The “Spacetime” from a Realist Perspective: On the Foundations of Lorentz Transformations

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Abstract

This article inaugurates a series dedicated to the title *The “Spacetime” from a Realist Perspective.* The objective of the series is to conduct a critical analysis of the Lorentz transformations and, consequently, of the Special Theory of Relativity (STR), proposing an alternative conceptual approach to spacetime based on principles considered more consistent with observable physical reality. In this first part, the physical, mathematical, and logical foundations associated with the Lorentz transformations are examined, with emphasis on conceptual and kinematic aspects of their construction. An initial concise presentation of the theory and its fundamental equations is provided, followed by an analysis of the points that, according to this approach, raise questions regarding the conceptual consistency of the relativistic formulation. It should be emphasized that this stage of the work focuses exclusively on the theoretical and conceptual structure of STR. The discussion of the experimental and observational aspects will be addressed in the third and fourth parts of the series. Although the considerations in this work focus on STR, they will also address some conceptual aspects of the General Theory of Relativity. It should be noted that the considerations presented here are not intended to diminish the fundamental contributions of the creators of Special and General Relativity, among whom Poincaré, Lorentz, and Einstein stand out.

Keywords: spacetime, realist perspective, Lorentz transformations, Special Relativity.

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1. Introduction

The nature of space and time has been the subject of different interpretations throughout the history of Physics. Among the theories developed to describe physical phenomena and the relationships between spatial and temporal quantities, Einsteinian relativity, in its special and general formulations, occupies a central position. However, the interpretation of its physical and conceptual foundations continues to raise questions that warrant a critical examination of the assumptions on which it is based.

This work adopts a foundational physical-conceptual approach. It is based on the principle that mathematics should serve as a language for describing physical phenomena, rather than as an ontological substitute for physical reality [1]. Accordingly, abstract geometric structures are treated here as useful representational tools, rather than as physical entities in themselves. The analyses presented prioritize physical consistency, objective causality, and ontological interpretability over formal elegance. The aim is not to reformulate notations, but to re-examine conceptual assumptions that, although widely accepted, are rarely made explicit or critically discussed.

In this context, the expression "space-time" is used to designate space and time considered jointly, as elements necessary for locating physical events. Its use does not, in itself, imply the adoption of the geometric structure of space-time introduced by Minkowski [2]. For this reason, the expression appears in quotation marks in certain passages to indicate that it is not necessarily being used in the specific geometric sense associated with the Minkowskian formulation. The approach developed here exhibits conceptual affinities with Newton's formulation, particularly with regard to the distinction between true and apparent motions and the rejection of an exclusively relational conception of motion. It does not, however, constitute a mere return to Newtonian mechanics; rather, it seeks to critically re-examine certain elements of the classical tradition and explore their implications for the interpretation of physical phenomena. The proposed conceptual distinctions and their foundations will be developed in the following sections.

The mathematical level adopted is deliberately tailored to the objectives of this work, prioritizing the explicit statement of assumptions, clarity of argument, and the relationship between mathematical description and physical meaning. The formalism employed seeks to ensure the rigor required for the analysis without resorting to more elaborate mathematical tools than those demanded by the problems under consideration. This choice is intended to facilitate the understanding and critical evaluation of the arguments by readers with university-level training in Physics, Engineering, or related fields.

2. The Special Theory of Relativity

Before the formulation of the Special Theory of Relativity by Albert Einstein in 1905 [3, 4], classical physics – grounded in Newtonian mechanics and Galileo's principle of relativity – conceived space and time as separate, absolute, homogeneous, and universal entities, independent of the reference frame adopted. STR, however, broke with this conception by proposing that time and space constitute interconnected dimensions within a spacetime structure, and that spatial and temporal (or spatiotemporal) measurements depend on the observer's state of motion [4, 5, 6, 7]. In this context, two observers in uniform relative motion may measure different durations for the interval between two successive events, a phenomenon known as *time dilation*. This reformulation of the notion of time became one of the conceptual pillars of STR and remains a central point of debate among different interpretations of modern physics.

2.1. The Emergence of the Special Theory of Relativity

The Special Theory of Relativity originated in the attempt to resolve the "incompatibility"[1] between the foundations of classical mechanics and the results of electromagnetism, particularly those expressed by Maxwell's equations [3, 8]. At the time of its development, Newtonian mechanics prevailed, based on the Galilean transformations[2], also known as *Galilean relativity*.

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1. The term *incompatibility* appears in quotation marks because, from the perspective adopted in this work, an in-depth analysis demonstrates that this problem is merely apparent. It is argued throughout this study that the perception of incompatibility resulted from a conceptual misunderstanding related to the propagation of light.

2. The Galilean transformations consist of a set of linear equations that relate the spatial and temporal coordinates of two inertial reference frames, one of which is in uniform rectilinear motion relative to the other [5].

At the end of the nineteenth century, the hypothesis that space was filled with a fluid medium called the *ether* was widely accepted. This medium was considered the physical support for electromagnetic phenomena and interpreted as an absolute reference frame [3, 9]. Several experiments were conducted with the aim of detecting the motion of the Earth relative to this hypothetical medium [9]. The systematic failure of these attempts motivated the search for new theoretical explanations, both for the negative experimental results and for the alleged incompatibility between classical mechanics and electromagnetism. Although Albert Einstein is widely recognized as the central figure associated with special relativity, he was neither the only nor the first contributor to its formulation. The development of the theory was preceded by fundamental contributions from several researchers, among whom *Hendrik Lorentz* and *Henri Poincaré* stand out [3, 4]. Lorentz systematically presented the equations that today bear his name – the *Lorentz transformations* – providing the mathematical foundation of special relativity. Einstein, in turn, proposed a new physical interpretation of these transformations [4, 5], formulating the following postulates, which consolidated the theory:

First postulate: The laws of physics are the same in all inertial reference frames [4, 5].

Second postulate (in two versions for the sake of clarity):

Version 1: "Light propagates through empty space always with a certain velocity V, independently of the state of motion of the body that emits it." [4, p. 158, Piattella's translation]

Version 2: "Each ray of light moves in a coordinate system 'at rest' with the velocity defined as V, regardless of whether this ray of light is emitted by a body at rest or in motion."[3] [4, p. 160, Piattella's translation]

According to STR [7, 10, 11], for two inertial reference frames S and S', with S' moving relative to S with velocity v, time passes differently, with the measurements of the time interval related by the expression:

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} \tag{1}$$

where Δt is the time interval measured in reference frame S and $\Delta t'$ is the proper time interval measured in reference frame S'. The expression $\gamma = 1/\sqrt{1 - v^2/c^2}$, known as the *Lorentz factor*, arises naturally in the formalism developed by Hendrik Lorentz in the context of his attempts to explain the observed properties of the propagation of light in the luminiferous ether [3, 9, 12, 13]. Although originally associated with this ether hypothesis, the Lorentz transformations became established as a fundamental part of special relativity [4, 5], describing how time and space coordinates are related between two inertial reference frames in relative motion. The following presents a derivation of the Lorentz transformations, based on references [14, 15]. Some steps that are not essential to this discussion have been omitted.

[3] . Some authors argue that Einstein's second postulate of special relativity did not explicitly imply that the speed of light would be independent of the reference frame in which it is measured [3]. However, a careful analysis of the context of the Portuguese version of the book *The Theory of Special and General Relativity by A. Einstein*, published in 1920, shows that, when referring to the "law of the propagation of light in vacuum," he established exactly what is stated in *Version 2* of the second postulate. This is evidenced by the use of the expressions $x = ct$ and $x' = ct'$ to describe the propagation of light in reference frames *K* and *K′* (according to the notation used in the book), respectively [4, 5].

2.1.1. The Lorentz Transformations (Non-Axiomatic Approach[4])

Consider an event observed in two inertial reference frames: in reference frame S, the event is described by the coordinates (x, y, z, t), and in reference frame S', which moves along the x axis with velocity v, by the coordinates (x', y', z', t'). The observers are located at the respective origins O and O', with the x', y', z' axes parallel to the x, y, z axes. Since S and S' are inertial reference frames, the laws of mechanics apply equally to both. Thus, if a particle moves with constant velocity along a straight line in S, it also moves with constant velocity in S'. Under these conditions, the spatiotemporal coordinates of the event are related by a system of linear equations whose general form is:

$$\begin{aligned} x' &= a_{11}x + a_{12}y + a_{13}z + a_{14}t + a_{15} \\ y' &= a_{21}x + a_{22}y + a_{23}z + a_{24}t + a_{25} \\ z' &= a_{31}x + a_{32}y + a_{33}z + a_{34}t + a_{35} \\ t' &= a_{41}x + a_{42}y + a_{43}z + a_{44}t + a_{45} \end{aligned} \tag{2}$$

where the coefficients a_{ij} are constants. Choosing v along the x axis simplifies the analysis without loss of generality. The constants a_{15}, a_{25}, a_{35} represent possible initial spatial displacements of S' relative to S along the x, y, and z axes, respectively, and a_{45} represents the initial temporal offset. If the clocks of S and S′ are synchronized at the instant when the origins coincide ($t = t' = 0$)), then:

$$a_{15} = a_{25} = a_{35} = a_{45} = 0$$

and, based on the homogeneity of space and the postulates of STR, the following reductions are obtained:

$$y' = y, \qquad z' = z, \qquad t' = a_{41}x + a_{44}t, \qquad x' = a_{11}(x - vt) \tag{3}$$

To determine the coefficients a_{11}, a_{41}, and a_{44}, the *principle of the constancy of the speed of light* is applied. It is assumed that, at the instant when the origins coincide, a spherical light wave emanates from the common origin and propagates with speed c in all reference frames. The wavefront is described in S and S', respectively, by the equations:

$$x^2 + y^2 + z^2 = c^2t^2 \tag{4}$$

$$x'^2 + y'^2 + z'^2 = c^2t'^2. \tag{5}$$

Substituting x' and t' into the equations above and equating the terms, we obtain:

$$a_{11} = a_{44} = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma \quad \text{and} \quad a_{41} = -\frac{v}{c^2}\gamma$$

Substituting these values into equations (3) yields the *classical Lorentz transformation equations*:

$$x' = \gamma(x - vt), \;\; y' = y, \;\; z' = z, \;\; t' = \gamma\left(t - \frac{v}{c^2}x\right) \tag{6}$$

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4. For this discussion, the approach adopted to determine the Lorentz transformations – whether non-axiomatic, axiomatic, or based on the postulates of the Special Theory of Relativity [see Ref. 14] – is irrelevant, since all of them share a fundamental aspect: the consideration of an invariant limiting velocity, independent of the reference frame. Even when the speed of light is not explicitly mentioned, this constant plays an analogous role and, in practice, is identified with c. Therefore, the determination of the coefficients a_{11}, a_{41}, and a_{44} depends, directly or indirectly, on this principle, which is an essential point of the analysis.

Einstein derived these equations starting from the two postulates of Special Relativity and using a similar analysis of the spherical light wave[5] [4, 5].

Up to this point, a brief summary of the emergence of the Special Theory of Relativity has been presented. In the following section, a critical analysis of the coefficients a_{11}, a_{41}, and a_{44} will be presented, highlighting points that, according to this approach, warrant reconsideration.

3. Analysis of the Fundamental Coefficients of the Lorentz Transformations

The objective of this section is to examine the validity of the coefficients a_{11}, a_{41}, and a_{44} of the Lorentz transformations. Critics of this work may argue that these coefficients arise naturally from the direct application of the two postulates of the Special Theory of Relativity. It is therefore appropriate to clarify that this study does not question the principle of relativity or the constancy of the speed of light in vacuum. The central point of the analysis, however, lies in the following fundamental question: *Are the two postulates being applied conceptually and kinematically correctly within the context of Special Relativity?* This question will be addressed and discussed in the following subsections.

3.1. On the Meaning of the *Principle of Relativity*

Let us again consider the case described in subsection **2.1.1**, involving a light wave emitted from the origin of a coordinate system. According to Einstein [4, 5], based on the postulates of the Special Theory of Relativity, the equation describing the propagation of light must have the same form in all inertial reference frames, which implies that the equation of the spherical wavefront retains the same form in S and S':

$$x^2 + y^2 + z^2 = c^2t^2 \quad \text{and} \quad x'^2 + y'^2 + z'^2 = c^2t'^2.$$

In the Einsteinian relativistic approach, these two equations describe the same physical wavefront using the coordinates defined in the reference frames S and S', respectively [4, 5, 16]. It is observed that two equations of spheres describe *the same light wavefront* (the same physical phenomenon); however, both remain *permanently centered at the origins* of their respective reference frames. However, this does not follow from the first postulate.

The principle of relativity states that the laws of Physics take the same form in all inertial reference frames. Thus, an experiment conducted in an inertial reference frame S, under certain physical conditions, should produce results consistent with those obtained in an inertial reference frame S', when the relevant physical conditions are equivalent [4, 6]. This principle implies the invariance of the form of the physical laws under the transformations relating inertial reference frames. However, in the case of a light wavefront, this principle does not imply that the centre of the spherical wavefront must remain at the origin of the coordinate systems of both reference frames.

From the perspective adopted here, the principle of relativity requires only the preservation of the geometric form of the wavefront – *not the permanence of the center of the sphere at the origin of each coordinate system*, nor the implicit assumption that the reference frame is at rest relative to the physical center of propagation in space.

How Relativists Respond to This Problem

According to Whittaker [9], some physicists had already considered the idea of a wavefront centered at a "subsequently unchanged position" strange from the outset. The classical relativistic explanation for this paradox is as follows:

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5. In Einstein's original article, the coordinates of reference frame k were represented by the symbols (ξ, η, ζ, τ), corresponding, in this text, to (x', y', z', t') of reference frame S' [5, 14]. The coefficients of the transformations, here denoted by a_{11}, a_{41}, and a_{44}, were symbolized by Einstein as $\varphi(v)\beta$, $-\varphi(v)\beta v/V^2$, and $\varphi(v)\beta$, respectively, with V being the speed of light (equivalent to c in this text) and $\varphi(v) = 1$. The term v represents the velocity of *k* relative to *K* [4, 5].

> "The paradox is explained when it is remembered that a wavefront is defined to be the locus of points which are simultaneously in the same phase of disturbance. Now events taking place at different points, which are simultaneous according to A's system of measuring time, are not in general simultaneous according to B's way of measuring: and therefore what A calls a wave-front is not the same thing as what B calls a wave-front. Moreover, since the system of measuring space is different in the two inertial systems, what A calls a sphere is not the same thing as what B calls a sphere. Thus there is no contradiction in the statement that the wave-fronts for A are spheres with A's origin as centre, while the wave-fronts for B are spheres with B's origin as centre." [9, p. 36]

In addition, another argument frequently presented in the relativistic literature states that the wavefront equation is not an arbitrary choice, but rather the geometric expression of the set of events that satisfy the null spacetime interval:

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = 0, \tag{7}$$

which is invariant in the geometry of Minkowski space [5, 14, 17].

According to this interpretation:

- The wavefront would represent an objective set of physical events [4, 18], not "what the observer sees";
- Each reference frame considers its own source to be at rest at the origin, reproducing the experiment under symmetric conditions [4, 5, 17];
- If the equation had different forms in distinct reference frames, this would imply either a violation of the constancy of c, or the existence of a privileged reference frame [4, 5, 19];
- Therefore, in order to simultaneously preserve the symmetry between reference frames, the isotropy of space, and the constancy of the speed of light, it would be necessary to require the formal invariance of the wavefront equation [5, 19];
- Mathematically, this requirement is equivalent to preserving the spacetime interval, which leads to the Lorentz transformations [4].

However, this work maintains that these arguments do not resolve the conceptual problems involved, nor do they eliminate the physical paradoxes associated with the relativistic interpretation of the propagation of light. The reasons for this will be analyzed in detail in the following subsections.

3.1.1. Mathematics as a Descriptive Language, Not as a Creator of Reality

Mathematics constitutes a formal language of description, rather than an agent that creates physical reality [20, 21]. Regardless of any conceptual intention associated with their formulation, equations (4) and (5) of the wavefront have a well-defined meaning within the usual mathematical language: for a fixed instant t (or t'), they represent the locus of points in three-dimensional Euclidean space at a distance ct (or ct') from the origin, that is, a spherical surface centered at the origin. This meaning is not interpretive; it follows directly from the algebraic and geometric structure of the language employed [22].

The Special Theory of Relativity was originally formulated using the language of three-dimensional Euclidean geometry with time added as a coordinate [4], employed to describe the propagation of light in vacuum relative to material reference systems. In this context, the equations presented must be interpreted according to the geometric and physical meaning that this mathematical language explicitly conveys. It is not methodologically consistent to maintain that such equations should be read primarily in light of conceptual intentions associated with their formulation, to the detriment of the objective geometric content they express. In science, physical interpretations must respect the internal semantics of the formalism employed. If the direct geometric meaning of mathematical expressions does not correspond to the physical phenomenon one intends to describe, the problem lies in the use of the adopted language, and not in the reader's interpretation. Such a discrepancy cannot be disregarded merely in the name of formal elegance or the internal coherence of the theoretical formalism. When introducing the equations in question, Einstein did not create a new mathematical language, but rather used already established formal tools. Subsequently attributing new geometric meanings to expressions that were already well defined, without explicitly

altering the underlying mathematical structure, does not constitute a physical deduction, but rather a conceptual reinterpretation. What governs the informational content of an equation is not the author's intention, but the formal and semantic rules of the language in which it is written [23].

Therefore, it is not methodologically consistent to assert, *a posteriori*, that such an equation "does not represent a sphere centered at the origin" without explicitly redefining the meaning of the symbols involved. From a semantic standpoint, doing so is equivalent to writing $y = 2x$ and maintaining that the line does not pass through the origin – which is incompatible with the internal meaning of the language itself. The subsequent recourse to the geometry of Minkowski space [2] arises precisely in this context as a change in the structural interpretation of the same mathematical expression, rather than as an inevitable consequence of the initial physical postulates. One begins with a formulation written in terms of ordinary spatial geometry, obtains conceptually problematic physical consequences, and then redefines the geometric framework in order to justify such results. Thus, the introduction of the Minkowski structure does not follow directly from an observational physical requirement, but constitutes a formal choice intended to preserve the internal consistency of the relativistic formalism. It is therefore a mathematical modeling decision, rather than a physical necessity logically imposed by the phenomena.

The Special Theory of Relativity Was Not Based on Minkowski Space Geometry

The creators of special relativity, such as Lorentz [12, 13] and Einstein [4], among others, used the mathematics available at the time to propose solutions to specific physical "problems." For example, Einstein [4] used classical mathematical tools such as algebra, elementary differential calculus, and analytic geometry. He obtained the Lorentz transformations by imposing linearity, symmetry, and invariance of the speed of light [4, 5]. This did not involve the creation of a new mathematics, a "relativistic mathematics," but rather the application of classical concepts under new physical postulates, replacing the Galilean transformations with the Lorentz transformations. Therefore, it is not historically appropriate to invoke the geometry of Minkowski spacetime as the original foundation of the theory. Einstein did not use this geometric formulation in 1905 [4]; Minkowski only formalized, years later, results that had already been established, reorganizing them into a geometric structure [10]. Using Minkowski geometry as the foundation of the theory constitutes an anachronistic interpretation of its origin, since it takes as its structural basis a subsequent reformulation of the formalism.

3.1.2. Einstein Describes a Single Physical Phenomenon, Not Distinct Phenomena

In the traditional derivation of the Lorentz transformations, the equations

$$x^2 + y^2 + z^2 = c^2t^2 \quad \text{and} \quad x'^2 + y'^2 + z'^2 = c^2t'^2$$

do not correspond to two distinct emission events (one occurring at the origin of S and the other at the origin of S'), but rather to the description of the same physical event in two different reference frames. In his 1905 paper, Einstein explicitly states:

> "At the time $t = \tau = 0$, let a spherical wave be emitted from the origin of the coordinates of both systems, which at this time coincide, and let it propagate in system K with velocity V. If (x, y, z) is a point that has just been reached by this wave, then we have:
>
> $$x^2 + y^2 + z^2 = c^2t^2 \ .$$
>
> We transform this equation with the help of our transformation equations and obtain, after a simple calculation:
>
> $$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2$$
>
> The wave under consideration is therefore a spherical wave with propagation velocity V also when observed in the moving system." [4, p. 163, Piattella's translation]

It is therefore clear that this concerns the same real physical phenomenon described in distinct reference frames [24]. Thus, even though the mathematical notions of "wavefront" and "sphere" depend on the

systems of time and space measurement adopted, this does not alter the fundamental physical fact: both observers refer to the same real phenomenon of light-wave propagation, which occurs in physical space (vacuum) in which both the inertial reference frames and the light wave itself are located and move.

Therefore, even considering that the wavefront represents an objective set of physical events, rather than the observer's view, the aforementioned equations are not merely linguistic conventions. They assert something objective about the set of events that satisfy these relations. For each fixed value of t (or t'), the set of points forms a sphere, and every sphere has a geometric center. This follows from the mathematical structure of the equation itself. Thus, if the equation is taken as a description of a real physical phenomenon, then it describes a spherical distribution of events around a central point. This conclusion does not depend on the observer's opinion or on relativistic or non-relativistic intention or interpretation; it follows directly from the geometry of the solution.

As for the physical status of this center, since the equation is a description of a real physical phenomenon, it makes no sense to treat it as something purely conventional. After all, the spherical symmetry described by the equation is not a convention; it is a property of the phenomenon represented. From this perspective, if a physical equation describes a spherically symmetric distribution of events, then it necessarily describes the existence of a geometric center associated with that distribution. The existence of this center follows from the objective structure of the mathematical solution and not from the subjective choice of a coordinate system. Consequently, if the wavefront corresponds to a single objective physical phenomenon, the geometric center associated with its symmetry must also constitute an objective characteristic of that phenomenon. Thus, although different coordinate systems may assign different coordinates to this center, it is not methodologically consistent to consider that the same wavefront remains centered at the origins of reference frames that move relative to one another. If the spherical symmetry of the wavefront is an objective physical property, the center of that symmetry must also be objective and, therefore, cannot be permanently tied to origins that separate as a result of the relative motion between the reference frames.

3.1.4. The Mathematical Description Should Reflect the Physical Phenomenon, Not the Observer's Convention

Physical phenomena occur in physical space (spacetime). In the thought experiment considered by Einstein in the derivation of the Lorentz transformations [4], a light wave is emitted from a point in space corresponding to the common origin of the systems at the initial instant. After emission, the light does not remain kinematically bound to the source or to the coordinate systems. The light wave, the source, and the coordinate systems (and consequently their origins) constitute distinct physical entities, all embedded in the "same" physical spacetime and possessing their own motions. Thus, regardless of each observer's freedom to consider themselves at rest, there are relative motions in physical space between the light wavefront and the coordinate systems, just as there is relative motion between the coordinate systems themselves. From this perspective, the observers do not describe mere coordinate conventions, but an objective physical phenomenon. It becomes inconsistent to represent the propagation of light by means of spherical wavefronts whose centers, over time, remain coincident with the moving origins of the respective coordinate systems, since these origins also move through physical space after the emission event. The mathematical description should reflect the physical phenomenon being observed, rather than impose an abstract geometric structure on nature merely to maintain mathematical consistency without physical correspondence. Coordinates label real physical positions and relations: they are arbitrary labels for these entities, and not merely mental constructions of the observer [24, 25, 26]. However, equations (4) and (5) contradict the physical scenario described above.

3.1.5. The Invocation of the Relativity of Simultaneity Does Not Resolve the Paradox of the Two Spherical Wavefronts

The relativity of simultaneity is frequently invoked to justify the coexistence of two spherical wavefronts centered at the origins of reference frames in relative motion [9]. However, this argument does not resolve the paradox for the following reasons:

- Even though observers in relative motion do not agree on the simultaneity of spatially separated events, this finding does not invalidate the fact that equations (4) and (5) describe spherical wavefronts centered at the respective origins. Therefore, even if what is simultaneous for S is not simultaneous for S′ and vice versa, through the aforementioned equations, each observer describes the wavefront as being permanently centered at the origin of their own reference frame.

- The equations used in this context are not intended to describe perceptual acts of observation, but rather the physical geometry of the propagation of the light phenomenon. In the scenario presented by Whittaker [9], it is assumed that *a light pulse is emitted* when the origins of the systems coincide and that, during the subsequent propagation, spherical wavefronts associated with each reference frame emerge. Such a formulation necessarily implies that the equations describe the geometry of the expansion of the light wave in physical space as attributed in each coordinate system, and not merely a reconstruction.

It could eventually be argued that the equations

$$x^2 + y^2 + z^2 = c^2t^2 \quad \text{and} \quad x'^2 + y'^2 + z'^2 = c^2t'^2$$

do not describe two spheres with centers at the origins of two reference frames that are moving apart, but merely represent conditions on the set of points under consideration, which may, in a later interpretation, be associated with the so-called light cones. However, even if this interpretation could correspond to a later formulation of the theory, it should not be used to retroactively attribute to the equations the geometric meaning of a language that was not part of Einstein's original formulation [27]. As mentioned in Section **3.1.1**, the authors of the theory of relativity used, in their original works, the mathematical and geometric language available at the time to formulate solutions to the physical problems they were investigating. This was not a language devoid of geometric meaning. The equations employed had specific meanings in the context in which they were introduced. In Einstein's case, this is particularly clear. When introducing

$$x^2 + y^2 + z^2 = c^2t^2,$$

he explicitly considers a spherical wave emitted from the origin and, after the transformation, obtains

$$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2,$$

the notation used in the 1905 paper. Einstein [27] then states: "Die betrachtete Welle ist also auch im bewegten System betrachtet eine Kugelwelle von der Ausbreitungsgeschwindigkeit V." [27, p. 901], that is, "The wave under consideration is therefore also, when considered in the moving system, a spherical wave with propagation velocity V."

Thus, in the original context, each equation represents a spherical surface centered at the origin of the respective coordinate system. If the first system is S, the first equation represents a spherical wavefront centered at the origin of S; if the second is S', the second represents a spherical wavefront centered at the origin of S'. These origins, however, correspond to distinct positions when the reference frames are in relative motion. Therefore, two spatially distinct points cannot represent the same spherical center, just as two distinct centers cannot, without additional justification, represent the same spherical wavefront.

The interpretation of these equations as "light cones" belongs to a later spacetime formulation [2] and should not be used to retroactively reinterpret the meaning they had in Einstein's original work. In the geometric language employed in that context, for each fixed instant, the equations describe spherical surfaces; the family of these surfaces constitutes an expanding spherical wavefront. Therefore, in their original formulation, these are not conical surfaces, but spherical surfaces.

The issue, therefore, is not to deny the possibility of a later mathematical interpretation of these relations, but to question the legitimacy of using this later interpretation to replace the geometric meaning of the original formulation and, in this way, circumvent the problem that emerges when the equations are interpreted according to the language in which they were originally presented.

3.2. On the Speed of Light

The discussion of the "speed of light" requires that we first understand what is meant by speed. Speed is the rate of change of an object's position in space over time relative to another object. It is, therefore, always a relational quantity [5, 7]. The term object here should be understood in the generic sense and may refer to a substrate. Albert Einstein [4, 5] stated that "the speed of light in vacuum is constant and does not depend on the motion of the source or the observer." However, there is a problem with the usual interpretation of this principle, since it involves an overlap of two distinct conceptual levels:

- the natural propagation speed of the light wave (relative to the propagation medium);
- the speed of the light wavefront relative to a reference body.

Understanding these two levels requires an analysis of the fundamental concepts of space, time, and motion, upon which the notion of speed is based.

3.2.1. The Concept of Space

Throughout the history of science, space has been conceived in different ways. These conceptions can be summarized, according to [28], into two fundamental positions:

- **Conception 1:** space does not exist independently of material objects; there is no absolutely empty space.
- **Conception 2:** space exists independently of material objects, admitting the possibility of absolute emptiness.

These conceptions are mutually exclusive: space cannot simultaneously depend on and be independent of matter. The first conception raises fundamental conceptual difficulties, such as:

1. What separates material objects?
2. What makes processes of displacement and expansion possible?
3. In what domain are geometric properties such as direction, orientation, and angle defined?

Such questions find a consistent answer only within the scope of the second conception. The distinction between material objects requires the existence of a non-material entity that makes this separation possible. From the perspective adopted in this work, partially aligned with the second conception, space is defined as:

A non-material physical reality independent of bodies, which constitutes the domain of extension in which objects exist and phenomena occur.

However, the possibility of absolute emptiness is not defended here. Space allows the definition of geometric properties – such as position, distance, and direction – but does not, in itself, possess interactive physical properties. Its characterization depends on the presence of material objects, which introduce measurable distinctions. In the absence of bodies, space does not cease to exist, but loses operational meaning, since there are no references for defining its geometric properties [5]. In this sense, there is a partial approximation to the perspective of Ernst Mach [29], according to which observable properties have a relational character [30]. However, such relations do not constitute space itself, but only the physical properties established within it. A useful analogy is that of a blank sheet of paper: without markings, there are no defined divisions. The introduction of a grid makes it possible to establish regions and geometric relations, but these constructions do not define the paper. When the grid is removed, the divisions disappear, but the underlying support remains. Similarly, space is not to be confused with objects or with the geometric structures defined by them.

On the Absolute Character of Space

The notion of absolute space was introduced by Isaac Newton [6] in the context of classical mechanics. In this formulation, space is conceived as a fixed and universal structure, independent of objects, which serves as a reference for defining states of motion. However, the ontological status of this space and its role in physical description have been widely debated [31]. In the approach proposed here, this notion is reformulated in the following terms:

Space is absolute in its existence, but not as a privileged material reference frame.

The absolute character of space refers to its *ontological independence, its invariance and universality, and not to the existence of an absolute state of rest associated with material bodies*. Thus, space constitutes a real and permanent domain in which physical phenomena occur, without, however, imposing a privileged material reference frame. In this way, a clear conceptual distinction is established:

- *Background structure (absolute)*: space as an independent physical domain;
- *Geometric description (relational)*: properties such as position, distance, and motion, defined in relation to objects or reference systems.

This separation makes it possible to preserve the objectivity of space as a "physical" entity while recognizing the relational character of measurable quantities. The result is a conceptually more consistent formulation that avoids ambiguities between the ontological substrate of phenomena and their mathematical description.

The Absolute Character of Space and the Principle of Relativity

It could be argued that the validity of the principle of special relativity excludes the possibility of absolute space. However, the concept of absolute space adopted in this work does not conflict with this principle. The principle of relativity establishes that the laws of physics take the same form in all inertial reference frames, implying the absence of privileged inertial systems for the formulation of these laws. In other words, all inertial reference frames are physically equivalent, with no body or material system being in a privileged condition in space. It is therefore a principle that refers to bodies and physical systems used as reference frames [4, 5], and not a statement about the nature of space itself.

In this context, inertiality constitutes a property attributed to bodies and physical systems, defined in relation to space, and is not a property applicable to space itself. Similarly, motion and rest are concepts attributed to bodies and physical processes. It makes no sense to state that space is in motion or at rest, since such states would require an additional spatial structure relative to which they could be defined. Thus, the principle of relativity does not directly address the existence or nonexistence of an absolute spatial structure. Accordingly, the admission of absolute space does not imply the existence of a privileged body or of a material reference frame in absolute rest [9]. In principle, space may possess an absolute character without any body being at rest relative to it, and it is possible that all bodies are in motion [7]. Consequently, the absence of privileged bodies or material reference frames in absolute rest does not imply the nonexistence of an absolute spatial structure.

3.2.2. Absolute Space and the Physical "Medium of Interaction"

In addition to absolute space, the existence of a distinct entity is proposed:

- *Medium of interaction:* a non-material physical medium (or non-material in the classical sense), responsible for making interactions between objects possible, including interactions at a distance.

This medium does not move through absolute space, does not produce drag effects, and is not carried along by bodies. It is a physical structure associated with space, homogeneous and isotropic, whose properties can be modified by the presence of matter. Unlike absolute space, this medium has a dynamic character and constitutes the substrate for the continuous and local propagation of physical interactions. Without it, the physical explanation of the continuous and local propagation of interactions between spatially separated

bodies would become problematic [5]. Matter alters the physical properties of this medium, giving rise to configurations that manifest themselves as physical fields, such as electromagnetic and gravitational fields. The idea of a physical medium underlying the propagation of electromagnetic interactions was already present in the work of James Clerk Maxwell [32, 33]. Subsequently, Albert Einstein, although he rejected the classical mechanical ether, admitted the existence of a physical structure associated with space, sometimes described as a kind of ether [1, 5]. The conception proposed here is partially close to Maxwell's in recognizing the physical reality of fields and interpreting them as states or configurations of a fundamental medium, rather than as ontologically independent entities. However, this formulation *does not recover the classical mechanical ether*, and *no detectable material medium is postulated here*.

3.2.3. Space as an Indispensable "Background"

Just as the Sun exists independently of the theories that describe it, the existence of space is independent of theoretical formulations and constitutes a necessary condition for any physical description [34]. Even in theories formulated in four-dimensional spacetime [2, 5], three-dimensional space remains indispensable as an underlying structure for the interpretation of phenomena; that is, its physical representation continues to depend on spatial directions. The coordinates x, y, and z are not merely mathematical labels, but represent physical directions in which objects exist, interact, and move. Without these dimensions, it would not be possible to define distances, volumes, orientations, or the propagation of waves. Thus, although the mathematical formalism reorganizes the description of phenomena, it does not eliminate the need for a domain of extension or the ontological reality of space. From this perspective, space should not be understood as a container. A container is a material structure that delimits a region in space and therefore depends on it. Regarding this matter, Barbour [35] states:

> "Space and time in their previous role as the stage of the world are redundant. There is no container. The world does not contain things, it is things. These things are Nows that, so to speak, hover in nothing." [35, p. 16]

However, from the perspective adopted in this work, this "nothing," in which the "things are Nows that, so to speak, float," corresponds to absolute space itself.

3.2.4. The Concept of Time

The notion of time is one of the deepest and most controversial issues in both physics and philosophy. Throughout history, different conceptions have been proposed. According to [28], these conceptions can be organized into two main conceptions:

> ***Conception 1:*** *Time is an order of events; a way of measuring the changes observed in the world. Time is not an autonomous entity.*
>
> ***Conception 2:*** *Time is an autonomous entity that flows independently of any change. If all changes were to cease, time would continue to flow.*

According to these conceptions, time has the following characteristics, considered critical here: it flows uniformly [6]; it does not exist by itself, being merely a relation between events [35, 36]; without changes, there is no time [35, 37]; time depends on the state of motion and the gravitational field [5]; time has a direction due to the increase in entropy [38]; time depends on the perception of consciousness [39, 40]; time emerges from more fundamental processes [41, 42]; past, present, and future coexist [43, 44].

Conceptual Problems with These Approaches

- **"Time flowing uniformly" and the "passage of time":** The idea that time "flows" or "passes" arises, to a large extent, from a metaphorical interpretation associated with the transformations observed in physical systems [45]. However, time cannot literally be conceived as something that moves, runs, or flows, since speed and motion already presuppose time in order to be defined. Thus, attributing a rate to time itself leads to a conceptual problem: the notion of temporal flow implicitly

uses that which it intends to explain [43]. When one states, for example, that "time passed quickly," this does not mean that time acquired a greater speed. What actually occurred was an intense succession of physical changes, material states, or subjective experiences within a given interval. Therefore, it is not time that flows; it is physical systems that undergo change.

- **Relational time:** If time were merely a relation between events, it would not be possible to speak of "before" or "after" the relation itself; nevertheless, this is done naturally. Therefore, admitting priority and posteriority implies recognizing time independently of relations.

- **Relativistic time: the identification of time with physical processes:** The central problem with the relativistic conception of time lies in the implicit identification between time and the behavior of physical processes [4, 5]. Clocks are physical systems, and their changes reflect specific material conditions; differences in their rates do not imply changes in time itself. What is experimentally observed are variations in the behavior of systems subjected to different conditions, such as different states of motion or different gravitational fields. Two coexisting systems may evolve according to different process rates without this implying time dilation. For example, while the Earth completes a certain number of revolutions around the Sun, Mars completes a different number. The difference between these cycles does not imply a temporal alteration, but merely differences between physical processes. Both systems coexist within the same objective context of events. It is not denied that processes may have different durations; only a distinction is made between time and the amount of process activity. The problem becomes even more evident when considering the very idea of the "transformation" (dilation) of time. Every transformation constitutes a process, and every process presupposes a "when" in which it occurs. Thus, stating that time transforms implies presupposing beforehand a temporal order in which that transformation would occur. This leads to a conceptual regression: if time could change, another time would be necessary to order that change. From this perspective, it makes no sense to attribute a proper and ontologically distinct time to each coexisting physical system.

- **Coexistence of past, present, and future:** The block universe theory [43] maintains that past, present, and future events coexist equally, differing only in their position within the four-dimensional structure of spacetime. From this perspective, the birth of Albert Einstein, his death, and any future events possess the same ontological reality. This interpretation, however, raises conceptual difficulties. First, it becomes necessary to clarify the meaning of *exist*. If a future event already objectively exists, in what sense can it be said that it will still occur? Likewise, if a past event continues to exist, in what sense can it be said that it occurred? Furthermore, how does an event cease to be present and become past? The assertion that all events are equally real does not answer these questions, but shifts the problem to the very definition of existence.
  Similar difficulties arise with respect to causality. In its usual meaning, to cause is to produce or bring about an effect. However, if cause and effect possess equal ontological reality, in what sense does one event cause the other? If Albert Einstein's death already exists in the structure of spacetime, in what sense did his illness cause it? The nature of the dependence between cause and effect also remains unexplained: is it a logical, physical, mathematical, or ontological relation? Furthermore, it is necessary to explain why causality is asymmetric. If both events coexist equally, why is it said that the illness causes the death, and not the reverse? Mere temporal ordering appears insufficient to justify this asymmetry within a conception in which time does not objectively elapse.
  Another difficulty concerns the passage of time. In the block universe, there is no objective transition between present, past, and future; all events remain fixed in their respective spatiotemporal positions. Thus, Einstein's birth never objectively becomes past, but is merely considered past by observers situated at later events. Consequently, the distinction between past, present, and future becomes relative to the position occupied within the block, rather than to an objective transformation of reality. The question then arises of how to understand the succession of events and the very dynamics of the universe if there is no objective process of change or actualization of events. Similarly, it becomes necessary to clarify how an observer situated at a given event can obtain information about events classified as past.

Conscious experience introduces an additional difficulty. Although the block universe attributes equal reality to all temporal states of an observer, consciousness appears to be invariably associated with what is called the present. The past is remembered and future events are anticipated, but different temporal stages are never experienced simultaneously. The question therefore arises as to why conscious experience is located at only one event if all events possess equal ontological reality. It is not sufficient to state that each point on the worldline corresponds to the present of the observer situated at that point, since this answer presupposes precisely what needs to be explained. If all states of the observer exist equally, what constitutes the unity of the conscious subject? What distinguishes the state currently being experienced from the other equally existing states? And how is the succession of experience to be understood if no objective transition between present, past, and future occurs?

Thus, if the block universe conception attributes equal ontological reality to all events, it must provide a conceptually precise explanation of the meaning of existence, causality, causal asymmetry, temporal transition, and conscious experience. As long as these concepts remain merely assumptions or are redefined without an explicit account of their ontological content, this interpretation appears to leave fundamental problems concerning the nature of time and physical reality unresolved.

- **Thermodynamic time and the confusion between process reversibility and temporal direction:** The interpretation according to which an increase in entropy demonstrates that "time only moves forward" constitutes one of the most widespread conceptions concerning the arrow of time in physics and the philosophy of science [38, 42, 43, 45, 46]. From this perspective, thermodynamic irreversibility is frequently taken as a manifestation of temporal direction [38, 42, 43, 45, 46]. The main problem with this approach, however, consists in identifying process irreversibility with temporal direction. When one states, for example, that "a broken glass does not spontaneously return to its original state, therefore time moves forward," one draws a conclusion about the nature of time from the behavior of a specific physical process. However, even if the glass were to reassemble naturally, this would not imply a temporal return; it would merely represent a new physical process occurring subsequently. The reassembly of the glass would not constitute temporal reversal or a return to the past, but rather a return to a new future physical state, a replica of a past state, which would become present. Priority, simultaneity, and posteriority would continue to exist among events. Thus, process reversal is not equivalent to temporal reversal.

  Even if certain physical systems exhibited a spontaneous reduction in entropy, an objective order among events would continue to exist. What would change would merely be the behavior of physical processes, and not temporal order. Thus, the so-called "direction of time" does not necessarily follow from an increase in entropy. The temporal ordering of events would remain even in hypothetical scenarios in which physical systems evolved in a direction opposite to that thermodynamically observed. Entropy describes properties of certain physical processes; it does not constitute a foundation for defining the nature or "direction of time."

- **Psychological time:** Perceptions vary among individuals; time, in the physical sense, does not depend on this. Time is not the way in which it is perceived.

- **Emergent time:** If time emerged from more fundamental processes, these processes would occur "before" time, which already presupposes time. Thus, the idea is contradictory, since every process presupposes a "when" in which it occurs and a "when" in which it does not occur.

3.2.5. Conception of Time According to the Proposed Approach

In view of the limitations of the previous approaches, the following conception is proposed:

Nature of Time

From the perspective adopted here, time does not constitute a physical process, a material configuration, a dynamic entity, nor does it consist of the number of activities or the rate of transformation of physical

systems. Nor does it constitute a property of clocks. Before defining the concept of time, it is appropriate to establish the concept of *physical existence*.

Definition of existence: *to exist is to occupy space.* A system exists if, and only if, it occupies a region of space. Objects exist by occupying a given spatial region. Every physical system is necessarily spatially localized. A system that does not occupy any region of space cannot interact, possess physical properties, or be part of physical reality. In this sense, the occupation of space constitutes a necessary condition for physical existence. For example, Albert Einstein no longer exists because the physical configuration that constituted him ceased to occupy space. The particles that composed his body continue to exist because they still occupy regions of space, but they are no longer organized in the configuration corresponding to the organism that was identified as Albert Einstein.

Time: is associated with the condition of existence of objects. It characterizes the "when" of the existence and the "when" of the non-existence of physical or non-physical systems, states, or configurations. Thus, it can be stated that an object did not yet exist at a given moment or that it ceased to exist at another. There is, therefore, a before, during, and after with respect to the existence of each system under consideration. Thus, *time can be understood as the "when" of the occupation or non-occupation of space; it is an abstract reference used to order the successive occupation of space.* Every event presupposes a "when" of occurrence, just as every material existence presupposes a spatial location. Time does not constitute a container in which objects exist. It merely orders the occupation of space.

Time does not depend on motion, matter, physical interactions, or numerical representations; it does not depend on changes. Changes do not create time; they merely make it possible to establish relations of priority, posteriority, and simultaneity or coexistence between events.

Past, Present, and Future

Past, present, and future correspond to distinctions relative to the order of physical occurrences or the occupation of space.

The past does not exist; it concerns the "when" of configurations or physical states prior to the current one, which occupied space but ceased to occupy it as a consequence of changes and transformations of the systems themselves.

Present is the "when" of the occupation of space, that is, of being in some region of space.

The future does not exist. The future corresponds to the "when" of states or configurations that do not yet occupy space but may come to occupy it.

Thus, past and future do not designate regions where objects exist, but qualify, respectively, configurations that have ceased to occupy space and configurations that do not yet occupy it. These distinctions are associated with the condition of existence of physical systems in space and with the objective order of their occurrences.

On the "Now"

From a mathematical point of view, the "now" can be represented as an idealized instant, that is, as a point on the temporal axis with no duration [47]. In physical reality, however, the "now" does not correspond to a strictly null instant (that is, a temporal point with a duration exactly equal to zero), but rather to a temporal interval in which events occur. This is because every experience, observation, or description of an occurrence necessarily involves a duration [39, 48]. For example, when someone states that "the car passed just now," this statement does not refer strictly to the exact instant at which the sentence is uttered, but to a temporal interval sufficiently close to be identified as current. The instant, in this sense, constitutes a useful mathematical idealization for the description of phenomena, but does not directly correspond to an observable physical occurrence. Thus, the practical "now" consists of past and present events.

From the perspective adopted here, objectively, *there is a universal "now" in the sense that multiple events coexist simultaneously in the universe.* While a physical system is in a given current condition, countless other systems objectively coexist in their respective conditions. This coexistence, however, does not imply immediate access to the current state of distant systems. The immediate sharing of this "now" between spatially separated regions is prevented by the finite speed of information propagation [4, 49]. Thus, observers located in different regions of space do not have instantaneous access to information about one another's "now." When information originating from a distant event is received, it already corresponds to a previous state (the past) of that system [49]. Thus, the perception of the "now" is relative to the

conditions of observation and information transmission. However, these limitations are epistemological in nature, since they concern access to events and not the objective existence of the events themselves. The coexistence of events therefore remains independent of the particular conditions of observation.

Events, Temporal Reference, and Operational Time

Each physical system is associated with a sequence of events that coexist with countless other events distributed throughout the universe. The physical description of time is based precisely on making explicit the objective relations of coexistence, succession, and coincidence between these events [4, 49]. Clocks are physical systems in which regular processes occur and are used as references for the temporal description of other events [4, 51]. When a clock is used to date an occurrence, only a coincidence is established between the observed occurrence and a particular state of the clock. For example, when it is stated that a ball fell to the ground at 7:30 p.m., this means only that the event "fall of the ball" occurred in coincidence with the event "the clock showing 7:30 p.m." Thus, the clock reading is not time itself, but a numerical convention used to represent the "when" of an occurrence. Identifying time with clock readings leads to confusion between existential order and its operational representation.

Thus, it becomes necessary to distinguish between *objective time (universal)* and *operational time*. *objective time* corresponds to the real order of physical occurrences [50], that is, to the objective "when" of events. This order is independent of perception, measurement instruments, and the reference frames adopted by observers. *operational time*, on the other hand, corresponds to the physical representation of this order through clocks, synchronization systems, and models of temporal description [4, 5, 50]. In other words, *operational time* corresponds to the quantification of *universal time* by means of a physical reference process (*how much in the when*). Temporal coordinates have an operational character [4]. Different observers may assign different values to the same event without this implying multiple objective times.

From the perspective adopted in this work, *objective time is absolute*, whereas *operational time is relative to the standard and to the conditions of the physical process used for its determination*. Universal time does not possess, in itself, a numerical scale or an intrinsic unit. Its quantification requires a temporal reference standard, just as the quantification of a spatial extension requires a unit established as a term of comparison. In both cases, the unit does not constitute the quantity itself, but merely the standard used to represent it quantitatively. This conception can be illustrated by what is referred to here as the *time diagram* **(see Fig. 1)**, in which the first line represents the same interval of universal time, while the lower lines represent the operational scales of two clocks, A and B. If the physical processes used as references by these clocks exhibit different rates of activity, the same interval of universal time may correspond to different numbers of subdivisions on their respective scales.

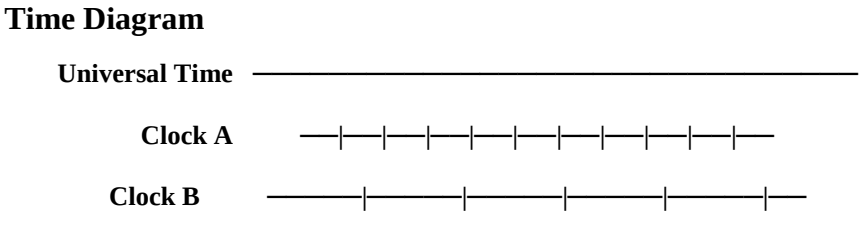


Figure 1 – Temporal diagram illustrating the distinction between objective (universal) time and operational time.

The equality of the extent of the three lines represents the same interval of universal time, while the difference in the number of subdivisions represents only the difference between their respective operational scales. It should be noted that *each vertical mark on the operational scale of B coincides, at the same point on the universal time line, with two marks on the operational scale of A*. This suggests that, within the same interval of universal time, clock A performs a greater number of activities than clock B. Thus, different clock readings may correspond to the same objective temporal interval, without this in itself implying different universal times. In this sense, the clock does not determine time nor directly measure universal time; its indication constitutes an approximate and relative quantitative representation of that time, obtained through a physical process taken as a reference. In contrast, the parameter t used in the formulation of physical laws ideally represents universal time, that is, the common parameter associated with the objective

when of physical events. The relationship between universal time and its different operational representations, as well as the consequences of this distinction for the description of physical phenomena, will be addressed in greater detail in later parts of the series.

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Epistemological Limitation and Propagation of Information

Although events possess objective relations of coexistence and succession, there is a fundamental limitation on access to these relations: information propagates at finite speeds [4, 49]. Observers do not have immediate access to distant events or to the totality of objective occurrences in the universe. Thus, perceived, measured, or inferred time depends on the propagation of physical signals, the observer's position, the conditions of the medium, the instruments employed, and the models used in the temporal reconstruction of events [49, 50]. In this way, operational time always constitutes a conditioned reconstruction of the order of occurrences.

On Simultaneity

The question of simultaneity occupies a central position in the foundations of modern physics [4, 5, 52]. In classical mechanics, the existence of an objective and universal simultaneity was implicitly assumed, valid independently of the state of motion of observers: two spatially separated events that occurred "at the same time" for one observer should likewise occur at the same time for any other observer. This conception was associated with the idea of absolute time, conceived as a physical entity independent of measurement processes and particular conditions of observation [6]. At the beginning of the twentieth century, Albert Einstein [4, 5] proposed a radical reformulation of this concept within the context of special relativity. According to his approach, simultaneity between spatially separated events does not constitute an absolute property of the events themselves, but depends on the observer's state of motion. The operational definition of simultaneity then becomes based on the propagation of light signals and on the hypothesis that the speed of light in vacuum has the same value in all directions for all inertial reference frames [4, 5].

To justify this new conception, Einstein introduces the well-known thought experiment involving two lightning flashes that simultaneously strike points A and B along a railway track [5]. An observer located at the midpoint M of the track receives the light signals from the two events simultaneously and therefore concludes that the flashes occurred at the same time. Next, a train moving along the railway track is considered, containing a second observer located at its midpoint M'. Since the train moves toward one of the light signals and away from the other, the observer at M' receives the signal from B first and then the signal from A. Based on this, Einstein concludes that the events are not simultaneous in the train's reference frame [5]. However, this argument presents a significant conceptual difficulty that is frequently overlooked in traditional presentations of the relativity of simultaneity. The problem lies in the fact that Einstein's explanation explicitly attributes the observed difference to the observer's motion relative to the light fronts. The observer at M' receives the signals at different instants because, according to the description presented, the observer moves toward one of the signals and away from the other during the propagation of light.

However, this explanation becomes problematic when analyzed in a rigorously symmetrical manner. The midpoint M of the railway track also has motion relative to numerous other possible physical reference frames, such as, for example, reference frames associated with the Sun, the galaxy, or any "inertial" system external to Earth [7]. Consequently, from the perspective of these external reference frames, the observer at M also moves relative to the light fronts originating from A and B, approaching one of them and moving away from the other during the propagation of light. In this case, the same causal mechanism used to justify non-simultaneity on the train should likewise produce an analogous effect for the observer on the track. In other words, if the relevant physical cause of non-simultaneity consists of the observer's motion relative to the light signals, then such an effect cannot be attributed exclusively to the train, since the track is also in motion relative to other reference frames; that is, the track is also a "train" in a reference frame external to Earth [5]. The conceptual difficulty arises from the fact that Einstein's argument implicitly treats the reference frame of the track as privileged with respect to light propagation. Although special relativity explicitly denies the existence of absolute reference frames, the thought experiment presented introduces an implicit physical asymmetry: the motion of the train relative to light is considered physically relevant,

whereas the motion of the track relative to light is ignored in the causal analysis of the phenomenon. Thus, the conclusion that the events are simultaneous on the track but not on the train follows from serious conceptual problems. The experiment, therefore, does not establish, in a logically necessary manner, the relativity of simultaneity, but merely shows that different observers may receive light signals at different instants due to the relative motion between the observer and the propagating signals.

An additional difficulty of the thought experiment proposed by Albert Einstein lies in the conceptual ambiguity present in the use of the term "simultaneity." Although special relativity is presented as a denial of absolute simultaneity, the very structure of the experiment appears to implicitly presuppose a distinction between the instant at which physical events occur and the instant at which light signals are received by different observers. In the experiment, Einstein initially considers two well-defined physical events: the lightning flashes occurring at points A and B along the railway track [5]. These events are initially presented as occurring simultaneously [5]. The propagation of the light signals emitted by these events is then analyzed as they travel toward different observers: the observer at M, located at the midpoint of the track, and the observer at M', located at the midpoint of the moving train. However, throughout the argument, it does not become conceptually clear to which set of events the term "simultaneity" is actually being applied. At certain points, simultaneity appears to refer to the physical occurrences of the lightning flashes at A and B themselves; at others, it refers to the temporal coincidence of the arrival of the light signals at the observers. This semantic oscillation produces a fundamental ambiguity in the experiment. Indeed, there are at least two distinct levels of occurrences in the problem:

1. the primary physical events, corresponding to the lightning strikes at A and B;
2. the secondary events of perception, corresponding to the arrival of the light signals at the observers.

These two levels are not equivalent.

The temporal coincidence between perceptual events does not necessarily imply temporal coincidence between the physical events being observed, just as a perceptual delay does not necessarily imply an objective delay between the original physical occurrences. The finite propagation of light signals necessarily introduces a separation between the physical occurrence and its perception by observers situated at different positions or in different states of motion [49]. However, Einstein's argument moves from the difference in the signal reception times to a conclusion about the simultaneity itself of the original physical events. In other words, the non-simultaneity of perceptual events is converted into non-simultaneity of the observed events. This transition, however, is not physically justifiable.

Consequently, the argument presents a fundamental conceptual ambiguity between the physical simultaneity of events and the perceptual simultaneity of light signals. This ambiguity compromises the logical clarity of the experiment and weakens its claim to establish, conclusively, the inexistence of absolute simultaneity. At most, the experiment establishes that different observers may receive light signals originating from the same pair of events at different instants, a fact that, by itself, does not constitute sufficient evidence that the original physical events themselves did not occur simultaneously. This difficulty is not restricted to the experiment in question, but is also present in various relativistic approaches to simultaneity, in which simultaneity between two events A and B (objective occurrence) is defined as a function of other events C and D, corresponding to the reception or perception of the light signals originating from A and B [7]. In other words, the simultaneity of the original physical events is inferred from the simultaneity of observation events. Thus, the definition of simultaneity becomes dependent on perceptual events rather than directly on the events whose simultaneity one seeks to establish. To avoid these ambiguities surrounding the concept of simultaneity, it becomes necessary to distinguish three levels of simultaneity:

- **Simultaneity of occurrences:** refers to the actual coincidence between the physical occurrences of events or to the *same order of occurrence*, independently of observers; it is absolute in nature. However, due to the finite propagation of information, this simultaneity is not directly accessible to observers.
- **Simultaneity of perception:** refers to the temporal coincidence in the reception of signals originating from the events, depending on the propagation of physical signals and the conditions of observation. It is relative.

- **Inferred simultaneity:** corresponds to the operational reconstruction of the instants of occurrence of events performed by the observer on the basis of observational information, depending on synchronization procedures, physical models, and reference systems. It is relative.

Thus, divergences between observers refer to operational reconstructions of simultaneity, and not necessarily to the objective order of physical occurrences.

3.2.6. Space and Time: Connected, but Conceptually Distinct

Certain approaches argue that space and time constitute inseparable components of a single physical entity – the *spacetime*. In this work, the operational coexistence of both in the description of events is not questioned, since space and time are indispensable for characterizing any physical phenomenon [2, 5, 49]. However, this does not imply that they are identical or interchangeable. The relationship between space and time may be compared to the relationship between the base and exponent in a power: there is no power without a base, nor without an exponent; nevertheless, the base is not the exponent, and the exponent is not the base. Similarly, space and time are jointly necessary for the definition of events and physical reality, constituting the domain of occurrences (the "spacetime"), without losing their own conceptual functions. Space defines the domain of extension, specifying "where" phenomena occur, whereas time orders occurrences and characterizes their duration, indicating "when" they occur. The trajectories of objects are interpreted here as successions of positions in space organized temporally. Although relativistic physics describes phenomena within a spatiotemporal structure [2, 5], this formulation does not eliminate the functional distinction between space and time. Recognizing this distinction avoids ambiguities arising from purely geometric interpretations of spacetime.

Limitations of the Ontology of Events

In contrast to the conception adopted here, some contemporary approaches propose replacing the classical ontology of space, understood as an independent domain whose points persist over time, with an ontology of events in spacetime [53]. A representative example is the proposal of Tim Maudlin [53], in which the distinction between absolute space and absolute time is abandoned in favor of a unified structure composed of events. From this perspective, an event is defined as "a place in space at a time" [53], being treated as a fundamental, point-like, and extensionless entity.

However, this reformulation faces significant conceptual difficulties. The very definition of an event relies on notions that presuppose what it seeks to eliminate. The expression "place in space" implies the existence of a previously identifiable and re-identifiable spatial structure. A place does not constitute an event, but rather a spatial position whose identity presupposes some form of persistence or continuity of space. Thus, the ontology of events ends up implicitly reintroducing an underlying spatial domain. Furthermore, characterizing an event as something that "occurs" presupposes a temporal order within which this occurrence takes place. If the event is treated merely as something that "exists permanently," its character as an occurrence is lost. A conceptual circularity then arises: the event depends on space and time to be defined, while space and time are presented as a set derived from the events themselves.

There is also a problem related to the individuation of events. Spatial location, by itself, is not sufficient to distinguish them, since the same place may be occupied repeatedly and under similar physical conditions. What prevents identity between successive occurrences is precisely temporal order: one may return to the same place, but not to the same moment. Even when a phenomenon is reproduced with identical physical properties and at the same spatial position, it constitutes a new event, distinct from the previous one by occurring at another instant. Thus, the individuation of events rests on the temporal asymmetry between "before" and "after," whereas space permits the re-identification of positions. In this way, the ontology of events does not consistently eliminate the dependence on an underlying spatiotemporal structure, nor does it, by itself, provide a sufficiently clear criterion for the individuation of occurrences.

On the Spatial Assumptions Implicit in the Lorentz Transformations and the Use of the Term "Event"

This section aims to make explicit the ontological assumptions underlying the Lorentz transformations. Although this approach may appear unconventional, its coherence becomes clearer when the non-axiomatic derivation of the Lorentz transformations is rigorously examined, as presented in Section **2.1.1**.

Initially, it is necessary to clarify the status of the sets (x, y, z, t) and (x', y', z', t'). Although frequently identified with the "event," these sets do not constitute the event itself, but rather numerical descriptions of a point in space-time [5, 12, 16, 48]. They are therefore distinct representations of the spatiotemporal location of the same physical event in different reference frames. In particular, (x, y, z) and (x', y', z') refer to the same objective location, whereas t and t' express distinct representations of the same objective moment. In this sense, the Lorentz transformations establish relationships between coordinate representations without implying any modification of the physical event being described and, consequently, of the associated space and time [5, 10, 12, 48]. Their function is to re-express the same physical reality in different coordinate systems. Furthermore, their derivation presupposes properties such as the homogeneity and isotropy of space, indicating the existence of a common underlying physical domain shared by the descriptions [10, 11]. This interpretation can be illustrated simply: a point drawn on a surface remains the same regardless of the coordinate system used to describe it. Changes in the system alter only its numerical representation, not the physical reality represented. Similarly, the Lorentz transformations operate on the coordinates of events, not on the physical events themselves or on the space and time inherent to them.

In the relativity literature, the term "event" is frequently used both to designate a point in space-time, geometrically identified by coordinates [12], and to designate a localized physical occurrence, such as the emission or reception of a light signal [5, 48]. Although these two meanings are related, they correspond to distinct concepts: the former belongs to the geometric structure of the theory, whereas the latter refers to an actual physical phenomenon. A physical occurrence can be associated with a point in space-time, but this association does not imply an ontological identity between the two. This distinction is particularly relevant to the discussion of the ontological status of space-time, since the geometric representation of an occurrence does not, in itself, demonstrate that the elements of the mathematical structure correspond to physical entities. For this reason, in the present work, the term "event" will be used exclusively to designate localized physical occurrences, distinguishing them from the geometric points employed in their representation.

3.2.7. Absolute Space and the Definition of Motion

At first glance, the notion of absolute space appears incompatible with the idea that all motion is relative [5, 6, 7]. This tension, however, is only apparent. Absolute space, as conceived in this work, does not imply absolute rest of bodies, but rather the existence of a domain with respect to which motion can be defined. Thus, motion remains relational, although the reference term need not necessarily be a material body. In this context, two types of motion are distinguished, in accordance with the classical formulation of Isaac Newton [6]:

- *Absolute (or proper) motion*: displacement of a body with respect to space;
- *Relative motion*: displacement of a body with respect to other material bodies.

This distinction follows from the conception adopted in this work, according to which motion is not reduced to relations between bodies, but corresponds to a state defined with respect to a common physical substrate. Bodies and material reference frames play an essential role in the description and measurement of motion [4, 5], but they do not constitute its ontological condition. To illustrate this idea, consider an observer who, in a vast isolated region of space, launches a projectile and then ceases to exist. It is natural to assume that the projectile continues in motion, even in the absence of other detectable material references. The impossibility of observation does not imply interruption of the state of motion, but merely an epistemological limitation regarding its detection. This highlights the distinction between physical existence and the possibility of observation. This conception does not conflict with the principle of relativity, since the latter refers to the operational equivalence between inertial reference frames as descriptions of motion [5, 10, 54], and not to the nonexistence of a background structure within which such descriptions are formulated.

An aspect that should be taken into account in the analysis of motion concerns the nature of coordinate systems. In some cases, the coordinate system may be established on the reference object, taking it as the immediate geometric support. For example, for motions on the ground, the coordinate system is established on the ground [5]; however, for the motion of planets or stars in the Earth reference frame, the coordinate system cannot be defined on Earth. Although the origin may be fixed on Earth, the points of the coordinate system extend throughout the spatial domain in which the celestial bodies move. In this case, the reference object or body (Earth) merely fixes the origin and orientation of the coordinate system, and does not constitute the support upon which this system is constructed. Thus, the reference body constitutes the object "with respect to which" motion is defined [7], whereas the common substrate (or background) constitutes the object "in which" the coordinate system is established and motion takes place. This distinction is fundamental. In general, motion does not occur in the reference body, but in a common substrate in which both bodies and coordinate systems are embedded. Thus, the difference between the expressions "with respect to which" and "in which" is not merely linguistic, but ontological. In general, reference bodies function as auxiliary elements of description, specifying with respect to what states of rest and motion are defined, but not the physical domain in which these processes actually take place. This distinction reinforces the thesis that motion is not reduced to relations between bodies.

In the example cited above, of stars moving in the Earth reference frame, the coordinate system is established on the common physical substrate, identified, in the present approach, with absolute space. However, since the reference body is moving through space, the coordinate system "moves" with the reference body, since its origin is attached to the reference body.

In the usual mathematical formulation of kinematics, (mechanical) motion is defined by the variation of spatial coordinates as a function of time [4, 6, 54]. A body may remain spatially at rest, that is, without variation in position, relative to an observer, without this implying an absence of temporal evolution. In other words, even at rest, the time coordinate continues to vary. Thus, rest and motion are distinguished not by variation in time (which is already presupposed), but by the presence or absence of spatial change. In this context, the coordinates (x, y, z) constitute state variables associated with the position of the body in the physical substrate, rather than properties defined within the reference body itself. The distinction between absolute and relative motion does not exhaust the analysis. It is also necessary to distinguish between physically real motion and merely descriptive motion, a question developed below.

True Motion and Apparent Motion (Reformulation)

The distinction between true motion and apparent motion originates in classical mechanics [6]. In the present approach, however, it is reformulated in terms of a more general ontological structure. Three cases are distinguished:

- *Apparent motion*: defined by the kinematic relation between bodies, not implying, by itself, a change in position with respect to the common substrate. For example, A may be in apparent motion relative to B, while its state with respect to the common substrate remains unchanged. This is a symmetric kinematic relation and, in this sense, purely descriptive, as in the description of the relative motion between two cars moving at different velocities on a road, independently of the description of their motions relative to the road itself, or in the description of the relative motion between a car moving along the road and a man standing still at its side.

- *True non-absolute motion*: defined with respect to a *common material medium*, such as a fluid, a solid surface, or another particular physical medium, in which the motion actually takes place. Here, the moving object changes its state (position) in the common medium.

- *Absolute true motion (in the strong sense)*: defined with respect to *absolute space*, understood as the fundamental substrate of motion.

Unlike the Newtonian formulation [6], in which true motion is exclusively associated with absolute space, here it is understood as being defined with respect to a common substrate, whether fundamental or not. Thus, the distinction between true and apparent motion is preserved, but on a broader ontological foundation. Apparent motion, as a kinematic relation between bodies, does not by itself produce dynamical

effects. A dynamical effect requires a change in the physical state of a body with respect to the common substrate or medium, independently of the existence of apparent motion between that body and another. In contrast, true motions are associated with physical interactions and make it possible to explain dynamical asymmetries between systems. On this basis, the following principle is formulated, hereafter referred to as the principle of true motions:

> ***Principle of True Motions (PTM):*** *let A and B be two physical bodies. If there is motion between A and B, then there exists a domain or element S, not necessarily material, that constitutes the common substrate in which their states of motion actually occur.*

In this principle, S does not simply represent an additional reference system with respect to which motion is described. S constitutes the physical common substrate itself in which the motions of A and B occur. A reference system may be established in this substrate to describe the states of motion, but it is not identical to the substrate. In this sense, motion is not a purely relational property between A and B, but a property defined with respect to S. The reciprocal description between the two bodies has only an operational and symmetric character, and does not imply physical equivalence between their states of motion. Although the kinematic relation between the systems remains symmetric and reciprocal, their dynamical states may be asymmetric due to their different relations with the substrate S. Acceleration constitutes a manifestation of a change in the state of motion of a body with respect to S, but does not eliminate the kinematic reciprocity between the bodies. This principle does not correspond to the arbitrary introduction of an additional reference frame. The element S designates the physical domain in which trajectories and dynamical states are defined and, in the present approach, is taken as an ontological condition for the consistent formulation of kinematics. Its necessity becomes particularly evident in situations involving dynamical asymmetries, such as the so-called twin paradox [14, 49].

Thus, it follows that *no physically real motion can be grounded exclusively in the relation between isolated bodies, although this relation is sufficient for the kinematic description of relative motion. Every true motion presupposes a common substrate, global or local, with respect to which velocities, accelerations, and physical interactions acquire dynamical meaning*. Once this has been established, it becomes necessary to distinguish the nature of this substrate. It may take two forms:

- *Fundamental substrate*: space (with its underlying physical medium), understood as the universal structure in which physical phenomena occur and positions, distances, and interactions are defined;
- *Non-fundamental substrates*: physical local media, such as surfaces, fluids, or fields, which make it possible to define motions in specific contexts.

Thus, every true motion is associated with a common physical "background" and originates in physical interactions, even though such interactions need not remain continuously active throughout the entire motion. An example of absolute true motion, in the present approach, is the propagation of light in vacuum. When a light wave is generated in vacuum, it propagates isotropically through physical space from the point of emission, independently of the presence of observers. This point is not associated with a privileged reference body, and the propagation does not depend on the choice of a reference body in order to exist, since it constitutes a physical process that occurs in the spatial substrate. The point of emission is physically determined by the wave-generation process itself, not by a geometric convention. It is therefore a motion physically defined in the fundamental substrate, although its description may be carried out using different coordinate systems.

The Notion of Speed and the Need for a Common Reference

The very definition of velocity provides an argument for the principle of true motions. Velocity is not determined by position itself, but by its variation with respect to time,

$$v = \frac{dx}{dt}$$

Thus, saying that *one body is faster than another* means quantitatively comparing their respective spatial variations during the same temporal interval. The expression "faster" therefore does not characterize merely a relation between two bodies, but a difference between their respective states of motion with respect to a common reference. If A and B have velocities v_A and v_B, stating that $|v_A| > |v_B|$ requires that both spatial variations be referred to a common domain in which distinct positions and their displacements can be defined and compared. The coordinate system used to describe motion does not constitute this domain, but merely provides a mathematical representation through labels assigned to positions. Even when the origin and axes are defined by means of a material body, the body does not thereby become the substrate upon which the other bodies move. The reference frame provides a standard of description; the substrate provides the common domain in which displacements occur. From the perspective adopted, this domain is space. Thus, the notion of speed presupposes space as the common substrate with respect to which different states of motion can be quantitatively compared.

The fact that velocity is a quantity defined by the relation between space and time does not eliminate this distinction. In the expression $v = dx/dt$, dx represents the spatial variation of position, whereas dt represents the duration during which this variation occurs. In mechanical motion, displacement corresponds to a change in position in space; time does not, in itself, constitute a domain through which the body moves, but provides the dimension by which spatial variation is ordered and quantified. Thus, although space and time are jointly necessary for the definition of velocity, they do not perform the same function in motion. The term "spacetime" (outside this approach) may designate a mathematical or physical structure that brings these two quantities together, but this does not imply that space ceases to constitute the proper domain of mechanical displacement. Consequently, the relative velocity between A and B does not exhaust the meaning of their motions. It expresses only the difference between their displacements, whereas the comparison between their individual states of motion requires reference to a common domain. It is in this sense that the principle of true motions maintains that relative motion constitutes a manifestation of the difference between states of motion occurring in space, and not the entirety of the physical meaning of motion.

It is important to emphasize that, although the existence of a physical medium for the propagation of light is proposed here, the conceptions of motion and velocity developed in this work do not depend on the existence of such a medium. Even if such a medium did not exist, these conceptions would remain consistent, since they are grounded in space, understood as the fundamental substrate of motion. The medium for the propagation of interactions constitutes, from this perspective, an additional physical element associated with space, whose function is distinct from that of space as the fundamental substrate of motion.

3.2.8. On Observational Limitations

From an observational point of view, it is not always possible to directly access the substrate of motion. In such cases, true motion may be inferred from dynamical effects, as illustrated by Isaac Newton's bucket experiment. For example, two observers in uniform relative motion, without access to an external reference associated with the substrate, may describe only the apparent motion between them. The absence of direct detectability, however, does not constitute a sufficient criterion for rejecting a physical entity. Contemporary physics admits entities that are not directly observed, but are postulated as explanatory components of empirically observed phenomena, as is the case with dark matter [55]. Analogously, rejecting absolute space does not necessarily eliminate the need for an underlying physical structure for the definition and explanation of states of motion; it may merely shift this function to another conceptual or physical structure [5, 7, 27]. In the present approach, the substrate of motion is explicitly identified as this structure. In this context, the absolute substrate is not introduced as an *ad hoc* hypothesis, but as a necessary condition for the consistent definition of states of motion. Moreover, every physical formulation explicitly or implicitly presupposes a support structure within which processes are defined. Even in theories that reject absolute space in the classical sense, a structure remains that functionally performs the role of a substrate [7, 27]. The difference, in the present approach, consists in making this structure conceptually explicit. Finally, an important consequence follows: if the emission of light occurs at an objective spatial point, belonging to physical space independently of observation and of the choice of coordinate system, its kinematic description should take this point as its primary reference, rather than the arbitrary origins of coordinate systems. The formalization of this idea will be developed in the following sections.

3.3. Galilean Kinematics under an Appropriate Physical Interpretation

The Lorentz transformations were historically introduced as a solution to a problem associated with the propagation of light [5, 12, 13]. It is argued here, however, that this problem arises from the failure to distinguish between two physically distinct levels: *electromagnetic propagation in the physical substrate* and *the kinematics of relative motion between bodies*. The equations of James Clerk Maxwell [32, 33, 54] describe the propagation of electromagnetic disturbances in a physical medium characterized by electric permittivity ϵ_0 and magnetic permeability μ_0. The speed of light in vacuum, given by

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{8}$$

follows from these properties and expresses the wave's propagation velocity in the substrate [33], independently of the state of motion of the source or the observer. On the other hand, Galilean kinematics [56, 57] describes motion relations between physical objects moving within this same physical domain. It is therefore a description of the kinematic relations between particles or physical systems, rather than of the propagation of fields in the substrate. This distinction makes it possible to introduce two conceptually different quantities:

- *proper velocity in the substrate* v_p: velocity of a body or physical process relative to the substrate of motion.

- *relative (apparent) velocity* v_{rel}: velocity defined by the kinematic relation between two moving objects in the substrate. It is termed *apparent* because it does not correspond to the proper velocity of either of the two objects considered individually, but results from the combination of their respective states of proper motion.

Each physical object has a proper velocity defined relative to the fundamental substrate, although this quantity cannot be measured directly. Between bodies, on the other hand, there are only apparent relative velocities, dependent on the system under consideration. Thus, the velocity of Earth relative to Mars differs from that relative to Jupiter, reflecting different states of motion in the common substrate. In the case of light, c corresponds to the proper velocity of wave propagation in the physical substrate. When light propagation is considered relative to moving bodies, the physically relevant quantity becomes its *apparent relative velocity*, rather than the velocity of wave propagation in the substrate itself. This velocity may differ from c without implying any change in electromagnetic propagation. The failure to distinguish between these levels leads to the interpretation that the constancy of c would require a modification of the structure of kinematics. In the present approach, by contrast, it is argued that the Galilean transformations remain sufficient when applied to the appropriate physical quantities, that is, to the relations between states of motion in the substrate, and not directly to local measurements associated with light propagation. In this context, the relative velocity between two systems can be expressed, in scalar form, as:

$$v_{rel}^{(1,2)} = v_p^{(1)} - v_p^{(2)} \tag{9}$$

Thus, expressions such as $x = ct$ are not interpreted here as descriptions of the kinematics of light relative to moving observers, but as relations associated with wave propagation in the physical substrate (or as the "kinematics of light *relative to the point of emission in the substrate*). A consistent kinematic description of light relative to moving objects requires consideration of the relative velocities between the wavefront and the material systems involved. It is therefore important to distinguish between electromagnetic propagation in the substrate and relative motion between bodies. The implications of this distinction will be explored in the following sections of this first part of the series, with application to the analysis of the spherical propagation of light and the kinematics of light rays, and in Parts 3 and 4.

3.4. The Kinematics of Light: General Analysis of the Spherical Wavefront and the Light Ray

In traditional derivations of the Lorentz transformations, light propagation is frequently analyzed under the assumption that emission occurs at the common origin of the coordinate systems at the initial instant [5, 4, 58]. This statement does not imply that all derivations of the Lorentz transformations explicitly introduce light propagation from the first step. In fact, there is an important distinction between two stages that can be identified in these derivations, either explicitly or implicitly. The first consists of obtaining a general structure for the transformations between inertial systems, a stage that is not questioned in the present approach and is corroborated. The second consists of determining the coefficients that particularize this general structure and lead to the Lorentz transformations. It is precisely this second stage that is the object of analysis. In the case of Ignatowski [59], it corresponds to the determination of the constant k (or n, depending on the notation adopted), whereas in other derivations it may take a different form. It is argued here that this determination does not follow exclusively from general kinematic principles, but requires additional physical information, obtained within the framework of electromagnetism and, specifically, related to the properties of electromagnetic propagation. Thus, the issue is not the validity of the general structure of the transformations, but the physical assumptions employed to select, within this structure, the Lorentz transformations rather than the Galilean transformations.

Regarding the emission of the light wave from the common origin, although this choice does not alter the functional form of the transformations, it may conceal fundamental conceptual inconsistencies. To avoid this problem, a more general treatment is adopted here, in which the emission of the light wavefront occurs at an arbitrary point in physical space, not necessarily at the origin of the coordinate systems. Before proceeding, it is necessary to distinguish two sets of quantities with conceptually distinct roles.

Additive Constants of the Linear Transformations

The terms $a_{15}, a_{25}, a_{35}, a_{45}$ of the standard linear transformations (2) exclusively represent the initial spatiotemporal configuration between the origins of the reference frames. In particular, a_{15}, a_{25} and a_{35} correspond to the spatial offset of the origin of S′ relative to S, whereas a_{45} represents a possible temporal offset between the clocks. When the origins coincide at the initial instant, we have:

$$a_{15} = a_{25} = a_{35} = 0$$

These terms are not related to the physical phenomenon of light emission.

Coordinates of the Physical Center of Emission

The variables x_c and x_c' represent the coordinates of the point in space where the light wavefront is emitted in reference frames S and S′, respectively. At the instant of emission, the position of the source coincides with this point, but from that instant onward, the wavefront propagates independently of the source and the reference systems. Thus, the coincidence of the origins ($x = y = z = x' = y' = z' = t = t' = 0$) does not, in general, imply that $x_c = x_c' = 0$, since the source may be located outside the origins. These coordinates are zero only at the instant of emission, if there is spatial coincidence between the source and the origins, since, after emission, the center of the wavefront remains at the spatial point (or physical position) at which the emission actually occurred, without following the subsequent motion of the source or the reference frame.

Let us then consider that the origins of systems S and S′ coincide at $t = t' = 0$, and that, at this same instant, a source located at a point P in space, with coordinates $x = x_0$ in S and $x' = x_0'$ in S′, emits a spherical light wave. This is a physically admissible configuration, and there is no principle requiring the emission to occur at the origin of the reference frames at the instant when locally coincident clocks indicate the initial time $t = t' = 0$. Thus, the generality considered does not concern the form of the linear transformations, but rather the physical description of the propagation of the light wavefront, without restrictions on the position of the emitting point. It should also be noted that the coincidence between points belonging to two inertial reference frames is not restricted to the origins. At a given physical instant, multiple pairs of points belonging to different reference frames may coincide in space, independently of

the presence of clocks at those locations. Clocks do not condition the occurrence of physical phenomena; their function is merely to record local occurrences temporally. Thus, the simultaneity of events distributed in space does not depend on operational conventions, but on the physical structure of the process itself. Under these conditions, the equation of the light wavefront along the X-axis can be written as:

in S: $$(x - x_c)^2 = c^2(t - t_0)^2 \tag{10}$$

in S': $$(x' - x'_c)^2 = c^2(t' - t'_0)^2 \tag{11}$$

where x_c and x'_c represent the coordinates of the emission point, and t_0, t'_0 the corresponding instants. According to the scenario considered: $t_0 = t'_0 = 0$. Applying the standard Lorentz time transformation,

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \tag{12}$$

to the emission event in S, characterized by $t = 0$ and $x = x_c \neq 0$, we obtain:

$$t'_0 = -\gamma\frac{v}{c^2}x_c \tag{13}$$

This result implies that, for reference frame S', the emission event does not occur at the instant $t'_0 = 0$, contrary to the initial physical condition of the scenario. This discrepancy does not arise from a limitation of the physical arrangement, but from the way in which the formalism relates the coordinates of events. In this context, the restriction imposed by the standard relativistic formalism, according to which only events occurring at the common origin can simultaneously satisfy the condition $t_0 = t'_0 = 0$, results from a structural characteristic of the coordinate transformations, rather than from a physical impossibility. The formalism thus proceeds to limit the class of physically admissible scenarios in order to preserve its own mathematical construction, thereby constituting a methodological inversion between theoretical description and physical reality. In the particular case $x_c = 0$, we obtain $t'_0 = 0$, as expected; however, validity in a particular case does not guarantee the general applicability of the formalism. It is therefore observed that a general kinematic analysis of light propagation, without restrictions on the position of the source, reveals a limitation in the generality of the Lorentz transformations in their usual form. This result motivates a critical reassessment of these transformations in the context of the description of light propagation. In particular, it becomes necessary to examine whether the form of the wavefront and the causal structure of the phenomenon can be preserved through a kinematic description directly anchored in propagation through physical space, without imposing restrictions on the position of the emitting point. This analysis will be developed below.

3.4.1. Reinterpretation of the Kinematics of the Spherical Light Wavefront

In light of the limitation identified in the relativistic treatment of the general case, the kinematics of the light wavefront is now formulated from the physical point of emission in space, using Galilean transformations under an appropriate physical interpretation. The objective is to determine whether the spherical form of propagation and the causal structure of the phenomenon follow directly from propagation in physical space, without previously resorting to the geometrization of spacetime.

Maintaining the scenario established previously, the emission of a spherical light wave from an arbitrary point P in space is considered at the instant $t = t' = 0$, when the origins of reference frames S and S′ coincide. The general equation of the wavefront in reference frames S and S' is, respectively, given by:

$$(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = c^2(t - t_0)^2 \tag{14}$$

$$(x' - x'_c)^2 + (y' - y'_c)^2 + (z' - z'_c)^2 = c^2(t' - t'_0)^2 \tag{15}$$

where (x_c, y_c, z_c) and (x'_c, y'_c, z'_c) represent the spatial coordinates of the center of the wavefront, and $t_0 = t'_0 = 0$ corresponds to the physical instant of emission.

Observation on the Center of the Wavefront.

In the present description, the center of the spherical wavefront is not treated as an event independent of the points that constitute the wavefront itself. At each instant t, the wavefront has a given spatial configuration, characterized by a center and the points that constitute it. For this reason, the center is described by coordinates dependent on the same temporal parameter of the wavefront, that is, $x_c = x_c(t)$, $y_c = y_c(t)$, $z_c = z_c(t)$, and not by a coordinate $x_c(t_c)$, $y_c(t_c)$, $z_c(t_c)$ associated with a supposed proper instant of the center. Thus, for a given t, $x_c(t)$, $y_c(t)$, $z_c(t)$ represents the position of the center and $x(t), y(t), z(t)$ the position of any point on the wavefront. Therefore, the spatiotemporal coordinates of the center and of any point on the wavefront are, respectively, $(x_c,\ y_c,\ z_c,\ t)$ and $(x,\ y,\ z,\ t)$. Consequently, when these positions are transformed at the same instant, one obtains directly

$$x' - x'_c = a_{11}(x - vt) - a_{11}(x_c - vt) = a_{11}(x - x_c)$$

without the need to introduce a t_c. The center and the points of the wavefront therefore belong to the same spatial configuration of the wave at each instant t.

The Point of Emission as a Kinematic Entity

The point P at which the emission occurs defines the geometric center of the light sphere. Although fixed in the substrate, this point is described as moving in the reference frames due to the proper motions of the origins of the coordinate systems. Thus, its coordinates evolve according to:

$$x_c = x_0 - v_p t \quad \text{and} \quad x'_c = x'_0 - v'_p t'$$

where v_p and v'_p represent the velocities of the origins of S and S′ relative to the substrate.

Transformation of the Coordinates of the Center of the Spherical Wavefront

Consider general linear transformations along the x-axis:

$$x' = a_{11}(x - vt), \quad t' = a_{41}x + a_{44}t \tag{16}$$

Applying them to the center of the wavefront, we obtain:

$$x'_c = a_{11}(x_c - vt), \quad t' = a_{41}x_c + a_{44}t \tag{17}$$

Compatibility Between the Descriptions of the Wavefront

To verify the compatibility between the descriptions of the wavefront in the two reference frames, the aim is not merely to verify whether the same geometric points satisfy both equations. A stronger condition is imposed: the equation of the wavefront obtained after the transformation must be algebraically identical to the equation originally established in S. In other words, the polynomials obtained in both cases must coincide term by term, which requires equality of the coefficients of the corresponding terms. Substituting the transformations of the coordinates of the point on the wavefront and of the physical center into the corresponding equations, we obtain:

$$a_{11}^2(x - x_c)^2 = c^2[a_{41}(x - x_c) + a_{44}(t - t_0)]^2. \tag{18}$$

Expanding and rearranging the terms:

$$(a_{11}^2 - a_{41}^2 \mathrm{c}^2)(\mathrm{x} - \mathrm{x_c})^2 - 2a_{41}a_{44}\mathrm{c}^2(\mathrm{x} - \mathrm{x_c})(\mathrm{t} - \mathrm{t_0}) - a_{44}^2\mathrm{c}^2(\mathrm{t} - \mathrm{t_0})^2 = 0. \tag{19}$$

For this expression to be identical to the polynomial representing the wavefront in S, the coefficients of the corresponding terms must be equal. This therefore results in the conditions

$$a_{11}^2 - a_{41}^2 \mathrm{c}^2 = 1, \quad a_{41}a_{44}\mathrm{c}^2 = 0, \quad a_{44}^2 = 1 \tag{20}$$

A question that equations (20) might raise is: Where does the requirement of term-by-term identity come from? This question is important and, although the answer is already implicit in the development preceding equations (20), it deserves attention and an explicit answer. The answer to this question has as its core the principle of relativity, as described below.

No inertial reference system has the status of a privileged reference frame in space. In particular, systems S and S′ have the same condition with respect to the phenomenon of light-wave propagation. The physical laws governing physical phenomena are the same for all inertial reference systems. Equations (14) and (15) represent exactly the same geometric object and the same physical phenomenon; S and S′ describe what occurs when an electromagnetic disturbance is generated in "neutral" physical space (in the sense that physical space does not privilege any material reference frame existing within it). Although they may receive the light signal at different instants, S and S′, through their respective equations, describe a single natural phenomenon. Therefore, *the requirement of term-by-term identity follows from the fact that S and S′ must describe the same law of propagation of the light wave and, consequently, the same physical phenomenon; therefore, both equations must represent the same geometric relation for the same points in spacetime (or in space and time)*.

From conditions (20), it follows:

$$a_{11} = a_{44} = \pm 1, \quad a_{41} = 0.$$

Polynomial equality, by itself, does not determine the signs. To select the branch physically corresponding to the continuous transformation between reference frames and to the conventional orientation of the axes and time, it is required that, in the limit $v = 0$), the transformation reduce to the identity transformation:

$$x' = x, \qquad t' = t.$$

This condition selects the positive signs, $a_{11} = a_{44} = 1$,

resulting in $$a_{11} = a_{44} = 1, \quad a_{41} = 0.$$

Thus, the transformation reduces to the form

$$x' = x - vt, \qquad t' = t,$$

that is, to the Galilean transformation along the x-axis. The result above shows that, when the wavefront is described relative to the physical point of emission and algebraic identity between the wavefront equations in the two reference frames is required, the spatial term in the time transformation disappears. The resulting temporal structure is therefore Galilean, without the need to introduce the Lorentz transformation beforehand.

It is important to clarify that, according to the interpretation adopted here, in the aforementioned equations, t and t' do not represent instants of perception or recording by observers, but temporal parameters that order the objective evolution of the events associated with the propagation of the light wave. It is also worth recalling that each event is unique, occurs only once, at a single spatial point and at a single objective "when," that is, at a single instant, independently of the observer or of clock readings. A phenomenon does not occur once for one observer and then again for another observer, and equations of *coordinate transformations* do not transform events (in the case of the light wave, they do not transform the spatial point P of emission of the wave, the form of wave propagation, or the when of objective occurrences), but only the *coordinates or labels* used for their description. Furthermore, the question of simultaneity between S and S′ does not arise here, since equations (14) and (15) do not describe perceptual acts, but rather a law of light propagation and unique events in space.

The argument above therefore provides an additional conceptual foundation in support of the result obtained thus far for the temporal transformation relation

$$t' = t.$$

since this concerns the time of the evolution of occurrences, rather than the time of their perception by observers.

The result obtained in this section will be independently verified in the analysis of the kinematics of the light ray, presented in the following section.

3.4.2. The Kinematics of the Light Ray Reinterpreted

A scenario similar to the situation considered by Einstein in obtaining the Lorentz transformations is now analyzed, in which a light ray travels longitudinally along the axis of motion, being considered in systems *K* and *k*. Unlike the analysis of the spherical wavefront, which describes the global propagation of radiation, this case corresponds to a local description of light propagation along a preferred direction, characterizing the light-ray limit.

Consider a carriage moving uniformly with velocity v along the *x*-axis relative to a station *K*. Associated with the carriage, consider reference frame *k*, which moves with the same velocity v relative to *K*. Inside the carriage, a light source is fixed at a material point A, while a plane mirror is fixed at another point B, separated by a proper distance L', measured in the carriage's reference frame. This arrangement is illustrated in **Figure 2**. The notations *K* and *k* are retained here, in accordance with Einstein's original treatment, equivalent to the systems **S** and **S′** used in the previous section.

At the instant $t = t' = 0$, when the origins of the reference frames coincide, a light pulse is emitted from A, propagates toward B, is reflected, and returns to the source. The analysis involves three distinct entities: the light ray, which propagates through space with velocity c, and points A and B, which move with the proper velocity of reference frame *k*.

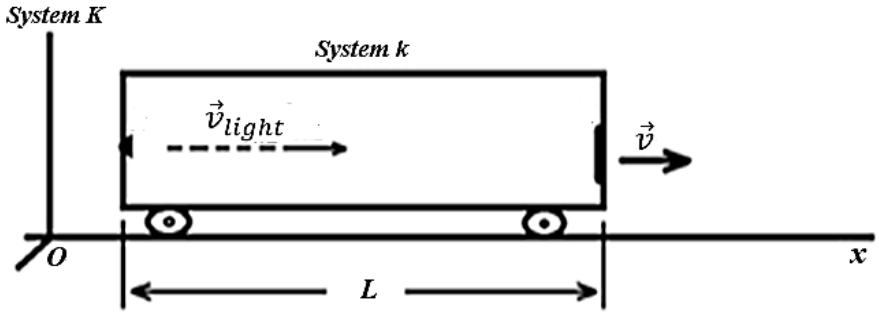


Figure 2 – A light ray traveling longitudinally between two points A and B in a carriage.

Temporal Asymmetry in the Carriage Reference Frame

Initially, consider the description in the carriage reference frame *k*. On the outbound path (from A to B), the ray leaves the spatial point P, where the source is located at the instant $t'_0 = 0$. From that instant onward, the ray propagates through free space with velocity c relative to the point P, while mirror B moves away from the emission point P through space with velocity v'_p. Note that B does not move away from A, but rather from P. Consequently, the ray does not simply travel the proper distance L', but must reach a moving target, such that the effective distance satisfies:

$$c(t'_1 - t'_0) = L' + v'_p(t'_1 - t'_0) \tag{21}$$

From this expression, it follows that

$$t'_1 - t'_0 = \frac{L'}{c - v'_p} \tag{22}$$

On the return path (from B to A), the opposite situation occurs: point A moves toward the reflected ray. Thus, the effective distance is reduced by the motion of A, and we have:

$$c(t'_2 - t'_1) = L' - v'_p(t'_2 - t'_1) \tag{23}$$

which yields:

$$t_2' - t_1' = \frac{L'}{c + v_p'} \tag{24}$$

As a direct consequence, we have

$$t_2' - t_1' < t_1' - t_0' \tag{25}$$

and therefore:

$$\frac{(t_0' + t_2')}{2} < t_1' \tag{26}$$

showing that, in the carriage reference frame, the outbound and return times are not symmetric when light propagation is explicitly described relative to free space. The temporal equality characteristic of the standard relativistic formulation follows, from this perspective, from the failure to distinguish between the proper propagation velocity of light in the substrate and its relative velocity with respect to material bodies, a hypothesis that, in the present approach, is considered physically unnecessary.

Equations of Motion in the Carriage Reference Frame

On the outbound path, the equation of motion of the ray in reference frame *k* can be written as:

$$x'(t') = x_0' + (c - v_p')t', \text{ with } x_0' = x'(0) = x_A'. \tag{27}$$

At the instant of reflection, we have:

$$x_B' = x_A' + (c - v_p')t_1' \tag{28}$$

And on the return path:

$$x_A' = x_B' - (c + v_p')(t_2' - t_1') \tag{29}$$

reaching the source's position again at the instant t_2'.

Description of Motion in the Station Reference Frame

For the observer in reference frame K, light also propagates through the spatial substrate with its proper velocity c, with the physical point P of emission as the center of the wavefront, while the station K itself also moves through the same space with velocity v_p, and the material points A and B move within the station with velocity v. Thus, the positions of these points in K vary with time. The motion of K through free space makes the velocity of light relative to the origin of K equal to $c - v_p$. Therefore, the following equation is obtained for the outbound path:

$$x_{B1} = x_{A0} + (c - v_p)t_1, \quad x_A(0) = x_{A0} \tag{30}$$

For the return path:

$$x_{A2} = x_{B1} - (c + v_p)(t_2 - t_1). \tag{31}$$

The motion of points A and B relative to station *K* is already incorporated into the coordinates x_{B1} and x_{A2}, which represent, respectively, the positions of B at instant t_1 and of A at instant t_2. Thus, $x_{B1} = x_{B0} + vt_1$ and $x_{A2} = x_{A0} + vt_2$. Equations (30) and (31) therefore express the spatial coincidence of the light with B at instant t_1 and with A at instant t_2, respectively, without the need to explicitly state separately the motion of these points in *K*.

It is worth recalling that the proper velocities satisfy the kinematic relation between reference frames:

$$v = v_p' - v_p, \quad \text{or equivalently,} \quad v_p' = v + v_p. \tag{32}$$

Compatibility with Linear Transformations

Applying the linear transformations

$$x' = a_{11}(x - vt), \quad t' = a_{41}x + a_{44}t$$

from equations (28) and (29), the following equations are obtained:

$$x_{B1} = \frac{a_{11}}{a_{11} - a_{41}(c - v - v_p)} x_{A0} + \frac{a_{11}v + a_{44}(c - v - v_p)}{a_{11} - a_{41}(c - v - v_p)} t_1 \tag{33}$$

and

$$x_{A2} = x_{B1} + \frac{a_{11}v - a_{44}(c + v + v_p)}{a_{11} + a_{41}(c + v + v_p)} (t_2 - t_1) \tag{34}$$

Imposing that equations (33) and (34) describe the same physical events described by equations (30) and (31), obtained directly in reference frame *K*, results in an algebraic system whose only consistent solution is:

$$a_{11} = a_{44} = \pm 1, \quad a_{41} = 0.$$

For the reasons already presented in the previous section, the solution corresponding to the conventional orientation of the coordinates is selected,

$$a_{11} = a_{44} = 1, \quad a_{41} = 0,$$

once again yielding the Galilean transformations. The agreement between the analysis of the spherical wavefront and that of the light ray reinforces the consistency of this interpretation and indicates that the preservation of the form of propagation, of c as the proper velocity of light in the spatial substrate, and of the causal structure of the phenomenon follows directly from the physical description of propagation through space, without the need to introduce the geometrization of spacetime beforehand.

4. CONCLUSION

This work presented a critical analysis of the foundations of the Lorentz transformations, with the aim of examining the assumptions that lead to their derivation and their interpretation as a necessary description of relativistic kinematics. The analysis focuses on the stage at which the coefficients of this structure are determined in such a way as to lead specifically to the Lorentz transformations. In this context, an attempt was made to determine whether this procedure follows exclusively from general kinematic principles or depends on additional physical assumptions concerning the properties of light propagation. The investigation was developed through an explicit analysis of the kinematics of light propagation, distinguishing the physical point of emission, material systems, and coordinate reference frames, as well as the motions of these elements relative to the common spatial substrate. Under this interpretation, it was shown that light propagation can be consistently described by means of Galilean transformations, preserving the principle of relativity, the speed c as the proper propagation speed in the substrate, and the causal structure of the phenomenon, without requiring the introduction of the Lorentz transformations. The introduction of the concept of absolute true motion, associated with the proper velocity v_p, defined relative to the fundamental substrate, provides a conceptual foundation for this interpretation. Within this framework, a distinction is made between the proper velocities of physical processes and systems relative

to the substrate and the relative velocities between processes and systems. In the case of light, the proper propagation speed c is determined by the properties of the medium, whereas the velocities between the wavefront and material systems depend on their respective states of proper motion. This distinction makes it possible to treat the physical propagation of light and the kinematic relationships between the systems involved separately.

The analysis of the spherical wavefront showed that, when emission is considered at an arbitrary point in space, distinct from the origins of the reference frames, the application of the Lorentz time transformation assigns different time coordinates to the emission event in the two systems when the reference frames are in relative motion. In the standard configuration, the condition $t = t' = 0$ in the relativistic formalism is associated exclusively with the event located at the common origin. In the present approach, this consequence is not interpreted as a physical impossibility of the scenario under consideration, but rather as a consequence of the specific structure of the transformations employed. The demonstration was supplemented by an independent analysis of the kinematics of a light ray in two inertial reference frames. In both frames, propagation was described using the same kinematic structure, distinguishing the proper velocity c from the relative velocities between the ray and material points. Imposing compatibility between the descriptions in the two frames once again yielded the coefficients corresponding to the Galilean transformations. The agreement between the two analyses therefore constitutes an independent result within the present approach and reinforces the interpretation according to which the form of light propagation can be preserved without resorting to the Lorentz transformations.

The analysis conducted in this first part was deliberately restricted to the physical and mathematical aspects of the kinematics of light propagation. At this stage, no attempt was made to analyse the behaviour of Maxwell's equations under Galilean transformations or to examine the principal experiments associated with Einsteinian relativity. These issues will be addressed in the subsequent parts of the series. Part 2 will further investigate the relationship between Galilean transformations and Maxwell's equations, as well as the conceptual implications of distinguishing the physical propagation of phenomena from their mathematical description. The analysis of the experimental and observational aspects associated with Special Relativity will be reserved for Parts 3 and 4. This work thus establishes the conceptual basis for a programme of critical reassessment of standard relativistic kinematics, proposing a description in which light propagation is treated as a physical phenomenon in space, while distinguishing the intrinsic dynamics of material systems from the relative kinematic relationships used to describe them.

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