A Geometric-Constitutive Equivalence for Propellantless Acceleration: Weyl-Slice Dynamics of a Thin Piezoelectric Cylinder and its Cylindrical Alcubierre Representation

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Abstract

We present a self-contained theoretical framework that accounts for the reported net acceleration of electrically isolated, asymmetric capacitor systems (taking the experimental claims of Buhler et al. as given) without mass ejection or directed radiation. The central postulate is an unacknowledged equivalence principle: a laboratory-generated, traceless strain distribution on a finite cylinder is observationally equivalent, for the motion of the centre of mass, to the tidal field of a distant mass. The cylinder is partitioned into infinitesimal Weyl slices; the axial acceleration gradient on each slice is $\mathrm{d}a_x/\mathrm{d}x = -c^2 W(x)$. Integration yields a net centre-of-mass acceleration. The same Weyl profile is shown to be identical to the axial tidal combination of a thin-walled cylindrical shift metric of Alcubierre type. All subsequent geometric quantities—Eulerian energy density, exterior evanescent tails, entrainment of nearby observers and of a contacting gas, and possible second-order inertial-mass corrections—follow by direct substitution. The construction never requires the capacitors to carry negative energy; the negative Eulerian density appears only as an observer-dependent reading of the pure-geometry metric. Momentum conservation is preserved globally through the relational character of the shift with respect to asymptotic inertial observers.

1 Introduction and working assumptions

We adopt the following experimental premise without further justification: certain asymmetric capacitor devices produce a measurable net force that survives electrical isolation and vacuum operation, scales with geometry and field strength, and is accompanied by neither measurable mass ejection nor directed electromagnetic radiation. Conventional explanations that rely on ion wind, electrostatic pressure differentials or QED vacuum forces are then excluded by total-momentum conservation for a closed system.

The present work supplies an alternative kinematic account. A thin-walled piezoelectric (or dielectric) cylinder is treated as a continuum of infinitesimal axial slices. Each slice experiences a traceless Weyl deformation. The integrated effect of these deformations is a net acceleration of the centre of mass relative to asymptotic inertial observers. The same deformation profile admits an exact rewriting as the wall of a cylindrical Alcubierre

metric. The resulting theory is therefore simultaneously a constitutive explanation of the laboratory effect and a concrete realisation of a shift geometry.

2 What the isolated observer sees, and why the effect enters the domain of general and special relativity

Consider a thin-walled piezoelectric cylinder equipped with capacitor-ring electrodes at both ends, placed in the gravitational field of a planet. Isolate the cylinder together with an observer inside a sealed falling box, so that neither can see the planet. Inside this box the observer records three simultaneous, axially dependent effects on the cylinder:

- progressive stretching along the axial direction,
- slight tapering (radial and hoop compression) on the end oriented toward the planetary centre,
- a longitudinal electric field whose intensity varies along the axis.

These are precisely the local manifestations of the electric part of the Weyl tensor acting tracelessly on the material. The observer never sees the planet, yet from the measured strain gradient, the induced electric field and the resulting deformation of the cylinder he can infer that he and the cylinder are accelerating relative to the distant stars. The background stars appear to drift, and the observer correctly concludes that the entire system is in accelerated motion with respect to the asymptotic inertial frame.

The same observational situation persists when the planetary mass is conceptually removed and the identical traceless strain profile is generated by the constitutive response of the piezoelectric crystal under applied voltage. The observer still measures the axial stretch, the taper and the longitudinally varying electric field; he still sees the stars drift; and he still concludes that the cylinder is entraining him into accelerated motion relative to the asymptotic frame. In this case the acceleration is produced entirely by the laboratory-engineered Weyl slicing inside the wall. The mathematics of the integrated tidal field then yields a net centre-of-mass acceleration without any external mass being present.

This is the operational content of the unacknowledged equivalence principle that underlies the present work. If the experimental claims of Buhler, Townsend Brown and related asymmetric-capacitor investigations are taken as given—i.e., if a closed, electrically isolated device exhibits a genuine residual acceleration—then the laboratory object is precisely a black box. The moment a black box begins to accelerate relative to the fixed stars while remaining electrically and mechanically isolated, the phenomenon has entered the domain of general and special relativity by definition: the only available description is a change in the identification of local inertial frames with respect to the asymptotic observers.

Conventional attempts to engineer propulsion by "curving spacetime" frequently adopt the wrong side of the Einstein equation. They begin from the planetary (or stellar) source term and attempt to reproduce a comparable curvature by supplying an energy density of order GM/r concentrated inside the vehicle. That approach is analogous to calculating the power required to turn a ship's propeller while treating the mass of the entire ocean as part of the engine. The arithmetic correctly returns an enormous figure, yet it answers

the wrong question. The practical problem is not to move the ocean past the ship; it is to move the ship past the ocean. Likewise, the relevant side of the gravitational problem for propulsion is the ship side—the local Weyl slicing experienced by the vehicle—rather than the planet side. Once the local traceless strain that a genuine tidal field would have produced is realised by constitutive means, the integrated effect on the centre of mass is already fixed by geometry, and the enormous planetary mass never needs to be reproduced inside the laboratory.

In short, the isolated observer sees a cylinder that stretches, tapers and polarises exactly as it would under a planetary tide; he sees the stars accelerate; and he attributes the motion to the cylinder itself. When the same local physics is produced without a planet, the same observational conclusions follow. The resulting acceleration is therefore a geometric-constitutive phenomenon that belongs, from the outset, to the relativistic description of inertial frames.

3 Weyl-slice kinematics of the thin cylinder

Consider a thin-walled cylinder of radius R, length L, wall thickness $\delta \ll R$ and surface mass density $\sigma(x)$. The total mass is

$$M = 2\pi R \int_0^L \sigma(x)\,\mathrm{d}x.$$

On each infinitesimal axial slice the only non-vanishing Weyl component generated by the constitutive strain is the axial tidal field $W(x)$. The postulated equivalence fixes the local acceleration gradient:

$$\frac{\mathrm{d}a_x}{\mathrm{d}x} = -c^2 W(x).$$

Integration from the blunt end yields

$$a_x(x) = a_x(0) - c^2 \int_0^x W(s)\,\mathrm{d}s.$$

The centre-of-mass acceleration is the mass-weighted average

$$a_{\mathrm{COM}} = \frac{2\pi R}{M} \int_0^L a_x(x)\,\sigma(x)\,\mathrm{d}x.$$

(The constant $a_x(0)$ may be set to zero by choice of asymptotic frame.)

4 Constitutive origin of $W(x)$

In the presence of a planetary tide the electric part of the Weyl tensor supplies

$$W_{xx} = \frac{2GM}{r^3}, \qquad W_{\rho\rho} = W_{\phi\phi} = -\frac{GM}{r^3}.$$

Geodesic deviation converts these components into a traceless strain on the wall:

$$\varepsilon_{xx} = \alpha W(x), \qquad \varepsilon_{\rho\rho} = \varepsilon_{\phi\phi} = -\frac{\alpha}{2} W(x).$$

The linear piezoelectric constitutive relations (strain-charge form)

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \qquad D_i = e_{ikl}\varepsilon_{kl} + \varepsilon_{ik}E_k$$

then generate an axial electric field. Under open-circuit conditions ($D_x = 0$) one obtains

$$E_x(x) = -\beta W(x),$$

where the material constant β collects the relevant piezoelectric and dielectric coefficients.

When the planetary source is removed, the identical traceless strain (and therefore the identical $W(x)$) can be realised by an applied voltage together with a controlled axial taper of the piezoelectric moduli. From this point the mathematical description is independent of the original planetary mass.

5 Derivation of the constitutive electric functions on the thin wall

We use the standard linear piezoelectric constitutive relations in the strain-charge form (the form most convenient for a prescribed geometric strain). All fields are evaluated on the wall $\rho = R$; the thin-wall limit means that only the axial coordinate x remains as an independent variable.

5.1 Linear piezoelectric constitutive equations

The independent variables are the strain tensor ε_{jk} and the electric field E_k. The dependent variables are the stress σ_{ij} and the electric displacement D_i:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \; D_i \qquad = e_{ikl}\varepsilon_{kl} + \varepsilon_{ik}E_k. \tag{1}$$

Here

- c_{ijkl} = elastic stiffness tensor at constant E,
- e_{kij} = piezoelectric stress coefficients,
- ε_{ik} = dielectric permittivity at constant strain.

(The opposite "stress-charge" form is equivalent after inversion of the stiffness matrix; we stay with the strain-charge form because the Weyl-induced strain is the primary geometric input.)

5.2 Traceless Weyl strain on the wall

The equivalence principle supplies a traceless strain that is an image of the Weyl field:

$$\varepsilon_{xx}(x) = \alpha, W(x), \; \varepsilon_{\rho\rho}(x) \qquad = \varepsilon_{\phi\phi}(x) = -\frac{\alpha}{2}, W(x), \; \varepsilon_{x\rho} = \varepsilon_{x\phi} = \varepsilon_{\rho\phi} = 0. \tag{2}$$

(The factor α has dimension length^2 and absorbs the elastic compliance; its precise value is fixed by the free-fall time or by the laboratory voltage that reproduces the same strain.)

5.3 Open-circuit (or prescribed-voltage) electric response

For a thin-walled tube with electrodes only at the two ends the radial and azimuthal components of $\mathbf{D}$ vanish on average (no free charge on the curved surface). The only surviving component is the axial electric displacement D_x.

In the open-circuit condition (no external current) one has $D_x = 0$ inside the crystal. Solving the constitutive equation for E_x then yields the induced axial field:

$$0 = D_x = e_{xjk}\varepsilon_{jk} + \varepsilon_{xx}E_x \quad \Rightarrow \quad E_x(x) = -\frac{e_{xjk}}{\varepsilon_{xx}}\,\varepsilon_{jk}(x).$$

Inserting the Weyl strain of section 2:

$$= e_{xxx}\big(\alpha W\big) + e_{x\rho\rho}\left(-\frac{\alpha}{2}W\right) + e_{x\phi\phi}\left(-\frac{\alpha}{2}W\right) = \alpha\left(e_{xxx} - \frac{1}{2}\big(e_{x\rho\rho} + e_{x\phi\phi}\big)\right)W(x).$$

Therefore the induced electric field is strictly proportional to the same function $W(x)$:

$$E_x(x) = -\frac{\alpha}{\varepsilon_{xx}}\left(e_{xxx} - \frac{1}{2}\big(e_{x\rho\rho} + e_{x\phi\phi}\big)\right)W(x).$$

We abbreviate the material prefactor by a single constant

$$\beta = \frac{\alpha}{\varepsilon_{xx}}\left(e_{xxx} - \frac{1}{2}\big(e_{x\rho\rho} + e_{x\phi\phi}\big)\right),$$

so that the constitutive electric function collapses to the elementary relation

$$E_x(x) = -\beta\, W(x).$$

5.4 Short-circuit or driven-electrode case

If the end-ring electrodes are connected to an external voltage source $V(t)$, the line integral of E_x is fixed:

$$\int_0^L E_x(x)\,\mathrm{d}x = V(t).$$

The constitutive equation is now solved for the strain (or for an additional piezoelectric strain that is superposed on any residual geometric strain). The axial electric field inside the wall becomes essentially uniform (or follows the prescribed electrode potential), while the strain acquires an extra piece

$$\varepsilon_{xx}^{\text{piezo}} = d_{xjk} E_k$$

(d_{xjk} = piezoelectric strain coefficients). The total strain that enters the definition of $W(x)$ is then the sum of the geometric (Weyl-like) piece and this piezoelectric piece. The function $W(x)$ is still recovered from the total measured strain by

$$W(x) = \frac{1}{\alpha} \varepsilon_{xx}^{\text{total}}(x).$$

6 Cylindrical shift metric and exact identification

The geometry that carries the same axial tidal field is the thin-wall cylindrical Alcubierre metric

$$\mathrm{d}s^2 = -c^2 \mathrm{d}t^2 + \big(\mathrm{d}x - c\, b\, \mathrm{d}t\big)^2 + \mathrm{d}\rho^2 + \rho^2 \mathrm{d}\phi^2,$$

with

$$b(x, \rho, t) = \frac{v_s(t)}{c} f(x)\, h\Big(\frac{\rho - R}{\delta}\Big).$$

The axial tidal combination extracted from the Einstein tensor is

$$W_x = b b_{xx} + b_x^2 + b_{tx} = \frac{v_s^2}{2c^2} \big(f^2\big)_{xx} + \frac{\dot{v}_s}{c} f_x.$$

The equivalence principle is the identification

$$W(x) = W_x \Big|_{\rho = R}.$$

Solving for the shift profile $f(x)$ yields, for the two elementary cases,

- steady shift: $f(x) = \pm (c/v_s) \sqrt{2 \int \int W + \cdots}$,
- purely accelerating shift: $f(x) = (c/\dot{v}_s) \int_0^x W(s)\, \mathrm{d}s + f(0)$.

Substitution into the COM integral produces the single linking formula

$$a_{\text{COM}} = -\frac{2\pi R}{M} \int_0^L \sigma(x) \left\{ \frac{v_s^2}{2} \Big[\big(f^2\big)_x(x) - \big(f^2\big)_x(0) \Big] + c \dot{v}_s \big(f(x) - f(0) \big) \right\} \mathrm{d}x.$$

7 Eulerian energy density

The future-directed unit normal to the spatial slices is $n^\mu = (1, b, 0, 0)$. Contraction of the Einstein tensor with this normal yields

$$G_{\hat{0}\hat{0}} = -\frac{3}{4} b_\rho^2.$$

Restoring units, the Eulerian energy density is

$$\rho_E = -\frac{3c^4}{32\pi G} b_\rho^2.$$

On the thin wall $b_\rho \sim (v_s/c) f/\delta$, so

$$\rho_E = -\frac{3c^2 v_s^2}{32\pi G} \frac{f^2}{\delta^2}.$$

This negative value is an artefact of the Eulerian observers of the pure-geometry metric; the constitutive electromagnetic and elastic energy density of the crystal remains positive.

8 Detailed derivation of the Eulerian energy density

We start from the metric (restoring c only at the end)

$$\mathrm{d}s^2 = -c^2\,\mathrm{d}t^2 + \big(\mathrm{d}x - c\,b\,\mathrm{d}t\big)^2 + \mathrm{d}\rho^2 + \rho^2\,\mathrm{d}\phi^2,$$

where $b = b(x, \rho, t)$ is dimensionless. For the thin-wall profile

$$b(x, \rho, t) = \frac{v_s(t)}{c}\, F(x, \rho), \qquad F(x, \rho) = f(x)\, h\left(\frac{\rho - R}{\delta}\right).$$

All calculations below are performed in units where $c = 1$; the factors of c are restored in the final expression.

8.1 Einstein-tensor components that enter the contraction

The relevant components (already computed from the Christoffel symbols of the metric) are

$$\begin{aligned}
G_{00} &= -b\, b_{\rho\rho} - \frac{b\, b_\rho}{\rho} - \frac{1 + 3b^2}{4}\, b_\rho^2, \\
G_{0x} &= \frac{1}{2}\, b_{\rho\rho} + \frac{b_\rho}{2\rho} + \frac{3}{4}\, b\, b_\rho^2, \\
G_{xx} &= -\frac{3}{4}\, b_\rho^2.
\end{aligned}$$

(The remaining components $G_{0\rho}$, $G_{\rho\rho}$, $G_{\phi\phi}$ are not needed for the Eulerian observer that is purely longitudinal.)

8.2 Eulerian normal vector

The unit normal to the spatial slices $t = \text{const}$ is the future-directed vector

$$n^\mu = (1, b, 0, 0)$$

(in coordinates (t, x, ρ, ϕ)). Its co-vector is

$$n_\mu = (-1, 0, 0, 0).$$

(The normalisation $n_\mu n^\mu = -1$ follows at once from the metric.)

8.3 Energy density measured by the Eulerian observer

By definition

$$G_{\hat{0}\hat{0}} = G_{\mu\nu}\, n^\mu n^\nu.$$

Expanding the contraction with the only non-zero components of n^μ,

$$\begin{aligned} G_{\mu\nu}\, n^\mu n^\nu &= G_{00}(n^t)^2 + 2G_{0x}\, n^t n^x + G_{xx}(n^x)^2 \\ &= G_{00} + 2b\, G_{0x} + b^2\, G_{xx}. \end{aligned}$$

Substitute the three Einstein components:

$$\begin{aligned} G_{00} + 2b\, G_{0x} + b^2\, G_{xx} = &\left(-b\, b_{\rho\rho} - \frac{b\, b_\rho}{\rho} - \frac{1+3b^2}{4}\, b_\rho^2 \right) \\ &+ 2b\left(\frac{1}{2}\, b_{\rho\rho} + \frac{b_\rho}{2\rho} + \frac{3}{4}\, b\, b_\rho^2 \right) \\ &+ b^2\left(-\frac{3}{4}\, b_\rho^2 \right). \end{aligned}$$

Collect like terms:

- terms with $b_{\rho\rho}$:

$$-b\, b_{\rho\rho} + b\, b_{\rho\rho} = 0;$$

- terms with b_ρ/ρ:

$$-\frac{b\, b_\rho}{\rho} + b \cdot \frac{b_\rho}{\rho} = 0;$$

- terms with b_ρ^2:

$$\begin{aligned} &-\frac{1+3b^2}{4}\, b_\rho^2 + 2b \cdot \frac{3}{4}\, b\, b_\rho^2 - \frac{3}{4}\, b^2\, b_\rho^2 \\ &= -\frac{1}{4}\, b_\rho^2 - \frac{3}{4}\, b^2\, b_\rho^2 + \frac{3}{2}\, b^2\, b_\rho^2 - \frac{3}{4}\, b^2\, b_\rho^2 \\ &= -\frac{3}{4}\, b_\rho^2. \end{aligned}$$

All other contributions cancel identically. Therefore

$$G_{\hat{0}\hat{0}} = -\frac{3}{4}\, b_\rho^2.$$

8.4 Conversion to the physical energy density

In geometric units ($G = c = 1$) the Einstein equation reads

$$G_{\mu\nu} = 8\pi T_{\mu\nu},$$

so the energy density measured by the Eulerian observer is

$$\rho_{\mathrm{E}} = T_{\hat{0}\hat{0}} = \frac{1}{8\pi}\, G_{\hat{0}\hat{0}} = -\frac{3}{32\pi}\, b_\rho^2.$$

Restoring c and G:

- b is dimensionless,
- b_ρ has dimension length^{-1},
- energy density has dimension energy/volume $= c^4/(G\,\mathrm{length}^2)$.

The unique combination that restores the correct dimensions is

$$\rho_{\mathrm{E}} = -\frac{3c^4}{32\pi G}\, b_\rho^2.$$

(The factor c^4/G converts a curvature of dimension length^{-2} into an energy density.)

5. Thin-wall limit

On the thin wall

$$b = \frac{v_s}{c}\, f(x)\, h\left(\frac{\rho - R}{\delta}\right).$$

The only radial derivative that survives is

$$b_\rho = \frac{v_s}{c}\, f(x) \cdot \frac{1}{\delta}\, h'\left(\frac{\rho - R}{\delta}\right).$$

Inside the support of the bump ($|\rho - R| < \delta/2$) one has

$$|h'| \sim O(1),$$

therefore

$$b_\rho \sim \frac{v_s}{c}\, \frac{f(x)}{\delta}.$$

Squaring and inserting into the expression for ρ_{E} yields

$$\rho_{\mathrm{E}} = -\frac{3c^2\, v_s^2}{32\pi G}\, \frac{f(x)^2}{\delta^2} \qquad \big(\rho = R\big).$$

(The precise numerical factor of order unity that multiplies f^2/δ^2 depends on the exact shape of h; the scaling $\propto f^2/\delta^2$ is universal.)

This completes the derivation: the Eulerian energy density reduces exactly to the pure radial-gradient term $-\frac{3}{4}b_\rho^2$ (in $c = 1$ units), and the thin-wall evaluation produces the stated formula.

9 Exterior evanescent field and entrainment

A realistic wall cannot support a compactly supported bump function. Continuity of the electrostatic potential, continuity of tangential $\mathbf{E}$, balance of elastic traction, and the atomic character of polarisation all generate exterior tails. Modelling the tail by an exponential decay of length $\lambda \sim \delta$ produces exterior fields

$$W_x^{\mathrm{ext}}(\rho) \propto e^{-(\rho-R)/\lambda} W(x).$$

A nearby observer at radial distance ϵ therefore measures an axial acceleration relative to the asymptotic stars

$$a_{\mathrm{obs}}^{\mathrm{stars}}(\epsilon) \propto e^{-\epsilon/\lambda} a_{\mathrm{COM}}.$$

The observer sees the background stars drift and correctly infers that he is being entrained by the cylinder. The same conclusion holds when the original planetary source is absent: the constitutive geometry continues to impress a diluted Weyl slicing on every nearby observer.

10 Gas flowing along the wall and in contact with it

We now place a continuous medium (ideal or viscous gas) in direct contact with the thin piezoelectric wall, so that the gas occupies the same radial support as the bump function h. All quantities below are therefore evaluated inside the layer

$$R - \frac{\delta}{2} \le \rho \le R + \frac{\delta}{2},$$

where the shift b, the tidal field W_x and the Eulerian energy density ρ_E attain their full, un-attenuated values.

10.1 Kinematic entrainment of the gas

An ideal-fluid element of the gas has four-velocity u^μ. In the cylindrical-shift metric the only non-zero connection terms that accelerate a fluid particle in the axial direction are those that contain W_x. The projected Euler equation (or the geodesic equation when pressure gradients vanish) reduces to

$$u^\nu \nabla_\nu u^x = -c^2 W_x + u^\nu \partial_\nu b + \cdots .$$

Because the gas is touching the wall, W_x is the full wall value

$$W_x = \frac{v_s^2}{2c^2} \big(f^2\big)_{xx} + \frac{\dot{v}_s}{c}\, f_x.$$

Consequently every fluid element acquires the same axial acceleration gradient that the wall itself carries:

$$\frac{\mathrm{d}a_x^{\mathrm{gas}}}{\mathrm{d}x} = -c^2 W_x.$$

Integration yields a local gas acceleration identical to the wall acceleration $a_x(x)$. The gas is therefore entrained at the same rate as the cylinder's own COM motion; there is no radial exponential suppression.

10.2 Effective inertial density of the gas

Inside the wall layer the Eulerian observers measure

$$\rho_E = -\frac{3c^2 v_s^2}{32\pi G}\big(F_\rho\big)^2 < 0.$$

A fluid element that co-moves with those observers therefore "sees" a negative energy-density background.

If one adopts the same second-order, potential-based definition of inertia used earlier, the effective inertial mass density ρ_{inert} of the gas acquires a correction

$$\rho_{\text{inert}} = \rho_{\text{gas}}\left(1 + \alpha\frac{\Phi}{c^2} + \cdots\right),$$

where Φ is the weak-field potential sourced by ρ_E. Because $\rho_E < 0$, $\Phi < 0$ and

$$\rho_{\text{inert}} < \rho_{\text{gas}}.$$

The fractional reduction is no longer exponentially small; it is of order the full wall value of $|\rho_E|$. (Whether this correction is numerically appreciable depends on the absolute magnitude of v_s and on the still-unspecified constant α that links energy density to inertia; the mathematics nevertheless permits a non-negligible reduction of the gas's inertial density while it remains inside the layer.)

10.3 Pressure and density response

The tidal field also stretches the gas. In the instantaneous rest frame of a fluid element the geodesic-deviation (or Raychaudhuri) equation for the expansion θ of the gas reads

$$\dot\theta = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - c^2 W_x + \cdots$$

(the shear $\sigma_{\mu\nu}$ is itself proportional to the traceless part of $W_{\mu\nu}$).

For a barotropic gas $p = p(\rho)$ the continuity equation then forces a density perturbation

$$\delta\rho_{\text{gas}} \propto -\int W_x \,\mathrm{d}t.$$

Thus the gas is rarefied or compressed according to the local sign of W_x, exactly as a material continuum would be stretched by a planetary tide. Because the gas is free to flow, this density perturbation can be advected along the wall, producing a measurable axial pressure gradient that co-moves with the cylinder.

10.4 Viscous boundary layer

If the gas has non-zero shear viscosity η, the no-slip condition at the solid wall forces the gas velocity to equal the local wall velocity. In the shift description that wall velocity is precisely the shift velocity c, b. The viscous stress tensor therefore contains a term

$$\pi_{x\rho} \propto \eta\,\partial_\rho(c\,b) \propto \eta\,\frac{v_s}{\delta}\,f(x).$$

The associated force per unit area on the gas is of order $\eta v_s/\delta$ and is directed axially. This force is an ordinary Navier–Stokes stress; it simply reinforces the kinematic entrainment already present from the tidal term $-c^2 W_x$.

11 Investigation of possible inertial-mass modification for a nearby observer

We remain inside the same cylindrical-shift geometry and the same constitutive interpretation. The only new question is whether an observer sitting in the exterior (or wall-adjacent) region experiences a modification of his inertial mass because he measures a negative energy-density domain on the cylinder.

11.1 What the nearby observer actually measures

Place the observer at radial coordinate $\rho = R + \epsilon$ with $\epsilon \sim \lambda$ (inside the evanescent layer). His four-velocity is approximately

$$u^{\mu}_{\text{obs}} = \gamma\,(1,\, b_{\text{ext}},\, 0,\, 0)\,,$$

where b_{ext} is the exponentially small exterior tail of the shift.

The energy density he attributes to the cylinder wall is the contraction

$$\rho_{\text{obs}} = T_{\mu\nu} u^{\mu}_{\text{obs}} u^{\nu}_{\text{obs}}.$$

Because the pure-geometry reading of the metric already gives

$$G_{\hat{0}\hat{0}} = -\frac{3}{4}\, b_{\rho}^{2} < 0$$

on the wall, any observer whose four-velocity has a non-vanishing overlap with the Eulerian normal will measure a negative contribution proportional to b_{ρ}^{2}. For the nearby observer this contribution is attenuated by the same exponential factor that multiplies W_x^{ext}:

$$\rho_{\text{obs}} \;\propto\; -e^{-2\epsilon/\lambda}\frac{v_s^2 f^2}{\delta^2}.$$

Thus the observer does "see" a negative-energy domain, albeit an exponentially diluted one.

11.2 Inertial response inside the metric

In the shift geometry the proper acceleration required to remain at fixed spatial coordinates (x, ρ, ϕ) is

$$a^{\mu} = u^{\nu}\nabla_{\nu} u^{\mu}.$$

Evaluating the only non-zero component for a near-wall observer yields

$$a^{x} = -c^{2} W_x^{\text{ext}} + O\left(b_{\text{ext}}^{2}\right).$$

The force F^x that must be applied to keep a test mass m at rest relative to the cylinder is therefore

$$F^{x} = m\, a^{x} = m\left(-c^{2} W_x^{\text{ext}}\right).$$

If one defines an effective inertial mass $m_{\rm eff}$ by the laboratory relation $F = m_{\rm eff} a_{\rm lab}$, where $a_{\rm lab}$ is the acceleration measured with respect to the asymptotic stars, one finds

$$m_{\rm eff} = m \cdot \frac{a^x}{a_{\rm stars}}.$$

But from the earlier entrainment calculation $a_{\rm stars} = a^x$ (both are equal to $-c^2 W_x^{\rm ext}$). Consequently

$$m_{\rm eff} = m.$$

To first order in the exterior tail the inertial mass is not renormalised; the same factor multiplies both the force and the stellar acceleration, leaving the ratio unchanged.

11.3 Second-order correction from the negative-energy region

A more refined definition of inertia takes into account the local gravitational potential generated by the energy density the observer measures. Even though we do not invoke the conventional Einstein equation, one can still form the Newtonian-like potential that would be sourced by $\rho_{\rm obs}$ in a weak-field reading:

$$\nabla^2 \Phi \sim 4\pi G \rho_{\rm obs} \;\propto\; -e^{-2\epsilon/\lambda}.$$

The solution that decays at infinity behaves as

$$\Phi(\rho) \;\sim\; -C\, e^{-2\epsilon/\lambda}, \qquad C > 0.$$

In any Mach-inspired or weak-field inertia formula the effective inertial mass acquires a multiplicative factor of the form

$$m_{\rm eff} = m \left(1 + \alpha \frac{\Phi}{c^2} + \cdots \right).$$

Because $\Phi < 0$ the correction is negative:

$$m_{\rm eff} \approx m \left(1 - \alpha \frac{C}{c^2} e^{-2\epsilon/\lambda} \right) < m.$$

The fractional change is proportional to the same exponentially small factor that multiplies the exterior tidal field. It is therefore present in principle, but numerically negligible once the observer is more than a few wall-thicknesses away from the cylinder.

12 Observer dependence and the asymptotic exchange

Both the classic Alcubierre metric and the present construction are relational. The shift b and the tidal field W_x are defined with respect to an asymptotic region in which $b = 0$. The COM acceleration is the relative acceleration between wall observers and asymptotic observers. The "exchange" that permits the equivalence principle to operate in the absence of a planet is precisely the re-identification of successive local inertial frames with that fixed asymptotic structure. Remove the asymptotic observers and the notion of net acceleration loses its meaning.

13 Conclusions

The framework supplies a geometrically consistent account of a net COM acceleration generated by a laboratory strain profile, free of the momentum-conservation defect that afflicts pressure- or vacuum-based explanations. The same profile is identical to the wall of a cylindrical Alcubierre metric, so every geometric consequence of the shift description is available. Exterior influence is real but evanescent; a contacting gas is fully entrained; inertial-mass corrections appear only at second order.

The decisive open task is experimental: a thin-walled piezoelectric cylinder must be instrumented so that the axial strain gradient, the induced electric field and the COM acceleration are recorded simultaneously and compared with the single linking formula derived above. Until that confrontation with data occurs, the construction remains a coherent theoretical candidate rather than a confirmed description of existing devices.

13.1 Observer dependence, asymptotic exchange, and the working of the equivalence

The suspicion that both the Alcubierre construction and the present theory depend on observers, and that an exchange with a global structure is required for the equivalence to operate, can be stated precisely inside the mathematics already developed.

13.2 What the metric actually does

The cylindrical shift line element is written with respect to a particular foliation whose asymptotic region ($\rho \to \infty$) is ordinary Minkowski space. In that region $b \to 0$, the four-velocity of the "fixed stars" is simply ∂_t, and the coordinate x measures proper distance for those observers.

Every non-zero value of b or of W_x is therefore a statement of relative motion or relative acceleration with respect to that asymptotic frame. The COM acceleration we derived,

$$a_{\mathrm{COM}} = -\frac{2\pi R}{M}\int \sigma(x)\left\{\frac{v_s^2}{2}\big[(f^2)_x - (f^2)_x(0)\big] + c\dot{v}_s\big(f - f(0)\big)\right\}\mathrm{d}x,$$

is not an absolute acceleration; it is the acceleration of the wall relative to the observers who sit where $b = 0$.

13.3 The exchange that makes the equivalence possible

When the planetary mass is present, the Weyl field $W_{xx} = 2GM/r^3$ is generated by the curvature that links the local free-fall frame to the distant stellar frame. The "exchange" is the gravitational interaction that keeps the asymptotic structure locked to the planet's centre of mass.

When the planet is removed and the same $W(x)$ is produced constitutively, the local geometry still has to be matched to an asymptotic Minkowski region in which $b = 0$. The only way the matching can occur is if the successive Weyl slices inside the wall are allowed to slide relative to that fixed asymptotic frame. That sliding is precisely the

shift b. In other words, the constitutive stress does not create an isolated, self-contained acceleration; it creates a local re-identification of the axial coordinate with respect to the distant observers.

The quantity that is "exchanged" is therefore the identification of the local inertial frames with the asymptotic ones. The cylinder does not push against a material medium; it pushes against the global structure of the inertial frames that the metric declares to be at rest at infinity. That is why the equivalence principle can operate without a visible planet: the asymptotic observers continue to play the role that the planet's gravitational field previously played.

13.4 Why both Alcubierre and the present construction are observer-dependent in the same way

In the classic Alcubierre metric the bubble wall moves at velocity v_s relative to the exterior observers. If one attempts to describe the same geometry from a frame that is co-moving with the bubble, the exterior stars appear to stream past and the negative-energy density changes its interpretation. Exactly the same frame dependence appears here: the negative ρ_E is measured by the Eulerian observers of the shift foliation; an observer who rides with the wall measures a different contraction of the stress-energy.

The COM acceleration itself is a relative quantity. It exists only because there is a distinguished asymptotic frame against which the wall's motion is registered. Remove every possible asymptotic observer (compactify the space, or declare that no distant inertial frame exists) and the notion of net acceleration of the cylinder loses its meaning. The theory therefore presupposes the existence of those observers; they are the other end of the exchange.

13.5 Mathematical expression of the exchange

The relative four-acceleration between a wall observer u_{wall} and an asymptotic observer u_{stars} is

$$a^{\alpha}_{\text{rel}} = u^{\beta}_{\text{wall}} \nabla_{\beta} u^{\alpha}_{\text{wall}} - u^{\beta}_{\text{stars}} \nabla_{\beta} u^{\alpha}_{\text{stars}}.$$

In the shift metric this reduces to

$$a^{x}_{\text{rel}} = -c^2 W_x.$$

The left-hand side is what the laboratory measures as a_{COM}. The right-hand side is generated by the constitutive profile. The equality is possible only because both four-velocities are defined inside one and the same metric that already contains the asymptotic region. That global metric structure is the medium with which the exchange occurs.

13.6 Conclusion inside the framework

The equivalence works because the local Weyl slicing is never a closed system; it is always a re-slicing relative to a fixed asymptotic inertial structure. The "something" that is exchanged is the alignment of successive local frames with that structure. Both the original Alcubierre construction and the present piezoelectric-cylinder construction rely on the same relational fact. Once that is granted, the observer dependence is not an optional philosophical extra; it is the operational content of the shift itself.

14 Acknowledgements

The present text is a systematic compilation and mathematical completion of an extended research dialogue. All derivations follow from the single equivalence postulate that a laboratory traceless strain realises the same COM kinematics as a genuine tidal field.