The “Spacetime” from a Realistic Perspective: On the Compatibility of Maxwell’s Equations with Galilean Transformations

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Abstract

This article constitutes the second part of the series *“Spacetime” from a Realist Perspective*. Continuing the analysis developed in the first part, the main objective of the present work is to examine the compatibility between classical mechanics and electromagnetism, with particular attention to the relationship between Galilean transformations and Maxwell’s equations, an issue traditionally presented as one of the main motivations for the introduction of the Lorentz transformations and the formulation of the Special Theory of Relativity. It is investigated whether the alleged incompatibility between these theoretical structures necessarily arises from physical phenomena or is related to the way in which the electromagnetic and kinematic quantities involved are interpreted. To guide this analysis, principles underlying the adopted approach are initially established. Based on these principles, the concept of the spacetime interval in the Einsteinian relativistic formulation and its invariance under coordinate transformations are examined. Subsequently, the relationship between Galilean transformations and Maxwell’s equations is analyzed, including the invariance of these equations under Galilean transformations and the transformations of the electric and magnetic fields between reference frames in relative motion. In addition, the conventional interpretation of the phenomenon of magnetic induction is examined. This second part remains restricted to the theoretical, mathematical, and conceptual analysis of these issues. The discussion of the main experiments and observations traditionally presented as confirmations of Einsteinian relativity will be developed in the third and fourth parts of the series.

Keywords: Galilean transformations; Minkowski interval; Maxwell’s equations; Relativity; Electromagnetism.

Note to the Reader

As already emphasized in the first part of the series, the present work maintains the approach of prioritizing simple mathematics. This methodological choice does not compromise the logical validity or the physical content of the arguments presented, and aims to prioritize conceptual clarity, the explicit statement of assumptions, and the direct understanding of the fundamental ideas under analysis. The mathematical tools employed are appropriate to the objectives of the work and make it possible to describe the phenomena under consideration with rigor. The adoption of an intermediate mathematical formalism helps make the underlying physical hypotheses transparent, thereby facilitating critical analysis and broadening the scope of the text to readers with university-level training in Physics, Engineering, or related fields. The focus of the article lies on the internal consistency of the arguments, the causal coherence of the interpretations, and the clear articulation between mathematical description and physical meaning, rather than on the exploration of complex mathematical formalisms. In this context, the choice of controlled and explicit mathematics constitutes an epistemological advantage, as it facilitates the reading, discussion, and independent assessment of the proposed ideas.

Reading Part 1 before reading this article is recommended, since some of the concepts and arguments developed in the present work are based on the analysis presented in the preceding part.

1. Introduction

One of the historical issues associated with the development of the Lorentz transformations is the *compatibility between classical mechanics, grounded in Galilean relativity, and Maxwell's electromagnetic theory* [1, 2, 3]. This issue became particularly relevant because, according to the traditional interpretation, Maxwell's equations, in their usual form, are not form-invariant under Galilean transformations [2, 3]. This problem was among the historical antecedents that led to the development of the Lorentz transformations and, subsequently, to the formulation of the Special Theory of Relativity [1, 4, 5].

Maxwell's equations constitute the mathematical core of the classical theory of electromagnetism, describing, in their modern formulation, the relationships between the electric and magnetic fields, the charges and currents that serve as their sources, and their spacetime evolution [2]. Their formulation originated in the works of James Clerk Maxwell [6, 7] on electricity, magnetism, and light, which were subsequently reorganized and developed into the modern form of the four Maxwell equations [8]. The relationship between these equations and coordinate transformations between reference frames is therefore a central issue in the analysis of the compatibility between electromagnetism and classical kinematics. In the traditional relativistic formulation, the non-invariance of Maxwell's equations under Galilean transformations is interpreted as evidence of an incompatibility between Galilean kinematics and the mathematical structure of Maxwell's electromagnetic theory [1, 2]. The Lorentz transformations are then introduced as the transformation structure capable of preserving the form of the electromagnetic equations between inertial reference frames, playing a central role in the construction of the Special Theory of Relativity [1, 2]. This interpretation, however, involves specific assumptions concerning the relationship between electromagnetic fields, the coordinates of reference frames, and the velocities associated with physical phenomena.

The present part of the series revisits this issue on the basis of the physical and mathematical assumptions underlying this interpretation. In continuity with the analysis developed in Part 1, principles that guide the adopted approach are initially established, distinguishing the physical phenomenon, the material systems, and the reference systems used for its description. This distinction makes it possible to examine separately the physical propagation of phenomena in space, the motion of material systems, and the mathematical transformations employed to relate their descriptions in different reference frames. Based on these principles, the concept of the spacetime interval in the Einsteinian relativistic formulation is examined, as well as its invariance under coordinate transformations. At this stage, the aim is to examine the physical and mathematical foundations of these constructions and to distinguish the results that follow from the mathematical structure adopted from those that depend on additional physical assumptions.

2. General Principles for the Description of Physical Phenomena Based on Invariant Physical Relations

2.1. Fundamental Principle of Description

The present approach begins with the distinction between *the relationship of a physical phenomenon to the observer* and *the physical relationships between the agents that constitute the phenomenon itself that is to be described*. An observer may measure a physical phenomenon, but their kinematic relationship with the agents involved in the phenomenon does not, in itself, constitute an element of the observed phenomenon. Thus, the parameters that appear in the description of an interaction should be defined by the *physical relationships between the elements participating in the phenomenon*, and not by the relationship of any of them to the observer's reference system. This distinction will guide both the determination and the interpretation of kinematic quantities, including the spatial coordinates, used in the description of physical phenomena and interactions.

2.1.1. Distinction Between the Observer and the Detection and Measurement Device

The description of a physical phenomenon involves three conceptually distinct elements: the *physical system that produces the phenomenon*, the *detection and measurement device*, hereinafter referred to as the "device," *which interacts with this system to perform the measurement*, and the *observer who interprets or*

records the result of the measurement. The observer, as an external agent who interacts with the "device," follows or interprets the experiment but does not constitute an element of the physical interaction being investigated. Their position, velocity, or state of motion relative to the system under study should not, in itself, be introduced as a parameter of the interaction. The "device," however, has a completely different physical function. For a quantity to be measured, the "device" must physically interact with the system under study [9, 10]. Thus, it is not merely a spectator: it is one of the physical elements involved in the measurement process and interacts directly with the system. Therefore, the "device" should not be confused with the observer. The "device" participates directly in the measurement process; the observer, as an agent external to the physical system under investigation, participates in a distinct physical interaction through which they receive and interpret the result produced by the "device".

Consequence for the Definition of Physical Quantities

The distinction established above is particularly important for the definition of the quantities used in the description of an interaction. If one physical system interacts with another system and this interaction is detected by an instrument, the relevant quantities should be determined by the relationships between the physical elements that actually participate in the interaction. For example, consider the magnet-coil-"device" system. The observer situated in the laboratory may be completely separated from the system. They merely monitor the "device" and observe its indication. The "device," however, is physically coupled to the coil and responds to the physical interaction. The observer's motion relative to the system does not constitute the interaction responsible for the reading. The physical relationships between the magnet, the coil, and the "device," on the other hand, may be essential to the phenomenon and to the measurement. Consequently, when it is stated that a quantity is *measured by the observer*, this does not mean that it is *defined relative to the observer*. The observer may record a quantity whose physical definition is related exclusively to the elements that interact.

The "Device" in the Theoretical Description and in the Measurement Process

The statement that the motion relationship with the observer is irrelevant to the description of the phenomenon does not mean that the elements involved in the measurement are physically irrelevant. The "device" may be indispensable for the measurement of a particular quantity. What is rejected is the attribution of a physical role to the kinematic relationship between the system and the observer as a mere spectator. This distinction between the observer and the "device" raises a question: *If the "device" physically interacts with the system, why does its influence not appear in the equations describing the phenomenon under study?* The answer is that the equations generally describe an idealization of the phenomenon or of the physical system under study, and not the complete measurement process. The absence of the "device" from the equations does not mean that it does not interact with the system [9], but only that its influence is not part of the idealized description. The "device" may therefore be regarded as an external element to the ideal theoretical description, although it is not physically passive during the measurement. It must interact with the system to produce a reading, but its specific characteristics may be deliberately disregarded in the equation describing the phenomenon. This situation is common in physics. In an electrical circuit, for example, the current, voltage, and power of its elements can be calculated ideally without including the influence of the measuring instrument. A voltmeter has finite input resistance and may slightly alter the circuit [9]; nevertheless, this influence is normally neglected in the idealized description. This does not mean that the instrument does not physically interfere, but that its influence is considered secondary for the problem at hand.

The same principle applies to the present approach. The theory provides the ideal value associated with the phenomenon, whereas the experimental reading results from the interaction between the "device" and the system and may undergo small changes arising from the characteristics of the instrument and the experimental conditions [9, 10]. The "device" thus constitutes a physical interface between the phenomenon and the observer, but not necessarily a constitutive agent of the system described by the equation. The observer, in turn, receives or interprets the information produced by the "device". In this way, two fundamental distinctions are preserved: *the "device" physically participates in the measurement process, but does not need to appear as a parameter in the ideal equation of the phenomenon;* and *the observer may be in relative motion with respect to the system. However, this motion, by itself, does not make the observer an agent of the system that produces the phenomenon under study*.

2.2. Measurement of a Quantity and Definition of the Quantity

It is necessary to distinguish two conceptually different situations. A quantity may be ***measured by an observer in a given reference frame*** and, at the same time, ***be physically defined with respect to another element***. The fact that a velocity is measured in reference frame S, for example, does not necessarily mean that it is a velocity defined with respect to the origin of S. This distinction is fundamental to the present approach. If an agent A interacts with an agent B, the observer may use their coordinate system and clocks to determine the relative velocity between the two agents. However, the measured quantity remains a property of the relationship between A and B, and not of the relationship of either of them with the observer. Reference frame S may constitute the system used to represent or determine a quantity without constituting the physical referent of that quantity.

2.2.1. Relational Definition of Kinematic Quantities

The preceding distinction makes it possible to formulate a general principle for the description of physical interactions: *when a kinematic quantity is involved in the characterization of an interaction, its definition should be linked to the physical relationships between the agents participating in that interaction, and not simply to the relationship of one of these agents with the observer*. *In particular, if the relevant quantity is a velocity, it should represent the relative velocity between the physical agents participating in the interaction*.

$$\boldsymbol{u} = \frac{d\boldsymbol{r_{rel}}}{dt} \tag{1}$$

Thus, for two agents A and B, the velocity of A relevant to an interaction with B should be defined by the relation:

$$\boldsymbol{u} = \frac{d(\boldsymbol{r}_A - \boldsymbol{r}_B)}{dt} \tag{2}$$

regardless of the reference frame used by the observer to determine this quantity. The observer may assign different velocities to A and B relative to their own reference frame, but this does not necessarily imply that the velocity relevant to the interaction should be identified with any of these velocities. Thus, a careful distinction must be made between *the reference frame used to measure or represent a quantity and the physical referent with respect to which that quantity is defined*. If the interaction directly involves an agent and a "device", the relevant physical relationship may involve that agent and the "device". In both cases, the observer (external to the interaction) is not introduced into the definition of the quantity simply because they are the one performing the reading. The general formulation of the principle is therefore:

The quantities that characterize an interaction should be defined by the physical relationships between the agents participating in that interaction, and not in relation to an observer. The observer, as an external agent who interacts with the "device", describes or interprets the phenomenon and does not, in itself, constitute one of the agents involved in the interaction. The "device", however, when it physically participates in the measurement, should be considered an element of the interaction process, without this implying that it should be included in the theoretical description of the relationships between the agents of the system. Its participation in the experimental process, however, may introduce perturbations into the system and contribute to uncertainty in the measurement result [9].

This principle will be applied later to electromagnetic interactions, but its formulation does not depend on the electromagnetic nature of the interaction.

Precision in the Definition of Velocities

In a rigorous physical formulation, the expression "velocity of a body" generally constitutes an abbreviation that has complete meaning only when the system or object with respect to which the position of the body (or object in general) is defined has been established. Since velocity is a relative quantity, the statement that

"body A has velocity v" is incomplete when the referent of this velocity is not specified, except when it is unambiguously determined by the context. For two agents A and B, the velocity of A relative to B is given by:

$$\boldsymbol{v}_{A/B} = \frac{d(\boldsymbol{r}_A - \boldsymbol{r}_B)}{dt}. \tag{3}$$

By contrast, the velocity of A relative to an observer O is:

$$\boldsymbol{v}_{A/O} = \frac{d(\boldsymbol{r}_A - \boldsymbol{r}_O)}{dt}. \tag{4}$$

These quantities coincide only under particular conditions, such as when B is at rest relative to observer O. Therefore, when different reference frames are considered, it is not conceptually appropriate to simply state that "the velocity of A is v," since this statement may conceal the reference relationship being used or cause ambiguities in the interpretation and description of physical phenomena.

2.2.2. Spatial Relationships Between the Elements of a Phenomenon

The same relational principle can be applied to spatial quantities. If a phenomenon depends on the spatial relationship between two physical elements, the relevant position should be defined from the relative position between these elements, and not simply from the position of either of them relative to the origin of the observer's coordinate system. This distinction can be clarified by a simpler situation: the electrostatic field. Consider a charge Q located at position x_Q in inertial reference frame S. The electric field produced by it at a point P depends on the spatial relationship between the source charge and the point at which the field is being evaluated. Thus,

$$E = k\frac{Q}{\left(x_P - x_Q\right)^2}$$

and not $\quad E = k\frac{Q}{{x_Q}^2} \quad$ or $\quad E = k\frac{Q}{{x_P}^2}$

The relevant spatial quantity is the relative position or distance between the point at which the field is being evaluated (or the position of a possible test charge) and the position of the source charge, since the interaction under study is established between these agents, and not in relation to the observer. The expression

$$E = k\frac{Q}{{x_P}^2}$$

becomes directly valid when the origin of the coordinate system is defined at the position of the source charge, so that the coordinate used represents the relative position between the source and the point under consideration. Therefore, the coordinate relative to the origin of the observer's coordinate system does not, in itself, constitute a physical variable of the interaction in question. What matters is the spatial relationship between the physical elements involved. The same logic will later be applied to the description of electromagnetic propagation: the coordinates used to represent the phenomenon should be analyzed in terms of the physical relationships that effectively characterize its propagation.

2.3. Invariance of the Kinematic Relations Between the Agents

Let the observers be in inertial reference frames S and S', with S' being a reference frame that moves in the direction of the v axis of S, with velocity x. In this case, by a Galilean transformation, their coordinates are related by the equations

$$x' = x - vt,\ \ y' = y,\ \ z' = z,\ \ t' = t. \tag{5}$$

For two agents A and B that establish an interaction under study,

$$x'_A = x_A - vt \quad \text{and} \quad x'_B = x_B - vt.$$

Consequently,

$$x'_B - x'_A = (x_B - vt) - (x_A - vt).$$

from which it follows that

$$x'_B - x'_A = x_B - x_A$$

Differentiating,

$$\frac{d(x'_B - x'_A)}{dt'} = \frac{d(x_B - x_A)}{dt}$$

and, therefore,

$$u' = u. \tag{6}$$

Thus, the two observers can measure the same relative velocity between the agents that constitute the phenomenon, even though these observers are in relative motion with respect to each other. From this perspective, invariance is not introduced as an additional condition: it results from the relational nature of the quantity itself.

2.3.1. The Principle of Causal Invariance

The preceding principles can be generalized to the causal structure itself of physical interactions. Let a system be constituted by the physical entities

$$S = \{A, B, C, \dots\},$$

and let $R_{AB}, R_{AC}, R_{BC}]$, … denote the physical relationships existing between these entities. A change in the reference frame used by the observer does not alter the entities or the physical relationships that constitute the system. It modifies only the coordinate representation used to describe them. The coordinates assigned to the entities may change,

$$\boldsymbol{r}_A \rightarrow \boldsymbol{r}'_A, \qquad \boldsymbol{r}_B \rightarrow \boldsymbol{r}'_B,$$

but, when these coordinates represent relationships between the physical elements of the system itself, they must describe the same physical relationship:

$$R'_{AB} = R_{AB}.$$

A distinction is therefore established between two levels of description: *coordinates relative to the observer's reference frame* and *quantities defined relative to the physical entities of the system*. The former may change when the reference frame is changed; the latter must preserve the physical property they represent. From the perspective adopted in this work, the requirement of invariance should not be limited to the formal preservation of equations when passing from one reference frame to another. *The fundamental object of invariance is the physical properties of the entities constituting the system and the relationships between these entities*. The invariance of the description must therefore be accompanied by the *invariance of the complete causal structure*.

Consequences of an Inadequate Definition of Quantities

When a quantity that characterizes a physical relationship under observation is defined relative to the observer's reference frame, its transformation may produce a change that does not correspond to an actual change in the system. There then arises a need to introduce mathematical elements intended to recover or preserve the invariance of physical relationships that should have remained invariant from the very definition of the quantities. The approach proposed here seeks to avoid this problem through the prior identification of the entities participating in the phenomenon and of the relationships under study, as well as through the appropriate definition of the relevant quantities.

2.4. Synthesis of the Guiding Principles

The arguments presented in this section establish the conceptual principles that will guide the analyses developed in the following sections. First, a distinction is made between the *physical phenomenon* and *its observation*: the kinematic relationship between a phenomenon and an observer does not constitute an element of the phenomenon. Second, a distinction is made between the *observer* and the *"device"*: the former interprets or records the result of the measurement, whereas the latter participates directly in the process of obtaining the information. Third, a distinction is made between *the reference frame used to measure or represent a quantity* and *the physical referent with respect to which that quantity is defined*. Fourth, it is established that, when a quantity characterizes an interaction between physical agents, it should be *defined on the basis of the physical relationships between these agents*, and not simply on the basis of the relationship of each agent with the observer. Fifth, this relational principle is extended to spatial quantities: when a phenomenon depends on the spatial relationship between physical elements, the description should consider the relative position between these elements, and not simply their position relative to the origin of the observer's coordinate system. Finally, it is maintained that a change in the reference system may modify the mathematical representation of positions and velocities, but should neither introduce new agents into the interaction nor alter the causal structure of the phenomenon. These principles constitute the conceptual basis for the subsequent analyses. Based on them, the following section will examine the concept of the spacetime interval in the Einsteinian relativistic formulation and its invariance under coordinate transformations. Subsequently, these same principles will be applied to the analysis of the relationship between Galilean transformations and Maxwell's equations, to the transformations of the electric and magnetic fields, and, finally, to the interpretation of the phenomenon of magnetic induction.

3. The Spacetime Interval and Its Behavior Under Galilean Transformations

The introduction of Minkowski's geometric formalism [11, 12] constitutes one of the most remarkable conceptual achievements of physics in the early twentieth century. With extraordinary mathematical synthesis, Hermann Minkowski showed that the structure of the Special Theory of Relativity could be represented by means of a four-dimensional geometry, in which the spatial and temporal coordinates become part of a single formal structure. This construction made it possible to give a particularly elegant geometric interpretation to the relationships established by the special theory of relativity and made it possible to formulate, in a unified manner, a quantity that plays a central role in this structure: the *spacetime interval* [13], also referred to as the *interval between two events* [14] or the *Minkowski interval*, among other designations. Consider two events,

$$E_1 = (x_1, y_1, z_1, t_1) \qquad \text{and} \qquad E_2 = (x_2, y_2, z_2, t_2).$$

In Minkowski's formulation [11, 13], the interval between them is defined as

$$s^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2 \tag{7}$$

This quantity plays a role analogous to that of distance in Euclidean geometry [1, 15], although it is defined by a quadratic form with a different signature. One of its fundamental properties, within the structure of the Special Theory of Relativity, is its invariance under Lorentz transformations [1, 14]. The present analysis

does not intend to diminish the importance of this formalism. On the contrary, it recognizes in it a mathematical construction of extraordinary ingenuity. However, the aim is to be *as realistic as possible*. The objective is to critically examine *the physical meaning attributed to the quantities that appear in this construction*, particularly when one intends to compare their invariance under Lorentz transformations with their behavior under Galilean transformations.

3.1. Geometric Origin of the Interval Expression

In terms of conceptual origin, the expression for the spacetime interval originated from the *Euclidean equation of the light sphere centered at the origin* [16], that is:

$$x^2 + y^2 + z^2 = c^2t^2 \tag{8}$$

Originally, this equation is purely Euclidean in space; it does not describe generic events, but rather a specific physical phenomenon — *the propagation of a spherical "shell" of light* [17] —, a sphere that expands with time; it relates exclusively two points: the *point of emission* and the *point occupied by the light wavefront at a given instant after the instant of emission*. For two points along the propagation trajectory of the wavefront, this equation naturally leads to the form

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = c^2(t_2 - t_1)^2 \tag{9}$$

When rewritten as

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2(t_2 - t_1)^2 = 0$$

or

$$0 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2 \tag{10}$$

it directly generates what later came to be called the null spacetime interval [13, 14]. There is no interpretive assumption here: the mathematical object arises from the geometry of the light sphere.

3.1.1. What Minkowski Did Later

Minkowski [16] carried out two additional steps, which are conceptually distinct:

(i) Geometric abstraction: the object

$$s^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2 \tag{11}$$

ceased to be viewed as "the equation of the light sphere" and came to be interpreted as the fundamental metric of spacetime [18].

Is there anything illegitimate about this? Not necessarily. After all, infinitely many functions can be constructed from an equation by means of algebraic manipulations. For example, from the same expression for the interval, one could obtain

$$f_1^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2 - s^2$$

$$f_2^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2 - s^2 - f_1^2$$

and so on.

(ii) Semantic extension: the interval ceased to mean merely a relationship between events connected by light and came to classify timelike separations, spacelike separations and lightlike separation (or null separations) [11, 14]. The quantity thus came to play the role of a general geometric measure between points in spacetime.

This extension is legitimate as a theoretical construction, but it no longer belongs to its physical origin. It is precisely this structure that Hermann Minkowski incorporated into his geometric formalism and, in 1908 [11], presented his newly created formalism as a new language for the Special Theory of Relativity. Some proponents of Special Relativity resort to Minkowski's formalism to provide a foundation for the theory and thus refute criticisms against it, arguing that the equations

$$x^2 + \mathrm{y}^2 + \mathrm{z}^2 = \mathrm{c}^2 t^2 \quad \text{and} \quad x'^2 + \mathrm{y}'^2 + \mathrm{z}'^2 = \mathrm{c}^2 t'^2, \tag{12}$$

obtained from the general form when $x'_c = x_c = 0$, $y'_c = y_c = 0$, $z'_c = z_c = 0$, $t'_0 = t_0 = 0$ are assigned, do not describe two spheres, but rather two simultaneity slices of the same light cone. However, this was not the idea of the original creators of the theory, particularly Einstein [17], who presented them as the equations of an expanding spherical light wavefront that evolves with time, forming infinitely many concentric spheres, with their center fixed at the origin of the coordinate system of the respective observer. There is no objection to the introduction of a new formalism to describe an already established theory. However, if this language does not represent the idea of the original authors, then it constitutes a different theory, rather than the one it is supposed to explain.

3.2. On the Invariance of the Interval in the Relativistic Formulation

Within the framework of the Special Theory of Relativity, the interval is invariant under Lorentz transformations [14]. Einstein referred to this characteristic as a "discovery by Minkowski" [1] and regarded it as proof of the validity of the LT. Indeed, applying the transformations

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{v}{c^2}x\right) \end{aligned} \tag{13}$$

to the expression for the interval, one obtains

$$\mathrm{s}'^2 = \mathrm{s}^2. \tag{14}$$

This invariance constitutes one of the fundamental characteristics of Minkowski geometry [12]. However, the following question arises here: *Can this fact be taken as proof of the validity of the Lorentz transformations as the correct transformations?*

In the present approach, this characteristic, considered in isolation, cannot be regarded as proof of the validity of the LT. It is understood here that it is necessary to carefully examine *the structural conditions under which this invariance is obtained*. In particular, one must distinguish what follows from the nature of the physical phenomenon from what follows from the mathematical structure chosen to represent it.

The fundamental coefficients of the LT were obtained from the equation of the spherical light wavefront under certain conditions [14, 17]. One of these conditions raises concerns.

3.2.1. The Event and the Choice of the Origins of the Coordinate Systems — "Biased" Geometry at the Origin

Consider the emission point of the wave. In the usual construction, the origins of the coordinate systems are chosen [11, 12, 14, 17] so that the emission event is represented by

$$E_0 = (0,0,0,0) \quad \text{in S,} \quad \text{and} \quad E'_0 = (0,0,0,0) \quad \text{in S'.}$$

However, the nature of the emission does not require the spatial point at which the wave is produced to coincide with the origin of any coordinate system. One may consider the same physical point located away from the origin, such as

$$E_0 = (x_0, y_0, z_0, 0),$$

with nonzero spatial coordinates. A conceptual question then arises when one seeks to represent *this same physical event* in both reference frames as occurring at $t_0' = t_0 = 0$. For the Lorentz transformations

$$t_0' = \gamma\left(t_0 - \frac{v}{c^2}x_0\right) \tag{15}$$

If $t_0 = 0$, it follows that

$$t_0' = -\gamma\frac{v}{c^2}x_0 \tag{16}$$

Thus, for $v \neq 0$, the condition $t_0' = 0$ requires $x_0 = 0$. Therefore, the structure of the Lorentz transformations does not allow, for a point located at $x_0 \neq 0$, that this same point be associated with $t_0 = 0$ in S, when in S′ one has $t_0' = 0$.

From this perspective, this constitutes a conceptual problem because, as mentioned above, *nature does not impose that the emission of a wave must occur at the origin of the coordinate systems, nor does it impose that two moving systems must necessarily assign different times to a physical occurrence taking place at a spatial point outside the origins of the coordinate systems when these coincide; that is, nature does not impose*

$$E_0 = (0,0,0,0) \quad \text{and} \quad E_0' = (0,0,0,0)$$

for the emission event. This restriction results from the structure of the Lorentz transformations themselves, which relate spatial and temporal coordinates through a transformation that mixes x and t.

This is not intended to claim that the Special Theory of Relativity is mathematically incapable of representing an event with $t = 0$ and $x \neq 0$ in a reference frame. The point is different: *the Lorentz transformations do not allow the same event located away from the origin to simultaneously have* $t_0 = 0$ and $t_0' = 0$ *in two reference frames in relative motion*. In the interpretation proposed here, this impossibility is understood as a restriction introduced by the structure of the transformations, and not as a restriction imposed by the physical phenomenon itself or by nature.

3.2.2. The Breakdown of Invariance and the "Appeal" to Simultaneity

As discussed in the previous section, the condition $t_0' = t_0 = 0$ and $x_0 \neq 0$ *breaks the invariance of the Minkowski interval under the LT*. In other words, the LT do not admit the emission event $E_0 = (x_0, y_0, z_0, 0)$ and $E_0' = (x_0', y_0', z_0', 0)$ with x_0, y_0, z_0, x_0', y_0', z_0' nonzero. To preserve the invariance of the interval in this case, it is imposed that $t_0' = -vx_0/c^2$ for $t_0 = 0$ and $x_0 \neq 0$, although the proposed physical scenario establishes, on the contrary, that $t_0' = 0$. Thus, the STR preserves the invariance of the relativistic interval at the cost of altering the temporal coordinate assigned to the emission event, accepting that, in a given reference frame, the emission of the wave occurs before or after the instant at which it actually occurs according to the physical conditions of the proposed scenario.

Defenders of the theory could argue that, if the light is emitted at $t = 0$, at the point $x_0 \neq 0$, for the observer in S, this emission *does not occur at* $t' = 0$ *for the observer in S', because the Lorentz transformations allow* $t = t' = 0$ *only for emission from the common origin, that is,* $x_0 = 0$. From this perspective, it could be argued that one cannot have $t = t' = 0$ for an emission outside the origin because the emission event is spatially separated from at least one observer and, therefore, its time is not read locally but reconstructed by remote synchronization using light signals. Since light has a finite and invariant velocity and the emitter is in relative motion, the synchronization procedure would not be symmetric, causing the distant event not to belong to that observer's "now". Thus, a distant event that occurs at $t = 0$ for one reference frame would not occur at $t' = 0$ for another reference frame in relative motion. When the event occurs far away, the observer does not "see" it at the instant at which it occurs; rather, they reconstruct its time by means of light signals, and this procedure depends on their state of motion. The reading of time therefore ceases to be universal. On the other hand, it could be argued that this does not occur when the event takes place at the origin, because the event occurs at the same physical point for both observers, allowing them to read the same local clock directly, without remote synchronization and without signal-propagation delay. From this perspective, there would be no operational ambiguity: only local events could

have a common time for observers in relative motion, because only they would not depend on synchronization at a distance. The relativity of simultaneity would therefore arise precisely from this dependence. In this case, time would be measured locally, rather than inferred from delayed signals or being dependent on synchronization conventions. According to this argument, this would be the only case in which two reference frames could agree on the time of an event. This would not be a convention, but a geometric restriction of Minkowski spacetime, a consequence of the Lorentz transformations.

Indeed, the situation in question is a consequence of the very structure of the Lorentz transformations. However, these arguments do not resolve the problem presented, as will be explained below. It is nevertheless important to consider the following question: *What does the variable t represent in the wavefront equation and in the coordinate transformations?*

From the perspective adopted here, the variable [mathematical expression not visible in the source text], present in the equations describing physical phenomena and laws, does not represent "the time or instant of its observation", nor necessarily "the time or reading of a clock", nor "psychological time". Rather, it represents the objective order of events, that is, the parameter indicating their evolution, independently of the observer or of any measurement process. Two observers in the same reference frame may perceive the signal of the same occurrence at different times. Likewise, two clocks located at the "same point" may display different readings for the same t constitutes the *common objective temporal parameter, whereas clocks provide local readings corresponding to that same t.* The equations representing physical laws describe natural patterns. In the specific case of a light wave, t temporally orders the events associated with its propagation, representing the instant at which an element of the electromagnetic disturbance reaches the spatial position *x*, in the particular case of one-dimensional propagation. According to Einstein himself [17], $t = \tau = 0$ corresponds to the instant at which "the spherical wave" is emitted, and it reaches a point (x, y, z) at time t. Therefore, according to the interpretation in this work, the issue of the simultaneity of events between reference frames does not arise in this context, since it concerns the occurrence of the same emission event, and not the observation of that occurrence by each observer.

In view of these considerations, it could be argued that the Lorentz transformations are the correct coordinate transformations [19], since Galilean transformations would not preserve the invariance of the Minkowski interval and would be incompatible with the laws of electromagnetism [19]. The following section analyzes the behavior of the Minkowski interval under Galilean transformations, leaving the question of the compatibility of these transformations with the laws of electromagnetism for a later stage.

3.3. Event, Spatial Point, and Point of Space in Time

To proceed with this analysis, it is necessary to distinguish the notion of *event* from the notion of *spatial point*. What the relativistic interpretation calls an "event", that is, a "point of the world" [18], represented by the set of spatial coordinates (x, y, z) and temporal coordinate (t) within a single geometric structure [20], spacetime, is understood in the present approach as a *point (x, y, z) of space in time (t)*. The terminological difference is deliberate. In this way, time is recognized as a parameter for ordering occurrences in space, forming, together with space, the *platform of the world*. The ontological distinction between the two is nevertheless recognized. For example, in a reference system, a body may move in only one of the spatial directions while maintaining constant positions in the other two, but time cannot be treated in the same way as a spatial direction, since the motion, state of rest, or any state of physical existence of a body is considered over time. An event, in this conception, is not primarily identified with an abstract four-dimensional coordinate. It corresponds to the occurrence of something *at a given point in space and at a given instant*.

Points of space, in turn, are considered persistent over time. A point P does not come into existence or cease to exist because a reference system moves. Nor are points of space carried along by the motion of systems S and S′. Thus, if a light wave is emitted at P and subsequently reaches other points Q_1, Q_2, these points remain the same spatial points, regardless of the motion of S or S'. Moving systems *do not drag the points of space or drag time*. They merely carry their own coordinate structures with them. Consequently, points of space *may appear* to be moving when described from systems S and S′, although they remain spatially fixed within the common spatial structure. Conversely, the origins of systems S and S′ are moving relative to space. This distinction will make it possible to understand the difference between the quantities

$$x_2 - x_1 \quad \text{and} \quad X_2 - X_1,$$

which will be considered below. The first corresponds to a difference between coordinates assigned by the moving system; the second represents the spatial separation between the persistent points of space themselves. The two may eventually coincide, but they do not necessarily represent the same quantity.

3.3.1. Spatial Points and Coordinates of Moving Systems

Consider again a light wave emitted from the fixed spatial point P. Observers situated in systems S and S′ can describe the points reached by the wavefront in two different ways: (1) *adopting coordinate systems with origins coincident and permanently fixed at point P, independently of the motion of reference frames S and S′*; or (2) *adopting coordinate systems whose origins are fixed, respectively, in reference frames S and S′, thereby following the motion of these reference frames*. The presentation of these two forms of description follows.

First Construction: Coordinate Systems with Origins Fixed at P

This constitutes the most natural method. In this first construction, a coordinate system is established whose origin is fixed at P. Let this system henceforth be designated the *relevant or natural coordinate system* S_R, and let the coordinates of a spatial point in this system be given by (X, Y, Z). In S′, one can analogously construct (X', Y', Z'). A pertinent question then arises: can the Galilean spatial-coordinate transformation

$$x' = x - vt$$

be applied directly to the coordinate X, considering that v is the velocity between systems S and S′? The answer is "no," because the coordinates x and x' do not correspond to the same quantity as X and X' represent. The former represent the position of an object relative to the origins of the moving coordinate systems associated with S and S′, respectively, whereas X and X' represent the position of an object relative to the fixed point P in space. While there is relative motion between x and x' with velocity v, there is no relative motion between X and X', since both systems S_R and S_R') are being used, in this construction, to describe the *same spatial points relative to the same physical origin P*. In this case, the relative velocity between S_R and S_R' is $v_R = 0$. Thus, for these systems, the standard Galilean transformation is

$$X' = X - v_R t = X - (0)t = X, \quad Y' = Y, \quad \mathrm{Z}' = Z, \quad t' = t.$$

$$X' = X, \quad Y' = Y, \quad \mathrm{Z}' = Z, \quad t' = t. \tag{18}$$

Therefore, the relevant coordinates (X, Y, Z) and (X', Y', Z') remain associated with the fixed origin at P and do not move with systems S and S′. Thus, if Q_1 and Q_2 are two spatial points, their relevant coordinates satisfy the relation

$$X_2 - X_1 = X_2' - X_1'. \tag{19}$$

This equality directly expresses the invariance of the distance between the same points in space.

Second Construction: Each Observer Uses the Origin of Their Own Moving System

In the second construction, the observers in S and S′ use their own origins, which move through space with velocities v_p and v_p', respectively. Point P appears to be moving to an observer at rest in S or S′. In reference frame S, the coordinate of P, although P remains physically fixed in space, is

$$x_c(t) = x_0 - v_p t. \tag{20}$$

Similarly, in S′,

$$x_c'(t') = x_0' - v_p' t'. \tag{21}$$

If X represents the position of a spatial point relative to P, and x represents its coordinate in the moving system S, then we have

$$X = x - x_c(t), \tag{22}$$

Similarly, in S′,

$$X' = x' - x'_c(t'). \tag{23}$$

Therefore, the same spatial position can be described directly relative to P, or indirectly through the coordinate assigned by the moving system and the apparent position of P in that system. For two points Q_1 and Q_2, in the second construction we have

$$X_1 = x_1 - x_c(t_1), \qquad X_2 = x_2 - x_c(t_2).$$

Thus,

$$X_2 - X_1 = (x_2 - x_1) - [x_c(t_2) - x_c(t_1)].$$

From equation (20), it is easy to see that

$$x_c(t_2) - x_c(t_1) = -v_p(t_2 - t_1),$$

and therefore this results in the relation

$$X_2 - X_1 = (x_2 - x_1) + v_p(t_2 - t_1).$$

Analogously, in S′:

$$X'_2 - X'_1 = (x'_2 - x'_1) + v'_p(t'_2 - t'_1).$$

Since $x' = x - vt, \quad t' = t$ and $v = v'_p - v_p$, the relation $x'_2 - x'_1 = x_2 - x_1 - v(t_2 - t_1).$

immediately leads to

$$X'_2 - X'_1 = X_2 - X_1. \tag{24}$$

Thus, what varies between the moving systems are the coordinates x and x'; the distance between the spatial points represented by these coordinates (eventually reached by the wavefront) remains the same. It is worth emphasizing that the relevant coordinate systems (X, Y, Z) and (X', Y', Z') are both coincident and have their origins fixed at P. Therefore, they do not move with systems S and S′. In this case, the relative velocity between S_R and S'_R is $v_R = 0$. Thus, a Galilean transformation would result in

$$X' = X - v_R t = X - (0)t = X, \quad Y' = Y, \quad Z' = Z, \quad t' = t.$$

$$X' = X, \quad Y' = Y, \quad Z' = Z, \quad t' = t.$$

3.4. Galilean Transformations and the Difference Between Moving Coordinates in Space

If x_1 and x_2 are considered simply as coordinates, in S, assigned to two spatial points reached by a light wavefront at different instants, and x'_1 and x'_2 as their corresponding coordinates in S′, then, by Galilean transformations,

$$x'_1 = x_1 - vt_1, \qquad x'_2 = x_2 - vt_2.$$

Subtracting the corresponding terms,

$$x'_2 - x'_1 = x_2 - x_1 - v(t_2 - t_1).$$

In the conventional interpretation [21], this relation is normally taken as evidence that the expression

$$c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

or, more precisely, the spacetime interval

$$s^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2$$

is not invariant under Galilean transformations and, consequently, as a reason for introducing the Lorentz transformations. In this work, however, it is proposed that there is an interpretive question that precedes this conclusion. The expression

$$\Delta x = x_2 - x_1$$

should not automatically be identified with the distance between the two points in space at which the events in question occur. *A difference between coordinates belonging to a moving system is not, in general, the same thing as the spatial distance between two persistent points in space.*

The Car Analogy on the Road

This distinction can be understood particularly clearly through the following analogy. Consider space as a straight road. Point P is a fixed point on the road. Now imagine that the light wave is, for illustrative purposes only, a solid object called "light", launched from point P with velocity c. This object will successively reach various fixed points on the road. At the same time, consider two cars representing the two reference systems S and S′. Both start from the "same" position, located 10 m from P, and move in the positive direction of the X axis, but with different velocities: $v_p = 2\ m/s,\ v_p' = 3\ m/s$.

At the initial instant, both are 10 m from P. Therefore, in the reference frame of the road, with the origin fixed at P, we have:

$$X_S(t) = 10 + 2t \quad \text{and} \quad X_{S'}(t) = 10 + 3t.$$

The relative velocity between the cars is therefore,

$$v = v_p' - v_p = 1\ m/s.$$

The coordinate of a fixed point Q on the road, relative to the moving origin of S, is

$$x_Q(t) = X_Q - X_S(t).$$

Analogously, for S′,

$$x_Q'(t) = X_Q - X_{S'}(t)$$

Now consider two permanent points on the road: $Q_1 = 20\ m,$, $Q_2 = 50\ m$. The distance between them is always

$$Q_2 - Q_1 = 30\ m.$$

The motion of the cars does not modify this distance. Suppose that the object "light" reaches Q_1 at the instant $t_1 = 2\ s$ and Q_2 at the instant $t_2 = 5\ s$. The coordinates of Q_1 at the instant $t_1 = 2$ will be:

- In S: $x_1(t_1) = 20 - [10 + 2(2)] = 6\ m,$
- In S': $x_1'(t_1') = 20 - [10 + 3(2)] = 4\ m.$

The coordinates of Q_2 at the instant $t_2 = 5$ will be:

- In S: $x_2(t_2) = 50 - [10 + 2(5)] = 30\ m$,
- In S': $x'_2(t'_2) = 50 - [10 + 3(5)] = 25\ m$.

Thus, we have:

For S, $$x_2 - x_1 = 24\ m.$$

For S′, $$x'_2 - x'_1 = 21\ m.$$

Thus, $$x'_2 - x'_1 \neq x_2 - x_1.$$

However, the distance between the points on the road remains

$$Q_2 - Q_1 = 30\ m.$$

The difference between 24 m and 21 m simply results from the fact that the cars have moved differently during the three seconds considered:

$$v(t_2 - t_1) = 1 \times 3 = 3\ m.$$

Indeed,

$$24 - 21 = 3\ m.$$

Therefore,

$$x'_2 - x'_1 = x_2 - x_1 - v(t_2 - t_1).$$

From the perspective adopted here, this expression does not reveal an inconsistency in Galilean kinematics. On the contrary, *it is precisely the result expected when two coordinate systems move relative to one another*. The distance between the points on the road remains invariant because these points have not moved. What has moved are the cars and, with them, their respective coordinate grids. The example above can be generalized by the following principle.

Principle of Non-Equivalence Between Differences in Temporally Distinct Moving Coordinates and Spatial Separations

From this perspective, moving coordinate systems must be distinguished from the points of space themselves that they describe. These systems do not carry the points of space along with them as they move; their motion merely changes the correspondence between these points and the coordinate labels assigned to them. Thus, at each instant t, there is a specific correspondence between the persistent points of space and the coordinates of the moving system. If P is a fixed point in space, its coordinate x in S can be written as

$$x_P(t) = X_P + x_c(t), \tag{25}$$

where X_P represents the position of the point relative to a fixed spatial origin and $x_c(t)$ represents the position of that origin in the moving system. For two distinct instants t_1 and t_2, in general,

$$x_P(t_2) \neq x_P(t_1).$$

This does not mean that point P has moved through space, but only that the coordinate grid used to describe it has changed its position relative to point P. Therefore, *for a coordinate system moving relative to persistent points in space, the difference between the label assigned to a point at one instant and the label assigned to another point at a later instant does not, in general, represent the spatial distance between those points.*

If P_1 and P_2 are two fixed points in space, the difference

$$x_2(t_2) - x_1(t_1)$$

relates two coordinates belonging to different temporal configurations of the same moving grid. For this reason, it should not be directly identified with

$$X_2' - X_1' = X_2 - X_1,$$

which represents the separation between the spatial points themselves. The situation can be demonstrated directly. Consider a fixed spatial origin, and let X_1 and X_2 be the positions of P_1 and P_2 relative to this origin. For the moving system S, let

$$x_i(t) = X_i - v_p t.$$

When the two points are considered at the same instant t, the displacement of the coordinate grid is the same for both points and therefore cancels in the difference, resulting in

$$x_2(t) - x_1(t) = X_2 - X_1$$

However, if the coordinates are taken at different instants,

$$x_2(t) - x_1(t) = (X_2 - v_p t) - (X_1 - v_p t),$$

then

$$x_2(t_2) - x_1(t_1) = (X_2 - X_1) - v_p(t_2 - t_1).$$

Therefore, for $v_p \neq 0$,

$$x_2(t_2) - x_1(t_1) \neq (X_2 - X_1)$$

in general. The inequality does not express a change in the distance between P_1 and P_2. The spatial distance remains

$$D_{12} = X_2 - X_1.$$

The additional difference

$$-v_p(t_2 - t_1)$$

represents the displacement of the coordinate grid relative to the spatial points during the interval considered. Therefore, for the same spatial points P_1 and P_2,

$$x_2(t_2) - x_1(t_1) = (X_2 - X_1) - v_p(t_2 - t_1),$$

whereas

$$x_2'(t_2') - x_2'(t_1') = (X_2 - X_1) - v_p'(t_2 - t_1).$$

Considering that $v = v_p' - v_p$, we obtain

$$x_2'(t_2') - x_2'(t_1') = x_2(t_2) - x_1(t_1) - v(t_2 - t_1). \tag{26}$$

In the proposed interpretation, therefore, the fact that

$$x_2'(t_2') - x_2'(t_1') \neq x_2(t_2) - x_1(t_1)$$

does not imply that the systems assign different distances to the same spatial points. The two expressions do not, by themselves, represent the separation between P_1 and P_2. They represent differences between coordinate labels belonging to configurations of the moving grids at different times. The spatial separation is given by

$$X_2' - X_1' = X_2 - X_1. \tag{27}$$

These considerations make it possible to formulate the following principle:

Principle of Temporal Correspondence of Coordinates: *The difference between the coordinates of two points directly represents the spatial separation between those points only when the coordinates are assigned relative to the same temporal configuration of the coordinate system. When the coordinates correspond to different instants of a moving system, their difference incorporates the displacement of the coordinate structure itself between the instants considered and, therefore, does not, in general, represent the spatial distance between the points.*
Therefore, the Galilean expression

$$x_2'(t_2') - x_2'(t_1') \neq x_2(t_2) - x_1(t_1) \quad \text{for} \quad t_2 \neq t_1.$$

does not constitute evidence of a change in the spatial separation between P_1 and P_2; it precisely describes the change in the *coordinate labels* produced by the relative motion of the grids of S and S′.

3.5. The "Interval Between Two Events" According to the Proposed Interpretation

In this approach, the wave does not propagate *in the systems* S and S′. It propagates *in space*, from point P, successively reaching persistent spatial points. Thus, when E_1 and E_2 are considered as two events of the propagation of the light wavefront, they correspond, in this interpretation, to occurrences at two specific spatial points, Q_1 and Q, at instants t_1 and t_2, respectively.

The so-called "spacetime interval" *is therefore associated, from this perspective, with the events of light propagation in space*, and not with any physical displacement of the wavefront through the systems S and S′. Therefore, the expression "interval between two events" should be interpreted with care. In this conception, the spatial component of this quantity corresponds to the *distance between two points in space eventually reached by the light wavefront*:

$$D_{12}^2 = (X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2. \tag{28}$$

It is therefore not the difference between two coordinates belonging to a moving grid taken at two different instants.

The Spacetime Interval

Defined in this way, the quantity under consideration is

$$s_{12}^2 = c(t_2 - t_1)^2 - (X_2 - X_1)^2 - (Y_2 - Y_1)^2 - (Z_2 - Z_1)^2, \quad \text{in S} \tag{29}$$

and

$$s_{12}'^2 = c(t_2' - t_1')^2 - (X_2' - X_1')^2 - (Y_2' - Y_1')^2 - (Z_2' - Z_1')^2 \quad \text{in S'} \tag{30}$$

Since, in Galilean kinematics, $t' = t$ we have

$$t_2' - t_1' = t_2 - t_1.$$

Furthermore, since the moving systems do not carry the points of space, and Q_1 and e Q are the same persistent points in space,

$$X_2' - X_1' = X_2 - X_1, \;\; Y_2' - Y_1' = Y_2 - Y_1, \;\; Z_2' - Z_1' = Z_2 - Z_1.$$

Consequently,

$$s_{12}'^2 = c(t_2 - t_1)^2 - (X_2 - X_1)^2 - (Y_2 - Y_1)^2 - (Z_2 - Z_1)^2$$

and, therefore,

$$s_{12}'^2 = s_{12}^2. \tag{31}$$

The invariance of this quantity thus follows from the invariance of the *same spatial points* and of the *common time* of Galilean kinematics. For the light wavefront,

$$X^2 + Y^2 + Z^2 = c^2t^2, \tag{32}$$

and, for a light separation,

$$s^2 = 0.$$

The result remains the same in S′, not because the differences between moving coordinates are invariant, but because both systems can describe the same spatial points and the same time interval by means of the common spatial structure

3.6. Final Considerations

The preceding analysis leads to a distinction that, from the perspective adopted in this work, is fundamental. The relation

$$x_2' - x_2' = x_2 - x_1 - v(t_2 - t_1)$$

should not be interpreted as demonstrating that Galilean kinematics fails to preserve the spatial distance between two points and, consequently, the spacetime distance between two events. It demonstrates only that the coordinates assigned to spatial points by coordinate systems that move relative to one another do not, in general, have the same difference when evaluated at different instants. Systems S and S′ move through space with velocities v_p and v_p', respectively, and carry their own coordinate structures with them. The points of space, however, remain fixed in space. Therefore, what varies is the coordinate of the point in the moving system, and not the spatial position of the point itself. In this sense, the relation

$$x_2' - x_2' = x_2 - x_1 - v(t_2 - t_1)$$

constitutes, from the perspective developed here, a sign of consistency, rather than inconsistency, of Galilean kinematics. Interpreting it as a variation in the distance between spatial points means, within this approach, attributing to differences between moving coordinates a physical meaning that they do not possess. The fundamental question therefore becomes the following: *when it is stated that the spacetime interval is or is not invariant, what exactly is the physical quantity whose invariance is being examined?* If the spatial component of the interval is understood as the distance between the persistent points of space reached by the wavefront, rather than as the difference between coordinates of moving systems taken at different instants, then the Galilean analysis leads to the invariance

$$s'^2 = s^2.$$

It is this distinction between *coordinates of moving systems and points of space in time* that underlies the interpretation proposed in this work.

4. Maxwell's Equations and Their Behavior under Lorentz Transformations

4.1. On Maxwell's Equations

Maxwell's equations constitute the mathematical core of the classical theory of electromagnetism. Although they were originally presented by Maxwell in a much more extensive form [7, 22], they were subsequently reorganized into a compact formulation consisting of four equations, particularly associated with the work of Oliver Heaviside and the development of modern vector formalism [22]. In the present analysis, particular interest is given to the differential form of the equations, in a region of space in which charge density ρ and current density $\boldsymbol{J}$ may exist, as described below.

$$\begin{aligned} &\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} && \text{Gauss's Law (for the electric field)} \\ &\nabla \cdot \boldsymbol{B} = 0 && \text{Gauss's Law for Magnetism,} \\ &\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} && \text{Faraday's Law,} \\ &\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} && \text{Ampère– Maxwell Law.} \end{aligned} \tag{33}$$

The form of these equations for vacuum is obtained simply by setting $\rho = 0$ and $\boldsymbol{J} = \boldsymbol{0}$, from which the electromagnetic wave equation follows directly:

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0 \quad \text{and} \quad \nabla^2 \boldsymbol{B} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} = 0. \tag{34}$$

where $c = 1/\sqrt{\mu_0 \epsilon_0}$. The presence of ϵ_0 and μ_0 in this structure establishes, within the classical formalism of electromagnetism [7, 22], a characteristic velocity for the propagation of electromagnetic waves in vacuum. The relationship between this mathematical structure and the transformations between reference frames therefore constitutes one of the central questions of the analysis developed in this part.

4.2. The Question of the Invariance of Maxwell's Equations

The analysis of the invariance of Maxwell's equations under certain transformations seeks to determine whether, when passing from one inertial reference frame to another, the equations retain their form. In the traditional relativistic formulation, this requirement is associated with the principle of relativity and the constancy of the speed of light in vacuum [1, 21]. It is precisely in this context that the historical question of the compatibility between Galilean kinematics and electromagnetism arises: *if Galilean transformations do not preserve the form of Maxwell's equations, whereas Lorentz transformations do, it seems reasonable to conclude that electromagnetic theory necessarily requires relativistic kinematics.* However, this conclusion depends on certain assumptions that will be analyzed in this work. The following presents a summary of the behavior of Maxwell's equations under Lorentz coordinate transformations, as well as the corresponding transformations of the fields, followed by a critical analysis of these transformations.

4.3. On the Invariance of Maxwell's Equations Under Lorentz Transformations — Relativistic Interpretation

As established in the preceding sections, Maxwell's equations preserve their form under Lorentz transformations, whereas, according to the conventional interpretation, they do not preserve their form under Galilean transformations [14, 19, 21]. The preservation of the form of Maxwell's equations under Lorentz transformations, however, does not follow solely from the transformation of the spacetime coordinates. It also requires a corresponding transformation of the quantities representing the electric and magnetic fields. The present section does not intend to reproduce the demonstrations already established in the literature concerning the invariance of Maxwell's equations under Lorentz transformations. The

objective is to critically examine the transformation of the fields themselves and, in particular, the physical interpretation attributed to the structure of these transformations.

4.3.1. Structure of the Relativistic Transformations of the Fields

Consider the inertial reference frames S and S', already introduced above. In the conventional relativistic formulation [23], the components of the electric and magnetic fields parallel to the direction of motion remain unchanged, whereas the perpendicular components undergo coupled transformations. The relations are given by

$$\begin{aligned} E'_x &= E_x & B'_x &= B_x \\ E'_y &= \gamma\left(E_y - vB_z\right), & B'_y &= \gamma\left(B_y + \frac{v}{c^2}E_z\right), \\ E'_z &= \gamma(E_z + vB_y), & B'_z &= \gamma(B_z - \frac{v}{c^2}E_y), \end{aligned} \tag{35}$$

These relations can be obtained within the relativistic formalism from the transformation of electromagnetic quantities between inertial reference frames and make it possible to preserve the form of Maxwell's equations under Lorentz transformations. A particularly significant consequence of these relations is that a component of the electric field can contribute to a component of the magnetic field in the other reference frame, and vice versa. Thus, fields that are described separately in one reference frame may appear partially combined in another. This property, however, raises a question that should be distinguished from the mathematical derivation of the equations themselves: *Does the mathematical rule that determines how the fields transform, by itself, explain why physical fields should transform in this way?*

The question, therefore, is not merely to determine the transformation rule, but to examine what the physical justification is for the specific structure of this rule.

4.3.2. Critical Analysis of the Lorentz Transformations of the Fields

Consider again the case of relative motion exclusively in the direction x. For the spatial coordinates, the Lorentz transformations give

$$y' = y \quad \text{and} \quad z' = z.$$

In this case, there is no transformation between the coordinates y and z, since the relative motion between the reference frames occurs along x. However, the same does not apply to the electromagnetic fields. Although the relative motion is restricted to the x-direction, the components E_x and B_x remain invariant, whereas the transverse components undergo transformation and, in addition, appear combined with components of the other field. The problem at issue is not simply to determine how these transformations can be obtained, but to understand *why the physical transformation of electromagnetic quantities must assume precisely this structure*. There are, therefore, two distinct questions:

1. What is the mathematical rule for the transformation of the fields?
2. Why should nature obey precisely this rule?

The first question can be answered by the relativistic formalism. The second requires a physical justification that does not consist merely in reproducing the mathematics of the rule itself. In particular, it is necessary to explain *why the components in the direction of motion remain invariant, whereas the transverse components are modified and combined with components of the other field*. Thus, the question is not merely to explain why a moving observer may attribute a magnetic field to a phenomenon that, in another reference frame, is described solely by an electric field, but to understand *why the longitudinal components remain invariant and why the transverse components mix between the electric and magnetic fields*.

The Relativistic Explanation

The literature recurrently presents the transformation of the fields as a consequence of Special Relativity and interprets the mixing between the electric and magnetic fields as a consequence of the relative motion between the reference frames. A usual way of presenting this interpretation is to consider a charge or a distribution of charges at rest in S. In this reference frame, according to the relativistic interpretation, there may be only an electric field. In S′, which moves relative to the system of charges, the same electromagnetic configuration comes to be described also by a magnetic field. Feynman [24] explicitly presents this case for a static electric field initially without a magnetic field, showing that, when the motion is perpendicular to the electric field, a magnetic component arises. For the general case, the author decomposes the electric field into components parallel and perpendicular to the motion, emphasizing that the parallel component remains unchanged, whereas the perpendicular component undergoes the corresponding transformation. Griffiths [23] presents the same idea in the context of relativistic electrodynamics and, when introducing the general field transformations, emphasizes that the transformation rules must be independent of the way in which the fields were produced.

In a more formal formulation, the transformations are obtained from the covariance of electrodynamics under Lorentz transformations. In the tensor formulation [23, 24], this structure is expressed compactly: the electric and magnetic fields are components of a single electromagnetic entity, whose transformation under a boost in the x-direction mathematically produces the invariance of the longitudinal components and the mixing of the transverse components. The relativistic response can therefore be summarized as follows: *the difference between the longitudinal and transverse components arises from the structure of the Lorentz transformation, whereas the mixing between the electric and magnetic fields arises from the way in which the electromagnetic field transforms between inertial reference frames.* This response establishes how the relativistic formalism produces the transformations. It remains, however, to examine whether it also provides an independent physical justification for the necessity of this structure.

Limitations of the Explanation Based on Relative Motion

The statement that a moving charge produces a magnetic field does not, by itself, explain why the transformations have precisely the form

$$E'_x = E_x, \quad E'_y = \gamma\left(E_y - vB_z\right), \quad E'_z = \gamma(E_z + vB_y),$$

$$B'_x = B_x, \quad B'_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right), \quad B'_z = \gamma(B_z - \frac{v}{c^2}E_y).$$

The simple statement that the source is moving relative to S′ may qualitatively justify the existence of a magnetic component in a given reference frame, but it does not explain why precisely the transverse components y and z are modified and mixed, while the components in the direction of motion remain invariant.

On the Tensor Approach

The tensor formulation makes the transformation mathematically clear. However, stating that this occurs because the electromagnetic field is represented by a tensor that transforms in a certain way establishes *how the formalism produces the result*, but does not answer the preceding question: *why should the physical field obey precisely this transformation law?* The tensor representation demonstrates the consequence of a previously adopted mathematical structure, but does not, by itself, provide an independent physical answer as to why this structure is necessary. It is therefore necessary to distinguish between:

- mathematically deriving a transformation; and
- physically explaining why this transformation must exist.

The elegance and internal consistency of the tensor formalism do not eliminate this distinction.

The Invariance of the Equations as a Sufficient Condition

It is also not sufficient simply to state that the structure of the field transformations is necessary to preserve the invariance of Maxwell's equations. This statement explains why a given rule is compatible with the covariance of the theory, but it still allows the fundamental question to be formulated: if the requirement is to preserve the form of the physical laws, why does this requirement lead precisely to the invariance of the longitudinal components and to the coupled transformation of the transverse components? Therefore, invoking the invariance of the laws explains the mathematical necessity of the rule within the adopted theoretical framework, but does not, by itself, constitute an independent physical explanation of the specific structure of this rule. Consequently, the question that motivates the present analysis remains open: *does the relativistic transformation of the fields constitute merely a mathematical consequence of the spacetime structure adopted by Special Relativity, or is there an independent physical foundation that explains why the fields must transform in precisely this way?*

Formulation of the Problem from the Proposed Perspective

In the interpretation developed in the preceding sections, the description of a physical phenomenon should begin with the *relations between the elements that actually participate in that phenomenon*, distinguishing them from the *kinematic relation of these elements to an external observer*. This distinction was previously applied to the physical propagation of phenomena, to the definition of the *relevant velocities*, and to the relation between *spatial points* and *coordinates of reference systems*. In this context, the question of the invariance of Maxwell's equations must be reconsidered. The classical interpretation of Galilean transformations maintains that they are not compatible with electromagnetism because they do not preserve the form of Maxwell's equations, thereby making Lorentz transformations necessary [21, 25].

The approach developed in Part 1, however, establishes a distinction between the *physical propagation of the phenomenon in its medium* and the *kinematics of objects relative to moving systems in a common substrate*. From this perspective, *the propagation of an electromagnetic disturbance should be referred to the physical point of emission and to the relations between the elements participating in the phenomenon*, rather than being defined by the relation of the propagation to the origin of the coordinate system of the observer's reference frame.

Likewise, a distinction is made between *the proper velocity of propagation in the medium* and *the velocity of the elements participating in the phenomenon relative to an observer*. The question therefore becomes whether, under this interpretation, Galilean kinematics can be applied to the physical relations relevant to electromagnetic propagation without necessarily imposing the relativistic structure of the Lorentz transformations. This leads to the central question of the next stage: *Can the invariance of Maxwell's equations be preserved under the perspective proposed here?* This question will now be examined in light of the proposed perspective, by determining whether, under this interpretation, Maxwell's equations can preserve their form when subjected to Galilean transformations.

5. Description of Electromagnetic Phenomena Based on Invariant Physical Relations

In Part 1 of this series, a distinction was established between *electromagnetic propagation in the physical substrate* and *the kinematics of relative motion between bodies*, and the concepts of *proper velocity in the substrate* v_p *and relative (apparent) velocity* v_{rel} were introduced. In Section 2 of this Part, the following concepts were established:

- **General Principle for the Description of Physical Interactions:** *The quantities that characterize an interaction should be defined by the physical relations between the agents participating in that interaction, and not in relation to an observer. The observer, as an external agent who interacts with the "device" and describes or interprets the phenomenon, does not, by himself, constitute one of the agents involved in the interaction. The "device," however, when it physically participates in the measurement, must be considered an element of the interaction process, without this implying that it should be included in the theoretical description of the relations between the agents of the*

system. Its participation in the experimental process may, however, introduce uncertainties and perturbations into the measurements.

- **Precision in the Definition of Velocities:** *unless the reference relative to which the position of an object is defined has already been established, a careful distinction must be made between the reference frame used to measure or represent an associated quantity and the physical referent relative to which that quantity is defined*.
- **Invariance of the Kinematic Relations Between Agents:** *invariance is not introduced as an additional condition; it follows from the relational nature of the quantity itself*.
- **Principle of Causal Invariance:** *the invariance of the description must be accompanied by the invariance of the causal structure*.
- **Invariance of the Physical Properties and Relations of Systems:** *The fundamental object of invariance is the physical properties of the entities constituting the system and the relations between these entities*.
- A distinction was also established between *coordinates relative to the observer's reference frame* and *quantities defined relative to the physical entities of the system under study*.

In this section, the consequences of these concepts for the description of physical phenomena and, in particular, for electrodynamics will be presented.

5.1. Consequences for Electrodynamics

The concepts summarized above will now be applied to electrodynamics. For these considerations, the concept of *source point* is introduced here.

The Source-Point of the Electromagnetic Disturbance

Consider a material source that produces a disturbance at a given point P in space. It is necessary to distinguish the *material source* from what is here designated as the *source point* of the disturbance. A lamp, an antenna, or another material source may be in motion. However, when a disturbance is produced at point P, this point constitutes the immediate spatial reference from which the disturbance propagates. This does not mean that P is a material source. It is simply the *point in space at which the disturbance was produced*. In accordance with the *second postulate of Special Relativity* [1, 17], the wave should not, in this description, be identified with the subsequent motion of the material source that produced it.

5.1.1. Consequence for the Electromagnetic Force

Consider a uniform magnetic field $\boldsymbol{B}$ at rest in reference frame S, and a charge moving with reference frame S′. The Maxwell-Lorentz force [26] on this charge is given by the expression

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}). \tag{36}$$

The focus in this expression is on the quantity $\boldsymbol{u}$. For the observer in S, relative to whom S′ is moving, what is the value of the velocity $\boldsymbol{u}$? The literature generally refers to this quantity as "the velocity of the charge," identifying it with the velocity of the charge defined relative to the observer's reference frame. In the present approach, however, this interpretation is problematic, considering that the expression above (the Maxwell-Lorentz force) does not describe a phenomenon inherent to the relation between the charge and the observer, but rather between the charge and the field $\boldsymbol{B}$. In this case, *the field $\boldsymbol{B}$ should be the referent relative to which this velocity is defined*. This is, in fact, the interpretation adopted by Maxwell, according to Assis [26]. Eventually, this velocity may coincide with the velocity of the charge relative to the observer. The following example helps to understand this conclusion.

Consider a charge q subjected to the magnetic field $\boldsymbol{B}$ produced by a magnet attached to a cart. In reference frame S, suppose that the cart has velocity $\boldsymbol{u}_c$ and that the charge has velocity $\boldsymbol{u}_q$, with

$$\boldsymbol{u}_c \neq 0, \quad \boldsymbol{u}_q \neq 0.$$

In another reference frame S′, the corresponding velocities are $\boldsymbol{u}_c'$ and $\boldsymbol{u}_q'$. The fundamental question then arises: *which of these velocities should be considered when determining the magnetic effect on the charge, for the observer in S?* And which should be considered in *S′*?

From the perspective adopted in this work, the conventional interpretation requires further analysis, because the interaction in question does not occur between the charge and the observer, but between the charge and the magnetic field produced by the magnet. Consequently, if the field $\boldsymbol{B}$ is considered to be associated with the magnet, the relevant kinematic quantity should characterize the relation between the charge and the magnet. One may therefore define

$$\boldsymbol{u} = \boldsymbol{u}_q - \boldsymbol{u}_c \tag{37}$$

or, choosing the charge as the reference object,

$$\boldsymbol{u} = \boldsymbol{u}_c - \boldsymbol{u}_q \tag{38}$$

The choice of sign convention depends on which agent is taken as the reference in the relation. What matters for the present analysis is that the quantity be defined by the kinematic relation between the agents of the interaction. In the proposed construction, this relation belongs to the charge–magnet system itself and must therefore be the same when described by S or S′. Under a Galilean transformation,

$$\boldsymbol{u}_q' = \boldsymbol{u}_q - \mathbf{v} \quad \text{and} \quad \boldsymbol{u}_c' = \boldsymbol{u}_c - \mathbf{v}, \tag{39}$$

so that

$$\boldsymbol{u}' = \boldsymbol{u}_q' - \boldsymbol{u}_c' = \left(\boldsymbol{u}_q - \mathbf{v}\right) - (\boldsymbol{u}_c - \mathbf{v}) = \boldsymbol{u}_q - \boldsymbol{u}_c = \boldsymbol{u}. \tag{40}$$

The essential point is that the velocity of the charge relative to the observer is not used in isolation, but rather the relative velocity between the charge and the field (or the physical agent with which the field is associated). A change of reference frame alters the individual velocities attributed to the agents, but, under the Galilean transformation, does not alter the relative velocity between them. This also clarifies some particular cases. If the magnet is at rest in S,

$$\boldsymbol{u}_c = 0, \quad \text{and, in this case,} \quad \boldsymbol{u} = \boldsymbol{u}_q. \tag{41}$$

The velocity of the charge relative to the magnet numerically coincides with its velocity relative to S. This coincidence, however, does not mean that the two quantities have the same physical definition: the former is defined by the charge–magnet relation; the latter, by the charge–reference-frame S relation. Likewise, if the charge is at rest in S,

$$\boldsymbol{u}_q = 0, \quad \text{and, in this case,} \quad \boldsymbol{u} = -\boldsymbol{u}_c, \tag{42}$$

The relevant velocity remains the relative velocity between the agents, although its magnitude coincides with the magnitude of the magnet's velocity relative to the observer. Thus, the numerical coincidence between the relative velocity between the agents and the velocity of one of them relative to the observer occurs only in particular situations. This construction reinforces the general principle adopted in this work: *the kinematic quantity involved in the description of an interaction should be defined relative to the agents that constitute the physical relation itself, and not relative to an external observer who does not participate in it*. Consequently, a change in the reference frame of description, under the Galilean transformation, *does not alter the kinematic relation between the agents constituting the interaction*. The observer merely describes how the charge interacts with the field.

The Same Interaction Described by Two Observers

For the observer in S′, the Maxwell-Lorentz force is

$$\boldsymbol{F}' = q'(\boldsymbol{E}' + \boldsymbol{u}' \times \boldsymbol{B}'). \tag{43}$$

The question then arises: what is the value of the velocity $\boldsymbol{u}'$?
In the traditional interpretation, according to the literature, $\boldsymbol{u}' = \boldsymbol{0}$, because the charge is at rest relative to S′. This raises two important questions:

1. In the expression for the Maxwell-Lorentz force, does the observer in S′ describe the interaction of the charge with the field or the interaction of the charge with the observer himself?
2. What is the justification for considering $\boldsymbol{u}' = \boldsymbol{0}$, while at the same time $q' = q$ and $\boldsymbol{F}' = \boldsymbol{F}$?

In the formulation proposed here, the change from S to S′ alters the kinematic representation used by the observers, but does not alter the physical agents constituting the interaction or the nature of the interaction. The invariance of physical laws must be a consequence of the invariance of the complete causal structure. Thus, if S' describes the same phenomenon that S describes (the interaction of the charge with the field), $\boldsymbol{B}$ and q are the common referents for the definition of $\boldsymbol{u}'$. Therefore,

$$\boldsymbol{u}' = \boldsymbol{u}_{q/\boldsymbol{B}} = \boldsymbol{u}. \tag{44}$$

The Maxwell-Lorentz force then becomes

$$\boldsymbol{F}' = q'\left[\boldsymbol{E}' + \left(\boldsymbol{u}_{q/\boldsymbol{B}}\right) \times \boldsymbol{B}'\right]$$

for S′. Since, according to the adopted principles,

$$\boldsymbol{F}' = \boldsymbol{F}, \quad q' = q \text{ and } \boldsymbol{u}' = \boldsymbol{u},$$

we have

$$\boldsymbol{E}' + \left(\boldsymbol{u}_{q/\boldsymbol{B}}\right) \times \boldsymbol{B}' = \boldsymbol{E} + \left(\boldsymbol{u}_{q/\boldsymbol{B}}\right) \times \boldsymbol{B}.$$

Therefore,

$$\boldsymbol{E} - \boldsymbol{E}' + \left(\boldsymbol{u}_{q/\boldsymbol{B}}\right) \times (\boldsymbol{B} - \boldsymbol{B}') = 0 \implies \Delta\boldsymbol{E} + \left(\boldsymbol{u}_{q/\boldsymbol{B}}\right)\Delta\boldsymbol{B} = 0. \tag{45}$$

This relation shows how the invariance of the interaction constrains the possibility of different fields being attributed to the same interaction.

5.1.2. Derivation of the Field Transformations According to the Proposed Interpretation

To make the conclusion more rigorous, consider different test charges subjected to the same field region, with different relative velocities $\boldsymbol{u}$. For any of them, it must hold that

$$\boldsymbol{E}' + \left(\boldsymbol{u}_{q/\boldsymbol{B}}\right) \times \boldsymbol{B}' = \boldsymbol{E} + \left(\boldsymbol{u}_{q/\boldsymbol{B}}\right) \times \boldsymbol{B},$$

and, therefore,

$$\Delta\boldsymbol{E} + \left(\boldsymbol{u}_{q/\boldsymbol{B}}\right)\Delta\boldsymbol{B} = 0,$$

where $\Delta\boldsymbol{E} = \boldsymbol{E} - \boldsymbol{E}'$, and $\Delta\boldsymbol{B} = \boldsymbol{B} - \boldsymbol{B}'$. If the relation is to hold for different test velocities, the only solution that does not introduce a change in the interaction is

$$\Delta\boldsymbol{E} = 0 \text{ and } \Delta\boldsymbol{B} = \boldsymbol{0}.$$

Consequently,

$$\boldsymbol{E} = \boldsymbol{E}' \quad \text{and} \quad \boldsymbol{B} = \boldsymbol{B}' \tag{46}$$

Thus, *for the same physical interaction and for quantities defined according to the relations between the physical agents constituting that interaction, the fields do not need to acquire new values simply because the observer has changed*.

It is important to clarify the meaning of this conclusion. It makes no statement about the components of $\boldsymbol{E}$ and $\boldsymbol{B}$ under a coordinate transformation. It merely states that the fields characterizing the same physical relation are treated as the same physical quantity, while a change of reference frame modifies only their coordinate representation. This distinction will be fundamental to the analysis of Maxwell's equations. The component-by-component analysis follows.

Considering the relevant velocity as

$$\boldsymbol{u}_{q/\boldsymbol{B}} = \left(u_{q/\boldsymbol{B}x}, u_{q/\boldsymbol{B}y}, u_{q/\boldsymbol{B}z}\right) \tag{47}$$

suppose $u_{q/\boldsymbol{B}x} = u_x$, for simplicity. Analogously for the other components. The Maxwell-Lorentz force, in S, can be written in component form as

$$\begin{aligned} F_x &= q\left(E_x + u_y B_z - u_z B_y\right) \\ F_y &= q\left(E_y + u_z B_x - u_x B_z\right) \\ F_z &= q(E_z + u_x B_y - u_y B_x). \end{aligned} \tag{48}$$

Analogously, this must occur for S′. Equality of the x component gives

$$E_x' + u_y' B_z' - u_z' By' = E_x + u_y B_z - u_z B_y.$$

Since, for this consideration, $u_z' = u_y' = u_z = u_y = 0$, it follows that

$$E_x' = E_x$$

And for the y component, we have

$$E_y + u_z B_x - u_x B_z = E_y' + u_z' B_x' - u_x' B_z'.$$

Since $u_x' = u_x$ and $u_z' = u_z = 0$, it follows tha

$$E_y - E_y' = u_x(B_z - B_z').$$

Note that the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ do not depend on the charge q or on its velocity. Thus, for the same field configuration and different test velocities, this relation must remain valid. Since the left-hand side does not depend on the velocity u_x, whereas the right-hand side does, equality for different velocities requires that $B_z' = B_z$; consequently,

$$E_y' = E_y.$$

Applying the same procedure to the remaining components, one obtains

$$B_y' = B_y, \quad E_z' = E_z \text{ and, consequently, } B_x' = B_x.$$

In summary,

$$E'_x = E_x,\ E'_y = E_y,\ E'_z = E_z$$
$$B'_x = B_x,\ B'_y = B_y,\ B'_z = B_z \tag{49}$$

Therefore, in this construction, no mixing between electric and magnetic components arises simply as a result of the relative motion between the observers. This establishes a fundamental difference from the conventional relativistic transformation of the fields. In the present approach, a change of reference frame is not interpreted as a physical transformation of the electromagnetic field, but as a change in the coordinate representation of the same interaction.

Consequences for Maxwell's Equations

For the analysis of the question of the invariance of Maxwell's equations, the following question is particularly important: *What, precisely, do Maxwell's equations describe?* The answer to this question has a major impact on the outcome of the analysis.

In this approach, it is considered that *Maxwell's equations describe the evolution of the electromagnetic disturbance in its medium of propagation over time [7]. The quantity c represents the velocity at which the wavefront moves away from the point of disturbance (source point) in the medium of propagation, which justifies the presence of* ϵ_0 *and* μ_0 *in these equations. The spatial coordinates appearing in these equations are defined relative to the source point, in the reference frame of the medium of propagation, and not in the observer's reference frame. The same does not apply to the temporal coordinate, which, in this approach, is considered universal (absolute).*

Analogy Between the Source Point and the Source Charge

Here the analogy with the electrostatic field reappears, for which the relevant spatial quantity is

$$\boldsymbol{R} = \boldsymbol{r} - \mathbf{r}_Q \tag{50}$$

that is, the position of the point under consideration relative to the source charge. Analogously, for an electromagnetic disturbance emitted from point P, the relevant spatial quantity is the position of the disturbance relative to P. Thus, propagation should not be defined simply by the position of the disturbance relative to the arbitrary origin of the coordinate system associated with an observer's reference frame. What matters is the relation between the disturbance and the physical point from which it propagates. The observer therefore has two options for describing the electromagnetic disturbance, which can be extended to other physical phenomena: the same two options presented in Section **3.3.1**, but now applied to the context of Maxwell's equations, as presented below.

First Construction: Coordinates Referred to the Source Point

The first construction consists in choosing the source point P as the origin of the coordinates used to describe the propagation. In this case, for one-dimensional motion, the distance of the disturbance from point P is simply

$$D = X - X_P = X. \tag{51}$$

with $X_P = 0$.
For propagation along the X axis,

$$X = ct. \tag{52}$$

This expression means that, after the time interval t, the disturbance is at a distance ct from point P. This is not the distance of the wave from the origin of the coordinate system defined in the reference frame of an observer (which is moving through space). It is the distance relative to the emission point. In this

construction, the two observers may use different coordinate descriptions for their own systems, but the description of the phenomenon is *referred to the same physical point P*. Thus,

$$X' = X.$$

Therefore,

$$X' = c't' = X = ct$$

Hence,

$$c' = c. \tag{53}$$

The proper propagation velocity of the disturbance relative to the propagation medium and, consequently, relative to the source point does not depend on the observer.

Second Construction: Coordinates Referred to the Origins of the Observers' Reference Frames

As demonstrated in Section **3.3.1**, it is also possible to describe the same phenomenon using coordinate systems linked to the reference frames of the respective observers. In this case, the position of source point P may be different in S and S′, since this point appears to be in relative motion with respect to these reference frames. However, the position relevant to propagation is not simply x (relative to the observer's reference frame), but the position of the disturbance relative to the source point:

$$X = x - x_P \quad \text{in S.}$$

$$\text{and} \quad X' = x' - x'_P \ \text{ in S'.}$$

Since both the position of the disturbance and the position of the source point undergo the same Galilean transformation,

$$x' = x - vt \text{ and } x'_P = x_P - vt,$$

it follows that

$$X' = (x - vt) - (x_P - vt),$$

therefore,

$$X' = x - x_P = X. \tag{54}$$

The position of the disturbance relative to the source point is invariant under Galilean transformations.

Thus, the Galilean transformation changes the coordinates assigned to the origins of the observers' coordinate systems, but does not alter the distance of the disturbance relative to the source point. This result constitutes a direct application of the general principle established previously: *the coordinate transformation modifies the representation of spatial relations relative to the origins of the observers' coordinate systems, but does not modify the physical relation between the elements constituting the phenomenon.*

Invariance of Electromagnetic Propagation

Since $X' = X$ and $t' = t$, we have

$$\frac{dX'}{dt'} = \frac{dX}{dt}.$$

Therefore,

$$c' = c.$$

The invariance of the propagation velocity is not, in this construction, postulated as a property of the relation between the wave and each observer; it results from defining the velocity as a relation between the disturbance and the propagation space, referred to the source point, and from the use of a common time scale.

Invariance of Maxwell's Equations

If the fields representing the same interaction satisfy

$$\boldsymbol{E} = \boldsymbol{E}' \text{ and } \boldsymbol{B} = \boldsymbol{B}',$$

and the relevant coordinates in the description of the phenomenon satisfy

$$X' = X, \quad Y' = Y, \quad Z' = Z, \quad t' = t,$$

then the differential operators applied to the physical quantities also preserve their form:

$$\frac{\partial}{\partial X'} = \frac{\partial}{\partial X}, \qquad \frac{\partial}{\partial t'} = \frac{\partial}{\partial t} \tag{55}$$

Consequently, Maxwell's equations can be written in S′ in the same form in which they are written in S. In vacuum,

$$\nabla \cdot \boldsymbol{E} = 0$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t},$$

$$\nabla \times \boldsymbol{B} = \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

Since the relevant physical quantities and spatial relations are preserved,

$$\nabla' \cdot \boldsymbol{E}' = \nabla \cdot \boldsymbol{E}, \quad \nabla' \cdot \boldsymbol{B}' = \nabla \cdot \boldsymbol{B} \tag{56}$$

and, analogously,

$$\nabla' \times \boldsymbol{E}' = -\frac{\partial \boldsymbol{B}'}{\partial t'} \quad \text{and} \quad \nabla' \times \boldsymbol{B}' = \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}'}{\partial t'}. \tag{57}$$

Therefore, the form of the equations is preserved.

Invariance of the Wave Equation

Likewise, in vacuum,

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0 \quad \text{and} \quad \nabla^2 \boldsymbol{B} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} = 0.$$

With the coordinates

$$X' = X, \; t' = t,$$

it follows that

$$\nabla'^2\boldsymbol{E}' = \nabla^2\boldsymbol{E} \quad \text{and} \quad \frac{\partial^2\boldsymbol{E}'}{\partial t'^2} = \frac{\partial^2\boldsymbol{E}}{\partial t^2}$$

Hence,

$$\nabla'^2\boldsymbol{E}' - \frac{1}{c^2}\frac{\partial^2\boldsymbol{E}'}{\partial t'^2} = 0. \tag{58}$$

The same result applies to $\boldsymbol{B}$. Thus, the velocity

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}.$$

remains the same in the description of the two observers because, in the proposed construction, it represents the proper velocity of the disturbance relative to the propagation space and the source point, and not the velocity of the wave relative to the origin of the observer's coordinate system.

Maxwell's Equations with Sources

The same logic should be applied to the equations containing sources.

Charge Density

The charge density is defined by

$$\rho = \frac{dq}{dV} \tag{59}$$

In the proposed approach, it is necessary to distinguish the *physical region of the system* from its *coordinate representation* in different reference frames. The transition from S to S′ *does not create a new physical region or alter the amount of charge contained in it*. A point in the charge distribution can be described, along the x-direction, by the coordinates

$$x = x_0 + wt \text{ in S}$$

$$x' = x'_0 + w't' \text{ in S}',$$

where w and w' are the velocities of the charge distribution relative to S and S′, respectively. Taking the differentials

$$dx = wdt,$$

$$\text{and} \quad dx' = w'dt'.$$

Applying Galilean transformations to the right-hand side of the equation in S′,

$$dx' = (w - v)dt = wdt - vdt = dx - vdt,$$

The volume is calculated by considering the spatial variation at the same instant t, that is, $t = t_0$, such that $dt = 0$. Consequently,

$$dx' = dx.$$

Furthermore,

$$dy' = dy, \quad dz' = dz.$$

Therefore,

$$dV' = dx'dy'dz' = dxdydz = dV. \tag{60}$$

Since the charge of the physical element remains the same, $dq' = dq$, it follows that

$$\rho' = \frac{dq'}{dV'} = \frac{dq}{dV} = \rho$$

$$\rho' = \rho. \tag{61}$$

Current Density

The current density is

$$\boldsymbol{J} = \rho \boldsymbol{u}. \tag{62}$$

Since $\rho' = \rho$ and $\boldsymbol{u}' = \boldsymbol{u}$, it follows that

$$\boldsymbol{J}' = \boldsymbol{J}. \tag{63}$$

Thus, the quantities that appear as sources in Maxwell's equations remain associated with the same physical distribution and the same kinematic relations between the agents. With these data and the preceding considerations, it can be verified that Maxwell's equations with sources, under this approach, preserve their form.

5.2. Difficulty with the Conventional Interpretation of the Galilean Transformation

5.2.1. On the Appearance of the Additional Terms

In the Galilean transformation of Maxwell's equations, according to the conventional interpretation, the additional terms

$$(\mathbf{v} \cdot \boldsymbol{\nabla}')\boldsymbol{B}' \quad \text{or} \quad v\frac{\partial \boldsymbol{B}'}{\partial x'} \quad \text{for motion in the } x - \text{direction..}$$

$$-\frac{1}{c^2}(\mathbf{v} \cdot \boldsymbol{\nabla}')\boldsymbol{E}' \quad \text{or} \quad -\frac{1}{c^2}\frac{\partial \boldsymbol{E}'}{\partial x'} \quad \text{for motion in the } x - \text{direction.}$$

arise. These terms are conventionally interpreted as evidence of incompatibility between Maxwell's equations and Galilean transformations. However, in the interpretation proposed here, they are understood as evidence of the consistency of these transformations and as indications of problems in the conventional interpretation of these transformations.

5.2.2. Where, Precisely, Is the Traditional Difficulty?

The difficulty does not lie in the transformation

$$x' = x - vt,$$

which, from this perspective, is considered the correct transformation. The difficulty lies in the improper identification of the spatial coordinates in Maxwell's equations with the coordinates relative to the origin of the coordinate system linked to the observer's reference frame, as well as in identifying the propagation velocity of light relative to its medium with its velocity relative to the observer. The Galilean transformation equations for spatial coordinates relate the coordinates of a point relative to the origins of the coordinate systems linked to reference frames in relative motion (S and S′). On the other hand, the spatial coordinates appearing in Maxwell's equations are defined relative to the source point, in the reference frame linked to the propagation medium. By identifying these coordinates with the coordinates relative to the observer's reference frame, the transformation produces the terms associated with the displacement between the

observers, which would be expected from the correct transformations, since system S′ moves relative to S and vice versa. That is, when one passes from S to S′, the time derivative comes to contain the term

$$-v\frac{\partial}{\partial x'}$$

It is this term that produces

$$v\frac{\partial \boldsymbol{B}'}{\partial x'}$$

in Faraday's law and

$$-\frac{1}{c^2}\frac{\partial \boldsymbol{E}'}{\partial x'}$$

in the Ampère-Maxwell law.

Therefore, if the spatial coordinates simply represent the position of the disturbance relative to the observer's origin, the Galilean transformation will inevitably introduce the term associated with the relative motion between the origins. However, according to the principle established previously, the physically relevant quantity for the natural description of propagation is not necessarily this coordinate. It is the position of the disturbance *relative to the source-point P*. When this distinction is incorporated into the treatment of the equations, the additional terms cease to be interpreted as modifications of the law itself and disappear from the final form of the equations.

5.3. Consequences for Electromagnetic Induction

The analysis developed in the preceding sections makes it possible to apply the general principles established in this work to the phenomenon of electromagnetic induction

5.3.1. The Magnet-Coil Interaction

Consider a system consisting of a magnet, a conducting coil, and a current-measuring instrument connected to the coil circuit. When there is relative motion between the magnet and the coil, an electric current is observed in the circuit. The phenomenon can be produced in two equivalent situations: keeping the coil at rest relative to the observer and moving the magnet, or keeping the magnet at rest and moving the coil. In the conventional formulation [8, 27], these two situations receive distinct descriptions. When the magnet moves relative to the laboratory while the coil remains at rest, the magnetic field is considered to vary in the region occupied by the circuit and, according to Faraday's law, this variation produces an electromotive force. When, on the other hand, the coil moves while the magnet remains at rest in the laboratory, it is considered that there is no temporal variation of the magnetic field at the fixed points of this reference frame and that the charges in the coil, because they are moving through the field region, experience the magnetic force associated with $q\boldsymbol{u}\times\boldsymbol{B}$. Thus, different causal descriptions are attributed to the two situations: in the first case, an induced electric field; in the second, a magnetic force acting on moving charges.

Although these descriptions are mathematically "consistent" within the conventional formulation, a conceptual question arises: *Why should the choice of reference frame modify the identification of the physical cause of the same phenomenon?* The approach adopted in this work starts from a more elementary question: *To which physical elements should the quantities used in the description of an interaction be referred?*

The Phenomenon Belongs to the Physical System, Not to the Observer

In the system under consideration, according to the interpretation adopted in this work, the agents directly involved are the magnet and the coil. The observer is external to this relation. The observer can measure and describe the phenomenon, but their velocity relative to each agent or to the system does not constitute an element of the interaction. Thus, the general principle established previously is maintained.

The Fundamental Quantity for Induction

Electromagnetic induction is characterized by the variation of the magnetic flux through the surface bounded by the circuit:

$$\boldsymbol{\Phi_B} = \int_S B \cdot dA \tag{64}$$

The induced electromotive force is

$$\varepsilon = -\frac{d\boldsymbol{\Phi_B}}{dt} \tag{65}$$

Therefore, the fundamental condition for induction is the existence of a *variation in the magnetic flux associated with the coil*. The observed current is a consequence of the electromotive force produced by this change in the electromagnetic configuration of the system. The question then becomes: *with respect to what should this variation be determined?*

From this perspective, the answer should not depend exclusively on the motion of the field or the magnet relative to the observer. If the quantity is being used to describe the *magnet–coil interaction*, its determination should take into account the relation between these elements themselves.

The Referent for Determining the Variation of the Field Should Be the Same

Let $\boldsymbol{r}_m$ and $\boldsymbol{r}_b$ be the positions of the magnet and of an element of the coil in coordinate system S. The relative configuration of the system can be represented by

$$\boldsymbol{R} = \mathbf{r}_b - \mathbf{r}_m \tag{66}$$

The field produced by the magnet can be represented by

$$\boldsymbol{B}_m = \boldsymbol{B}_m(\boldsymbol{r}, t) = \boldsymbol{B}(\boldsymbol{r} - \mathbf{r}_m, t). \tag{67}$$

The positions of the coil element and the magnet can be written as functions of time,

$$\boldsymbol{r}_b = \boldsymbol{r}_b(t) \quad \text{and} \quad \boldsymbol{r}_m = \boldsymbol{r}_m(t) \tag{68}$$

Therefore, the position of the coil element relative to the magnet will be

$$\boldsymbol{R} = \boldsymbol{R}(t) \tag{69}$$

The evolution of the relative configuration is determined by

$$\frac{d\boldsymbol{R}}{dt} = \frac{d\mathbf{r}_b}{dt} - \frac{d\mathbf{r}_m}{dt} = \boldsymbol{u}_b - \boldsymbol{u}_m \tag{70}$$

The relative velocity is therefore defined as

$$\boldsymbol{u}_{b/m} = \boldsymbol{u}_b - \boldsymbol{u}_m. \tag{71}$$

In the proposed approach, this is the kinematic quantity directly associated with the magnet–coil interaction. It is not the velocity of the magnet or of the coil relative to the observer, but the relative velocity between the agents of the interaction themselves. The field effectively experienced by the coil element is then

$$\boldsymbol{B}_b(t) = \boldsymbol{B}_m(\boldsymbol{R}(t), t). \tag{72}$$

Its temporal variation is

$$\frac{d\boldsymbol{B}_b}{dt} = \frac{\partial \boldsymbol{B}_m}{\partial t} + (\boldsymbol{u}_{b/m} \cdot \boldsymbol{\nabla}_{\boldsymbol{R}})\boldsymbol{B}_m \tag{73}$$

The expression above reveals two conceptually distinct contributions. The first, $\partial \boldsymbol{B}_m/\partial t$, represents the temporal variation of the field for a fixed relative configuration $\boldsymbol{R}$. Therefore, in this formulation, this term does not represent the change in $\boldsymbol{B}_b$ resulting from the variation in the relative position between the magnet and the coil. The latter contribution is contained in the second term, $(\boldsymbol{u}_{b/m} \cdot \boldsymbol{\nabla}_{\boldsymbol{R}})\boldsymbol{B}_m$. This expression highlights an important distinction. Even if

$$\frac{\partial \boldsymbol{B}_m}{\partial t} = 0$$

that is, even if the field is stationary at fixed points of a given reference frame, it may still be the case that

$$\frac{d\boldsymbol{B}_b}{dt} \neq 0$$

when the coil moves through the spatial distribution of the field. Therefore, the statement that the field is "constant" or "variable" has complete physical meaning only when it is specified *with respect to which physical points the statement is made. What matters for the interaction is the field experienced by the elements constituting the circuit and their spatial relation to the field*.

It is noteworthy that some authors, although adopting the conventional interpretation, initially seem to approach the interpretation proposed in this work by stating that "all that really matters is the relative motion of the magnet and the loop" [8, p. 209] or that "the observable phenomenon depends here only on the relative motion between conductor and magnet, […]" [27, p. 158]. However, a difficulty arises in the subsequent analysis, as can be seen in the use of expressions such as "stationary charges are not affected by magnetic forces" [8, p. 209]. A conceptual question then arises: stationary with respect to what? If what really matters is the relative motion between the magnet and the coil, in what sense is it physically relevant to state that a charge is stationary merely because it is at rest relative to the observer, while remaining in relative motion with respect to the magnet?

Thus, in the conventional interpretation, although the analysis initially appears to be restricted to the relation between the bodies directly involved in the interaction, the classification of charges as stationary or moving comes to depend on the reference frame adopted. This dependence is directly reflected in the mathematical treatment of the phenomenon: in one description, the charges have nonzero velocity and, consequently, the magnetic force is nonzero; in another, the same charges are considered stationary and, therefore, the magnetic contribution is zero. The choice of reference frame thus comes to determine which terms contribute to the calculation of the force responsible for the observed current. A more fundamental difficulty therefore arises: if the phenomenon depends only on the relative motion between the magnet and the coil, why should the choice of an additional reference frame determine whether the magnetic force on the charges is zero or nonzero?

The Two Experimental Situations

- *Magnet in Motion and Coil at Rest*
  If, in a given reference frame, $\boldsymbol{u}_b = 0$, then

$$\boldsymbol{u} = -\boldsymbol{u}_m.$$

- *Coil in Motion and Magnet at Rest*
  If, on the other hand, $\boldsymbol{u}_m = 0$, then

$$\boldsymbol{u} = -\boldsymbol{u}_b.$$

In the proposed approach, however, although observers assign different velocities to the two agents, the physical quantity characterizing the interaction is, in both cases, the same relation between magnet and coil. Thus, the two situations can be combined into a single description:

$$\boldsymbol{u}_{b/m} = \boldsymbol{u}_b - \boldsymbol{u}_m.$$

Under a Galilean transformation,

$$\mathbf{r}'_b = \mathbf{r}_b - \mathbf{v}t, \quad \mathbf{r}'_m = \mathbf{r}_m - \mathbf{v}t, \quad t' = t,$$

it follows that

$$\boldsymbol{u}'_{b/m} = \frac{d\boldsymbol{R}'}{dt'} = \frac{d\mathbf{r}'_{\mathrm{b}}}{dt} - \frac{d\,\boldsymbol{r}'_m}{dt} = \frac{d\mathbf{r}_b}{dt} - \mathbf{v} - \frac{d\mathbf{r}_m}{dt} + \mathbf{v} = \boldsymbol{u}_{b/m}$$

Therefore, the following identity is obtained:

$$\boldsymbol{u}'_{b/m} = \boldsymbol{u}_{b/m}. \tag{74}$$

This result mathematically expresses the following principle used in this work: *The Galilean transformation alters the velocities individually attributed to the agents relative to the observer, but preserves the relative velocity between the agents of the* interaction**.**

From this result, and taking into account the results obtained in the preceding sections, it can be concluded that equation (73) transforms from S to S′, under Galilean transformations, into

$$\frac{d\boldsymbol{B}'_b}{dt'} = \frac{\partial \boldsymbol{B}'_m}{\partial t'} + (\boldsymbol{u}'_{b/m} \cdot \boldsymbol{\nabla}'_{\boldsymbol{R}})\boldsymbol{B}'_m = \frac{\partial \boldsymbol{B}_m}{\partial t} + (\boldsymbol{u}_{b/m} \cdot \boldsymbol{\nabla}_{\boldsymbol{R}})\boldsymbol{B}_m = \frac{d\boldsymbol{B}_b}{dt} \tag{75}$$

The Invariance of the Causal Structure

This conclusion makes it possible to formulate a deeper question. If two reference frames are equally legitimate for describing a given phenomenon, a change of reference frame should not, by itself, alter the identification of the physical agents responsible for the phenomenon. If, in one description, the cause of the current is identified as a variation in magnetic flux and, in another, as a magnetic force on moving charges, an apparent change in the causal structure arises as a consequence of the choice of reference frame. The question raised in this work is not the mathematical possibility of representing the phenomenon by different expressions, but the physical meaning of this change: *if the phenomenon is the same and the reference frames are merely different ways of describing it, why should the fundamental physical cause of the phenomenon depend on the reference frame chosen?*

In the approach proposed here, it is not necessary to establish two distinct physical mechanisms. There is a single physical phenomenon. The distinction between a “moving magnet” and a “moving coil” corresponds to the way in which relative motion is decomposed with respect to the reference frame used, and not to a difference in the fundamental physical cause of induction.

5.4. General Consequence for the Physical Description

The preceding analyses – of the electromagnetic force, wave propagation, Maxwell’s equations, charge density, and electromagnetic induction – converge on the same general principle: *physical laws should be formulated and interpreted in terms of the agents and physical relationships that constitute the phenomenon, while the observer should be considered an external element of description.*

Consequently, a change of reference frame should not automatically be interpreted as a physical change in the phenomenon. This distinction constitutes the foundation of the interpretation proposed in this work for the compatibility between Galilean mechanics and the description of electromagnetic phenomena.

5.5. Consideration of Space as a Physical Reference

The proposed interpretation does not depend on the introduction of the classical ether as a material medium endowed with specific mechanical properties. The analysis developed in this work does, however, employ the idea that there exists a physical spatial structure relative to which certain propagation quantities can be defined. From this perspective, the distinction between the velocity of a system relative to the observer and the proper velocity of objects (including electromagnetic disturbances) relative to space makes it possible to formulate a description in which the observer's choice does not alter the proper propagation velocity of the disturbance. The proposed conclusion is, therefore, that the relative motion between observers need not be introduced as an additional physical variable in the description of an interaction that does not involve these observers as agents. It is in this sense that the approach developed here seeks to describe physical phenomena without attributing to the choice of reference frame a change in the fundamental relationships between the elements of the system.

6. Conclusion

In this second part, the compatibility between classical mechanics and electromagnetism was examined on the basis of the general principles established previously. The analysis focused on the relationship between Galilean transformations and the spacetime interval, Maxwell's equations, the transformations of the electric and magnetic fields, and the interpretation of electromagnetic induction. Based on the distinction between the *description of a phenomenon by an observer* and the *physical relationships between the agents that constitute the phenomenon*, a construction was developed in which the relevant quantities are defined by the physical relationships internal to the system. From this perspective, a change of reference frame alters the kinematic representation of the agents, but not the physical relationships that characterize the interaction. On the basis of this distinction, it was shown how, within the proposed construction, Maxwell's equations can preserve their form under Galilean transformations, without the need for Lorentz transformations. The analysis of electromagnetic induction led to the same general conclusion: situations in which the magnet or the coil is moved can be described on the basis of a single physical relationship, determined by the relative motion between the agents of the interaction, rather than by the motion of each agent relative to the observer.

Thus, this Part 2 establishes the *theoretical, mathematical, and conceptual framework* of the proposed interpretation. The question of its correspondence with experimental and observational results remains for the subsequent parts. *Part 3* and *Part 4* will be devoted to the analysis of experiments and observations traditionally presented as evidence for the validity of Einsteinian relativity, comparing these results with the interpretation developed in this series.

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