories

Source: Ref. [1] Sections 6.3–6.4, source Eqs. (87)–(90). Remove applied support. The Hamiltonian determines the trajectory through $\dot{\mathbf{x}} = \partial_{\mathbf{p}} H$ and $\dot{\mathbf{p}} = -\partial_{\mathbf{x}} H$.

From Eq. (F6), the clock and ruler comparisons can be recorded as

$$c_0^2 d\tau^2 = f_s^2 c_0^2 dt^2 - b_s^{-2} d\mathbf{x}^2. \tag{I1}$$

With $x^0 = c_0 t$, define the bookkeeping matrix $g_{\mu\nu} = \mathrm{diag}(-f_s^2, b_s^{-2}, b_s^{-2}, b_s^{-2})$. Greek indices run over time and three spatial coordinates; repeated indices are summed. Its inverse is $g^{\mu\nu}$. The phase equation is

$$g^{\mu\nu} p_\mu p_\nu + m_0^2 c_0^2 = 0, \qquad p_0 = -H/c_0. \tag{I2}$$

Multiplying by $f_s^2 c_0^2$ recovers Eq. (F2). The matrix records propagation already specified by the Hamiltonian.

Let λ_{aff} be an affine trajectory parameter and define $\mathcal{H} = (g^{\mu\nu} p_\mu p_\nu + m_0^2 c_0^2)/2$. Its characteristic equations give

$$\frac{dx^\mu}{d\lambda_{\mathrm{aff}}} = g^{\mu\nu} p_\nu, \qquad \frac{dp_\mu}{d\lambda_{\mathrm{aff}}} = -\frac{1}{2}(\partial_\mu g^{\alpha\beta}) p_\alpha p_\beta. \tag{I3}$$

Insert $p_\mu = g_{\mu\nu} dx^\nu / d\lambda_{\text{aff}}$ into the second equation and differentiate:

$$\frac{d^2 x^\mu}{d\lambda_{\text{aff}}^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\lambda_{\text{aff}}} \frac{dx^\beta}{d\lambda_{\text{aff}}} = 0. \tag{I4}$$

Here ∂_μ differentiates x^μ and $\Gamma^\mu_{\alpha\beta} = g^{\mu\nu}(\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})/2$. These coefficients summarize how comparison standards vary. No second force has been added. Ref. [1] establishes this characteristic relation on the common branch; the same algebra applies to the two-factor Hamiltonian of its Section 8.2.

For a massive clock, proper time is an affine parameter up to constant rescaling. Equation (I4) states zero proper acceleration: a released proof mass and accelerometer casing have no support load between them. Ideal test bodies with the same initial position and velocity obey the same trajectory law, independent of composition within the shared branch.

In a sufficiently small freely falling laboratory, local clocks and rulers give $d\tau^2 = dT_{\text{ff}}^2 - d\mathbf{X}_{\text{ff}}^2 / c_0^2$, up to tidal terms. The subscript ff denotes freely falling coordinates. This is the other part of equivalence. It does not require the clock's comparison rate relative to a distant supported laboratory to return to one.

I.1 Tidal limits

The equivalence statement retains spatial variation that cannot be removed throughout an extended laboratory. Ref. [1] Section 7.3 gives, in the weak-field limit,

$$\Delta a^i = -\sum_j \frac{\partial^2 \Phi}{\partial x^i \, \partial x^j} \xi^j. \tag{I5}$$

Here $\boldsymbol{\xi}$ is the small separation between neighbouring freely falling bodies, Δa^i their relative acceleration, and i, j spatial component indices. A laboratory is locally inertial to the required accuracy when these effects are negligible over its size and duration. Zero support acceleration does not remove the source's influence or the accumulated clock difference.

I.2 Accumulated readings on specified histories

1. **Choose the reference.** Specify the clock defining t, the coordinate chart, and the events at which readings will be compared. If two clocks start together, record their common initial event and readings.

2. **Choose the source profile.** Obtain $f_s(\mathbf{x})$ and $b_s(\mathbf{x})$ from the selected OQG source application. Use $f_s = b_s$ only where the common branch is justified. A given A/A_0 fixes $f_s b_s$; it does not generally determine both factors separately.

3. **Choose the motion.** For free fall, solve the source Hamiltonian. For driven motion, include the applied force and physical trajectory. For an accelerating laboratory, use its explicit worldlines or Appendix H's coordinates.

4. **Convert the velocity.** Calculate $\mathbf{V} = \mathbf{v}/(f_s b_s)$, then $s = \sqrt{1 - \mathbf{V}^2/c_0^2}$ at every point.

5. **Integrate the clock once.** For clock j on path $\mathbf{x}_j(t)$, compute

$$\Delta\tau_j = \int_{t_1}^{t_2} f_s\left(\mathbf{x}_j(t)\right) \sqrt{1 - \frac{|\dot{\mathbf{x}}_j(t)|^2}{f_s^2 b_s^2 c_0^2}}\, dt. \tag{I6}$$

All factors in the integrand are evaluated on that same path. The index j labels a trajectory, not a universal frequency or watt baseline. This expression uses the static isotropic source approximation; rotating or more general time-dependent environments require their corresponding complete propagation law.

6. **Compare the readings.** At reunion take $\Delta\tau_2 - \Delta\tau_1$. For separated clocks specify synchronization or signal transfer. Received frequency includes propagation and Doppler factors; it is not automatically the ratio of accumulated times.

7. **Audit the response interpretation.** If quoting supporting powers, state the reference receiver, time standard and mapped environment. If applied forces or control perturb the clock internally, include those effects or state the ideal-clock approximation.

I.3 Weak field limit

For $|\Phi|/c_0^2 \ll 1$ and $|\mathbf{v}|/c_0 \ll 1$, expand Eq. (F6) to first retained order. Differences between local and comparison velocities enter beyond this leading expression:

$$\frac{d\tau}{dt} \simeq 1 + \frac{\Phi}{c_0^2} - \frac{\mathbf{v}^2}{2c_0^2}. \tag{I7}$$

The neglected dimensionless orders are Φ^2/c_0^4, $\Phi\mathbf{v}^2/c_0^4$ and $\mathbf{v}^4/c_0^4$. For two paths with the same comparison-time endpoints,

$$\Delta\tau_2 - \Delta\tau_1 \simeq \frac{1}{c_0^2}\int_{t_1}^{t_2}[\Phi_2 - \Phi_1 - \frac{\mathbf{v}_2^2 - \mathbf{v}_1^2}{2}]\,dt. \tag{I8}$$

The gravity and velocity terms can reinforce or offset each other. If one clock remains at rest at reference potential Φ_0 and the other satisfies $\mathbf{v}^2 = 2(\Phi - \Phi_0)$, the leading rate difference vanishes. A separate force calculation must establish that path; a clock-rate equality does not prescribe propulsion.

I.4 Released clocks and power profile gradients

The following results are consequences of the static Hamiltonian and clock law already proved in Appendices F and I. Take a freely moving small body released from rest at x_A, with time-independent source factors, no applied work, and fixed internal reference energy E_0. Let $f_A = f_s(x_A) > 0$ and $\Pi_A = f_A^2$. Equations (F2)–(F4) give a conserved Hamiltonian $H = E_0 f_A$ and the motion factor

$$\frac{E_0 f_s(x)}{s(x)} = E_0 f_A, \qquad s(x) = \frac{f_s(x)}{f_A}, \qquad \frac{V(x)^2}{c_0^2} = 1 - \frac{\Pi(x)}{\Pi_A}. \tag{I9}$$

These relations apply on the accessible unforced portion of the path, where $f_s \le f_A$. The stationary clock at release height records $d\tau_A = f_A\,dt$. Combining this with $d\tau_{\text{fall}} = f_s s\,dt$ gives

$$\frac{d\tau_{\text{fall}}}{d\tau_A} = \frac{f_s s}{f_A} = \frac{f_s^2}{f_A^2} = \frac{\Pi(x)}{\Pi_A}. \tag{I10}$$

The same formula follows for an internal transition by taking the phase difference of levels transported at the common velocity. It uses no isotropic substitution for the moving longitudinal factor. The energy relation selects the motion, while the clock law evaluates the phase along it.

The rates in (I10) use the same static comparison time to select events on the separated upper and falling clocks. The ratio is not the raw frequency received over a moving signal link. A practical comparison must specify that link and its Doppler correction, or complete both histories to reunion using (I6). The fall-only lag in (I11) ends before impact and excludes any later return segment.

For a short weak-field downward fall, let z be downward distance, take constant acceleration magnitude g, and set T to the upper clock time to the retained order. Then $z = gT^2/2$ and the rate relative to the upper clock is $1 - 2gz/c_0^2$. If the final drop is h, the leading flight time and accumulated difference before impact are

$$T_f = \sqrt{\frac{2h}{g}}, \qquad \tau_{\text{fall}} - \tau_A \simeq -\frac{g^2 T_f^3}{3c_0^2} = -\frac{2ghT_f}{3c_0^2}. \tag{I11}$$

At $h = 10\,\text{m}$ and $g = 9.80665\,\text{m}\,\text{s}^{-2}$, the flight time is approximately $1.42809\,\text{s}$, the endpoint fractional rate deficit is 2.18227×10^{-15}, and the accumulated lag is approximately 1.0388 femtoseconds. These are weak-field numerical illustrations, with tidal, finite-size, stopping, and signal-transfer corrections excluded from that specified segment.

Finally, $\ln\Pi = 2\Phi/c_0^2$ to first weak-field order. Substituting this in the relative-acceleration result (I5) yields

$$\Delta a_i \simeq -\sum_j \left(\partial_i \partial_j \Phi\right) \xi_j \simeq -\frac{c_0^2}{2} \sum_j \left(\partial_i \partial_j \ln\Pi\right) \xi_j. \tag{I12}$$

The derivatives use the weak-field stationary spatial comparison coordinates, which agree with local Cartesian distance to the retained order. The identity expresses the leading source tidal response; it is not a replacement for the complete clock-and-ruler propagation law at higher order or in a general time-dependent environment.

Appendix J OQG comparisons and numerical checks

The stationary fractional-frequency law has a direct null test. Compare two different transitions using the same signal-transfer procedure and the same fixed comparison standards at two stationary locations. Their frequency ratio is unchanged by a common environmental factor. Ordinary differential shifts must be controlled before a nonzero residual can be assigned to a failure of universal scaling. This is the ratio criterion already identified in Ref. [4] Section 5, expressed below as a dimensionless residual.

$$D_{jk} = \frac{\dfrac{\nu_j(x_2)}{\nu_k(x_2)}}{\dfrac{\nu_j(x_1)}{\nu_k(x_1)}} - 1 = 0. \tag{J1}$$

The corresponding free-fall test compares two compositions with the same initial position and velocity in the same prescribed field. The complete reference energy cancels between force and inertia in Eq. (E7), and the full common Hamiltonian gives the same trajectory in Appendix I. In the weak-field test

with nonzero common acceleration, the usual dimensionless differential-acceleration residual therefore vanishes.

$$\eta_{jk} = \frac{2|a_j - a_k|}{|a_j + a_k|} = 0. \tag{J2}$$

The accelerations in (J2) are measured along the same specified direction. Equation (J1) tests the common transition response of Ref. [4, Section 5]; (J2) tests the complete-body energy cancellation of Ref. [1, Section 7.3]. Together with the height and work relations in Appendix E, these specify the corresponding OQG observables. The microscopic interpretation is supplied by the response and exchange construction in Appendices A–D.

The supporting-exchange statement has an additional operational requirement. The numerator and denominator must refer to the same receiver channels with their stated time standards, environmental matching, and spectral domain. Electrical supply power, dissipated heat, or one isolated spectral line cannot be substituted for the complete balanced exchange without an explicit measurement model.

These tests have different evidential roles. A nonzero transition or free-fall residual, after ordinary instrument effects are controlled, would test the universal correspondence. A null result is compatible with both OQG and the corresponding relativistic laws. Testing the additional microscopic account requires independent response information: for example, a source strength calculated from independently specified spectra and couplings, or an exchange-sensitive observable with a complete measurement model. Reconstructing supporting power from a clock ratio alone is a consistency inference, not that independent test.

J.1 Clock and motion checks

Use $c_0 = 299792458\ \mathrm{m\,s^{-1}}$ and the illustrative standard acceleration $g_0 = a_0 = 9.80665\ \mathrm{m\,s^{-2}}$. A day means 86400 seconds on the reference clock. The examples below are numerical checks of the stated laws, not new observational datasets.

Check	Input	Result
Supported gravitational clocks	1 m height difference	Leading upper-minus-lower fractional rate $1.09113696722 \times 10^{-16}$.
Accumulated height comparison	1 m for one reference day	Upper clock gains approximately 9.42742 ps.

Check	Input	Result
Accelerating laboratory	Floor acceleration a_0; ceiling 1 m higher	Same fraction $1.09113696722 \times 10^{-16}$ by Eq. (H5).
Inertial coasting	$V = 0.6c_0$ for 10 s of local laboratory time	$s = 0.8$, $\gamma = 1.25$; travelling clock records 8 s.
Corresponding moving support	Same speed; Ref. [5]'s mapped receiver comparison	$P(v \mid B)/P(0 \mid B) = 0.64$; not a heating rate.
Constant proper acceleration	Start at rest; accelerate at a_0 for 10 s of inertial time	Travelling clock lags by approximately 0.178340 ps.

The last result follows from Eq. (G9), or $T - \tau = a_0^2 T^3/(6c_0^2) + O(a_0^4 T^5/c_0^4)$. A picosecond, ps, is 10^{-12} second. These checks use the same law for any ideal clock in the correspondence; no nuclear cutoff frequency is needed for the ratios.

J.2 Scattering and source checks

Calculation	Inputs	Result
Resonant dipole reception, Eq. (A5)	Dimensionless coupling 0.01; separation parameter 1	Reception ratio 0.999245360517; deficit 0.07546395%
Passive material coupling, Eq. (B4)	Equal couplings 0.2; environmental frequency 2; reference material frequency and reduced Planck constant 1	Interaction energy 0.013627779555; contraction load 0.004385153827; imbalance stiffness approximately 0.00330351
Earth source benchmark, Eq. (E9)	Gravitational parameter $3.986004 \times 10^{14}\ \mathrm{m^3/s^2}$; radius 6.3781×10^6 m; reference light speed $299792458\ \mathrm{m/s}$	Surface deficit $1.390704935 \times 10^{-9}$; gradient $2.180437647 \times 10^{-16}$ per metre
Earth clock and work benchmark	Same source inputs; height change 1 m; reference mass 1 kg	Upper clock gains 9.41949 ps per reference day; reversible work 9.79840 J

The dipole value checks a fixed-response scattering calculation. The passive-model values use its three exact positive normal-mode frequencies, subtract the uncoupled ground energy, and differentiate at the reference scale and zero electric–magnetic imbalance. The Earth value uses the calibrated weak source profile. Its acceleration differs slightly from the conventional standard acceleration used in J.1; the two numerical examples retain their stated inputs.

Proof and source map

Appendices	Principal JOATP source locations
A–B Scattering and receiver equilibrium	[1], Sections 2–3; [2], Section 2; [3], Sections 3–5; [4], Sections 2–3
C–D Stable scaling and power	[1], Section 4; [3], Sections 2–3 and 6; [4], Sections 2–3
E–F Work and propagation	[1], Sections 5–7 and 8.2; [2], Sections 3 and 5; [5], Section 4
G–H Motion and local equivalence	[5], Sections 4–5; [1], Sections 5.1, 6.2, and 7.3; [3], Section 4. Hyperbolic construction also follows [7].
I–J Free fall and comparison protocols	[1], Sections 6.3–6.4 and 7.3; [4], Section 5; [5], Section 5

Statements and Declarations

Funding. This work was self-funded by the author. No funds, grants, or other external support were received during the preparation of this manuscript.

Conflict of Interest. The author is the founding editor and publisher of the Journal of Alternative Theoretical Physics. This editorial interest is disclosed. Independent assessment and the journal's conflict-management process are required before acceptance; the handling and decision will be recorded separately.

Author contributions. Todd J. Desiato conceived the OQG program and the material-scaling interpretation, directed the theoretical development, reviewed the calculations and literature, and prepared the manuscript.

Data availability. This theoretical study generated no new experimental or observational datasets. The analytical derivations, assumptions, numerical parameters, and consistency checks are contained in the article.

Code availability. The analytical identities and numerical examples can be reproduced directly from the equations and parameters in the article without proprietary software. No new microscopic source-response simulation is claimed.

Ethics approval and consent to participate. Not applicable. This study involved no human participants, human data, animals, or biological material.

Consent for publication. Not applicable.

Use of generative artificial intelligence. The author used OpenAI ChatGPT and Codex for literature discovery, mathematical cross-checking, numerical evaluation, organization, drafting, and editing. The author is responsible for reviewing the scientific arguments, equations, references, and final text and for the content of the manuscript.

Supplementary information. The analytical derivations and supporting calculations are included in the article. No separate supplementary files accompany this article.

References

[1] Desiato, T. J. Operational Quantum Gravity: From Electromagnetic Power Balance to Gravitational Response. Journal of Alternative Theoretical Physics, Vol. 1, Issue 1, founding collection, manuscript version 2.3, 21 October 2026. https://www.joatp.com/articles/power-balance-gravitational-response

[2] Desiato, T. J. Scattering as the Cause of Gravity in Operational Quantum Gravity. Journal of Alternative Theoretical Physics, Vol. 1, Issue 1, founding collection, manuscript version 1.1, 23 October 2026. https://www.joatp.com/articles/scattering-supporting-power

[3] Desiato, T. J. Matter Wave Scaling and Recoverable Energy Storage in Operational Quantum Gravity. Journal of Alternative Theoretical Physics, Vol. 1, Issue 1, founding collection, manuscript version 1.5, 21 October 2026. https://www.joatp.com/articles/matter-wave-scaling

[4] Desiato, T. J. Nuclear Linear Response Scaling and Gravitational Correspondence in Operational Quantum Gravity. Journal of Alternative Theoretical Physics, Vol. 1, Issue 1, founding collection, manuscript version 1.4, 21 October 2026. https://www.joatp.com/articles/nuclear-response

[5] Desiato, T. J. Correction of the MEGA Propulsion Interpretation and Physical Lessons about OQG. Journal of Alternative Theoretical Physics, Vol. 1, Issue 1, correction manuscript version 0.3, 23 October 2026. https://www.joatp.com/articles/mega-gravity-assist

[6] Desiato, T. J. Warp Drive Constraints, Gravitational Energy Storage, and a Proposed Proper-Time-Matched Flight Experiment. ResearchGate preprint, version 1.2 (2026). https://www.researchgate.net/publication/415198684

[7] Rindler, W. Hyperbolic Motion in Curved Space Time. Physical Review 119, 2082–2089 (1960). https://doi.org/10.1103/PhysRev.119.2082