Strong Field Matter Waves in Operational Quantum Gravity and Bohmian Dynamics

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Manuscript version 0.6 • Revised 3 November 2026

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Abstract

Operational Quantum Gravity (OQG) describes gravity through the response of matter to its shared supporting environment. Changes in supporting electromagnetic exchange change material transition energies and dimensions, which determine how matter waves propagate. This paper connects that description to the wave-guidance formulation associated with de Broglie, Madelung, and Bohm. Starting from OQG's established Schwarzschild and Kerr exterior descriptions, we construct the corresponding relativistic wave equation for a spin-zero test field. Writing its complex wavefunction in terms of amplitude and phase gives an exact local equivalence to a quantum Hamilton–Jacobi equation and a conserved wave current. The transformation retains the full gravitational coefficients and uses no weak-field or small-spin expansion. We distinguish the energy change associated with the supporting environment from Bohm's quantum potential, which depends on the shape of the receiving wave. In the rotating case, a wave with no azimuthal phase momentum nevertheless has a current that turns at the exact Kerr dragging rate. Numerical checks independently verify the gravitational coefficients and the amplitude-and-phase identities. The result establishes a strong-field OQG formulation of Bohm–Madelung wave dynamics, connecting the material origin of gravitational propagation to the receiving wave's phase flow and conserved current. The Dirac appendix supplies electron-field propagation, conserved-current guidance, and the exact separation of dragging from local flow.

Keywords: Operational Quantum Gravity; strong gravity; matter-wave dispersion; quantum potential; Bohmian dynamics; Schwarzschild exterior; Kerr exterior

1 The physical connection and the results established here

How can the environment that changes a material clock's rate also direct the motion of that material system? OQG answers through the dynamics of matter. Scattering changes the exchange that supports a bound state. The state responds by changing its energy and dimensions. Those changes alter the energy–momentum relation of its matter wave and therefore its propagation [1–4]. We use **dispersion** in this

matter-wave sense: the relation between the wave's energy and momentum. Geometry records the resulting clock, ruler, and motion comparisons.

Bohmian mechanics describes motion through a wavefunction's amplitude and phase. The phase determines a momentum flow, while spatial variation of the amplitude produces the term called the quantum potential [5,6]. Madelung's earlier transformation supplies the mathematical rewriting of the wave equation into these real quantities [7]. The connection to OQG is consequently a question about the same evolving matter wave: can its exact gravitational propagation be written in this amplitude-and-phase form?

We show that it can, for the relativistic spin-zero wave equation specified below. Wherever the wavefunction is smooth and nonzero, the complex equation and its Bohm–Madelung form describe exactly the same wave evolution. Strong gravity changes the coefficients in both descriptions together. The same amplitude-and-phase transformation therefore applies throughout the strong-field calculation.

Two further results make the physical connection explicit. The energy associated with the supporting environment is generally different from Bohm's quantum potential. Their connection lies in the evolution of the wave, not in an equality between the two potentials. For a rotating source, the wave current gives the exact dragging angular rate when the wave has zero azimuthal phase momentum. Thus the calculation connects OQG's material dispersion directly to both quantum guidance and rotational dragging.

The strong-field backgrounds used here are already part of OQG's Einstein-equivalent classical branch [1–4]. Schwarzschild describes the selected spherical vacuum exterior; Kerr describes the selected stationary, uncharged rotating black-hole exterior. We retain their exact exterior coefficients. Appendix A recalls the classical correspondence, including the independent GR derivation of the clock, ruler, and supporting-power comparisons.

Bohmian work on gravity predates this paper. Shojai and Shojai discuss relativistic quantum potentials and curved-background motion [12]. Tumulka constructs a Bohmian model in Schwarzschild spacetime [13]. Pinto-Neto and Struyve review Bohmian quantum gravity and cosmology [14]. Our contribution is the explicit connection of OQG's supporting environment and material scaling to these wave dynamics.

The argument proceeds from the environment to the wave and then to its current. Sections 2 and 3 define the material response and the propagation equation. Sections 4 and 5 establish the Bohmian connection and distinguish the potentials. Sections 6 and 7 apply the result to the exact Schwarzschild and Kerr exteriors. The appendices retain the detailed proofs and operator checks.

2 What the supporting environment changes

All comparisons use a stated reference environment. The constants c_0 and G_0 are the reference light speed and gravitational constant; $\hbar$ is the reduced Planck constant, which remains invariant. The receiver's reference rest mass is m_0, and its complete reference rest energy is denoted by E_0. This energy includes the internal and binding contributions assigned to the body, so $E_0 = m_0 c_0^2$.

Two dimensionless factors describe the stationary material response. The positive factor f is the ratio of a material transition energy, or clock frequency, to its reference value. The positive factor b is the corresponding ratio of an isotropic material length to its reference value. The equilibrium supporting power is P, and its reference value is P_0. Here power means the common incoming and outgoing exchange at equilibrium.

The supporting-power theorem relates the clock factor to that exchange [1,2]:

$$\frac{P}{P_0} = f^2. \tag{1}$$

Thus the power ratio fixes the clock and transition-energy ratio through its positive square root. A lower supporting-power ratio corresponds to a lower transition-energy ratio in this comparison. The wave's propagation then follows from the complete clock, ruler, and phase dynamics developed below.

Let E_{rest} denote the corresponding stationary rest energy in the comparison standards. Define V_{OQG} as its change from the reference energy. The exact storage-energy relation is

$$E_{\mathrm{rest}} = fE_0, \qquad V_{\mathrm{OQG}} = E_{\mathrm{rest}} - E_0 = E_0\left(\sqrt{P/P_0} - 1\right). \tag{2}$$

This potential measures how the environment changes the receiver's stored energy. Moving matter also responds to the spatial ruler factors and, for a rotating source, an angular shift. Those effects enter the full energy–momentum relation in Section 3.

The clock factor and ruler factor should be kept separate. The common scaling branch has $f = b$, but the exact Schwarzschild exterior generally has distinct f and b. Their first weak-field terms agree. Their higher-order differences must be retained in a strong-field calculation. The exact profiles used below follow from the established exterior correspondence.

3 The exact equation that propagates the wave

A stationary environment has coefficients that do not change with the chosen coordinate time t. Let x^i denote its three spatial coordinates, with Latin indices i, j running over the spatial directions. Repeated

upper and lower indices are summed. The symmetric coefficients γ_{ij} describe spatial ruler comparisons, and γ^{ij} are the components of their inverse matrix. The shift components w^i describe motion of the local spatial frame relative to the reference coordinates. For rotation, the azimuthal shift is an angular rate.

The symbol ds^2 denotes the interval that packages these comparisons. With the clock factor f already defined, the stationary interval is

$$ds^2 = -f^2 c_0^2 dt^2 + \gamma_{ij}(dx^i - w^i dt)(dx^j - w^j dt). \quad (3)$$

Let δ_{ij} denote the identity matrix in Cartesian coordinates. In a static isotropic environment the shift vanishes and $\gamma_{ij} = b^{-2}\delta_{ij}$. The inverse ruler coefficients are then $\gamma^{ij} = b^2\delta^{ij}$. The interval is a compact representation of the material response used in OQG.

Let p_i be the receiver's spatial canonical momentum components, meaning the momenta paired with the coordinates in Hamilton's equations. Let H denote its coordinate-time energy. Choosing the positive local energy branch gives the exact ray Hamiltonian [1,4]:

$$H = w^i p_i + f\sqrt{m_0^2 c_0^4 + c_0^2 \gamma^{ij} p_i p_j}. \quad (4)$$

The square root is the local relativistic energy, converted by the clock and ruler factors. The first term accounts for the frame shift. Hamilton's equations give the packet paths when the wavelength is short enough for a ray description. This energy relation contains no expansion in momentum or gravitational strength.

To retain interference and amplitude variation, we now specify the wave equation itself. The complex scalar field Ψ represents a spin-zero receiving matter wave. It is treated as a test field: the source fixes the gravitational environment, and the receiver's backreaction on that environment is neglected. The calculation follows the receiving wave through that gravitational environment while retaining its interference and amplitude dynamics. The scalar proof uses a single amplitude and phase. Appendix E gives the Standard Model electron field's Dirac equation, spin connection, and conserved current in the same exact material coefficients [15,16].

For the covariant notation, let x^μ denote four coordinates, with Greek indices running over time and space. We use the signature $(-+++)$, so the time contribution is negative. When all four coordinates have dimensions of length, $x^0 = c_0 t$. The coefficients $g_{\mu\nu}$ are those of the interval, $g^{\mu\nu}$ are their inverse, and g is their determinant. The derivative ∂_μ means differentiation with respect to x^μ. The operator $\Box_g$, called the covariant wave operator, combines the time and spatial derivatives with the exact gravitational coefficients.

We use the minimally coupled Klein–Gordon equation, the standard relativistic scalar propagation prescription [8]:

$$\left(\square_g - \frac{m_0^2 c_0^2}{\hbar^2}\right)\Psi = 0, \qquad \square_g = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\, \partial_\nu\right). \tag{5}$$

The derivative acts on the coefficients as well as on the field. Keeping those coefficient derivatives matters in a nonuniform environment. Equation (5) specifies the minimally coupled scalar wave dynamics. Its coefficient derivatives carry the spatially varying gravitational response into the evolution of the complete wave.

An optional scalar coupling to Ricci curvature is included in the proof in Appendix C. It vanishes in both vacuum exteriors used here because their Ricci scalar is zero. Appendix B gives the full static wave operator directly in the material factors f and b, verifies its ray limit, and checks its positive static energy operator.

4 The same wave written in Bohmian form

The complex field contains two real functions. Its amplitude $\mathcal{A} = |\Psi|$ describes the wave's magnitude. Its phase action S describes the phase in units of $\hbar$, so S has units of action. Let i denote the imaginary unit. On a smooth region where the wave does not vanish, we can choose a continuous phase and write

$$\Psi = \mathcal{A} e^{iS/\hbar}, \qquad \mathcal{A} > 0. \tag{6}$$

This is polar decomposition: it expresses a complex number through its magnitude and phase at each event. It changes the variables used to describe the wave, while retaining the wave itself.

The covariant momentum components p_μ are the derivatives of the phase action. The relativistic amplitude term Q is obtained by applying the same gravitational wave operator to the amplitude and dividing by that amplitude. We use Q for a quantity with units of momentum squared; the familiar nonrelativistic quantum potential, introduced in Section 5, has units of energy. The definitions are

$$p_\mu = \partial_\mu S, \qquad Q = -\hbar^2 \frac{\square_g \mathcal{A}}{\mathcal{A}}. \tag{7}$$

Let ∇_μ denote the covariant derivative, which accounts for changes in the coordinate basis. On a scalar such as S, it equals the ordinary derivative. Raising its index uses the inverse coefficients: $\nabla^\mu S = g^{\mu\nu}\, \partial_\nu S$. Define the wave current J^μ as the flow vector $\mathcal{A}^2 \nabla^\mu S$. Its overall constant normalization is immaterial for the conservation equation and the current ratios used below.

The pair below starts with the quantum Hamilton–Jacobi equation. It relates the phase momentum to the rest-mass term and the amplitude-dependent quantum term. The second equation says that the wave current is conserved: flow entering and leaving a region balances its change within that region. These are the relativistic amplitude-and-phase equations associated with the Madelung and Bohm formulations [5–7,9].

Exact equivalence. On every such nonzero-wave region, the Klein–Gordon equation is equivalent to the following pair of real equations:

$$g^{\mu\nu}p_\mu p_\nu + m_0^2c_0^2 + Q = 0, \tag{8}$$

$$J^\mu = \mathcal{A}^2\nabla^\mu S, \qquad \nabla_\mu J^\mu = 0. \tag{9}$$

The proof separates the wave equation into its real and imaginary parts. The reverse substitution reconstructs the original complex equation. Appendix C supplies both directions explicitly. No approximation in gravitational strength, clock scaling, ruler scaling, or source spin enters this identity.

Here local refers to a smooth region on which the amplitude is nonzero and a continuous phase can be chosen. At a wave node the amplitude is zero, so division by the amplitude is undefined. A continuous phase may also require more than one patch around a node. The original initial and boundary conditions, and the requirement that the reconstructed wave be single-valued, must accompany the real equations. Appendix C states the phase-circulation condition.

The conserved current defines a family of flow lines and gives the wave's covariant guidance law. For the Klein–Gordon field, the time component of this current is generally signed. A particle interpretation uses an admissible class of states and its probability prescription [8–10]. The result established here is the exact scalar-wave guidance law in the OQG gravitational environment.

5 Why the two potentials differ

The supporting environment and the receiving wave enter the dynamics in different ways. The quantity V_{OQG}, defined in Section 2, measures a change in the receiver's corresponding rest energy. The quantum term depends on the receiving amplitude and its derivatives. The source environment governs propagation, while the initial wave data specify the receiving state.

To connect with Bohm's usual energy-valued potential, take the weak-field, slow-motion limit for a receiver with positive reference mass $m_0 > 0$. Let Φ be the Newtonian gravitational potential per unit mass, with units of speed squared. Its leading clock factor is $f = 1 + \Phi/c_0^2 + \cdots$, and the leading storage potential is $V_{\mathrm{OQG}} = m_0\Phi$. Let ψ denote the slowly evolving wave after the rapid reference rest-energy phase has been removed. The full wave is then $\Psi = e^{-im_0c_0^2t/\hbar}\psi$.

In this limit, ∇ is the ordinary Cartesian spatial gradient, and ∇^2 is its Laplacian, the sum of the three second spatial derivatives. The leading wave equation is [1,8]

$$i\hbar\,\partial_t\psi = \left[-\frac{\hbar^2}{2m_0}\nabla^2 + m_0\Phi\right]\psi. \tag{10}$$

Let $\mathcal{A}$ now denote this slow wave's amplitude and S_{NR} its phase action; the subscript NR means nonrelativistic. The nonrelativistic quantum potential Q_{NR} measures the spatial curvature of this amplitude relative to its magnitude:

$$Q_{\mathrm{NR}} = -\frac{\hbar^2}{2m_0}\frac{\nabla^2\mathcal{A}}{\mathcal{A}}. \tag{11}$$

For a slowly varying amplitude, the relativistic term reduces to this energy through $\mathcal{Q}/(2m_0) \to Q_{\mathrm{NR}}$. Writing the Schrödinger equation in amplitude and phase gives

$$\begin{aligned} \partial_t S_{\mathrm{NR}} + \frac{|\nabla S_{\mathrm{NR}}|^2}{2m_0} + V_{\mathrm{OQG}} + Q_{\mathrm{NR}} = 0, \\ \partial_t \mathcal{A}^2 + \nabla\cdot\left(\mathcal{A}^2\frac{\nabla S_{\mathrm{NR}}}{m_0}\right) = 0. \end{aligned} \tag{12}$$

The first line contains two distinct energy terms: the environmental storage change and the quantum potential of the particular wave. The second line conserves the flow of the positive density $\mathcal{A}^2$. These equations use V_{OQG} at its leading weak-field order.

Define $\mathbf{v}$ as the nonrelativistic guidance velocity, equal to the phase gradient divided by m_0. The derivative $\partial_t + \mathbf{v}\cdot\nabla$ follows that flow. Differentiating the phase equation gives its acceleration law, away from nodes and phase singularities:

$$\mathbf{v} = \frac{\nabla S_{\mathrm{NR}}}{m_0}, \qquad m_0(\partial_t + \mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\big(V_{\mathrm{OQG}} + Q_{\mathrm{NR}}\big). \tag{13}$$

This is the familiar force form of the Bohmian connection. Both terms contribute, but their contributions are not generally equal. A free Gaussian packet in a uniform reference environment has zero environmental storage potential and a nonzero quantum potential. Appendix D gives this counterexample explicitly.

The negative gradient of the storage potential equals the canonical momentum rate at zero spatial canonical momentum. For moving matter, the Hamiltonian of Section 3 gives the full relativistic interaction, including the ruler response and frame shift.

Bohm's quantum potential appears when the original Schrödinger kinetic term is written in amplitude and phase. The covariant wave operator similarly contains the gravitational response through its material coefficients. Each contribution is already present in the propagation equation and emerges explicitly in the corresponding amplitude-and-phase description.

6 The exact spherical exterior

Consider a source of mass M. Its Schwarzschild radius is denoted by r_s and equals $2G_0M/c_0^2$. The isotropic radius ρ is a radial coordinate in which the spatial ruler conversion has the same factor in every direction. The areal radius r is defined by the area $4\pi r^2$ of a symmetry sphere. These two radii label the same spheres differently.

Let $u = r_s/(4\rho)$ be a dimensionless radial parameter. In the exterior, $0 \le u < 1$. The established exact spherical correspondence gives [1,2]

$$f = \frac{1-u}{1+u}, \qquad b = (1+u)^{-2}, \qquad r = \rho(1+u)^2. \tag{14}$$

These expressions specify the clock and ruler response without truncating a series in gravitational strength. The radial conversion derivative is $dr/d\rho$. Direct substitution gives the associated power ratio and radial ruler identity:

$$\frac{P}{P_0} = f^2 = 1 - \frac{r_s}{r}, \qquad \frac{dr}{d\rho} = \frac{f}{b}, \qquad \frac{f}{b} = 1 - u^2. \tag{15}$$

The last relation displays the finite-field difference between the clock and ruler factors. Their equality holds at the common first weak-field order, while the exact dispersion retains both.

To write the propagation equation in areal coordinates, define $F(r) = 1 - r_s/r$. Let θ and ϕ be the polar and azimuthal angles, and let Δ_{S^2} denote the angular Laplacian on the unit sphere. This operator accounts for variation with those two angles. Derivatives labelled by t and r are coordinate-time and radial derivatives. The exact exterior wave equation is

$$\left[-\frac{1}{Fc_0^2}\partial_t^2 + \frac{1}{r^2}\partial_r(r^2F\,\partial_r) + \frac{1}{r^2}\Delta_{S^2} - \frac{m_0^2c_0^2}{\hbar^2}\right]\Psi = 0. \tag{16}$$

The time, radial, and angular parts all retain their exact coefficients. The Bohmian amplitude term is obtained by applying the derivative operator $\square_g$, without the rest-mass term, to $\mathcal{A}$. The full operator therefore carries the gravitational response into the wave's amplitude curvature.

Table 1 shows how large the exact response can be. The first column is the radius measured in Schwarzschild radii; the other columns give the power, clock, and ruler ratios. The ruler values follow from $b = \left[\left(1 + \sqrt{F}\right)/2\right]^2$.

Table 1 Exact spherical material factors

Areal radius r/r_s	Power ratio f^2	Clock factor f	Ruler factor b
10.0	0.900000	0.948683	0.949342
3.0	0.666667	0.816497	0.824915
1.5	0.333333	0.577350	0.622008
1.1	0.090909	0.301511	0.423483

For example, at 1.5 Schwarzschild radii the supporting-power ratio is one third, the clock factor is about 0.577, and the ruler factor is about 0.622. The wave equation and its polar transformation retain these finite differences. Both descriptions therefore apply directly at finite gravitational strength.

The displayed stationary exterior comparison applies for $r > r_s$. The horizon marks the boundary of this static coordinate and observer description. In a regular coordinate and observer description, the same covariant wave equation yields the same amplitude-and-phase identity. Its form is preserved wherever the field and its derivatives are regular.

7 Rotation and the exact dragging current

Rotation adds a direction-dependent ruler response and an angular shift. The earlier OQG rotational calculation fixes the leading dragging normalization from conserved source momentum and represents the selected exact Kerr exterior in the same material variables [4]. We now use that exterior to propagate the receiving wave.

Let J_s be the source angular momentum, distinguished by its subscript from the wave current J^μ. Define the gravitational length $r_g = G_0 M/c_0^2$ and spin length $a = J_\mathrm{s}/(Mc_0)$. We use Boyer–Lindquist coordinates: r is their radial coordinate, θ the polar angle, and ϕ the azimuthal angle. Their zero-spin radial coordinate becomes the Schwarzschild areal radius.

Three auxiliary functions shorten the exact coefficients. The functions Σ and Δ have units of length squared, while A_{K} has units of length to the fourth power. The latter is a Kerr coefficient, not the wave amplitude. Their definitions are

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 - 2r_g r + a^2,$$
$$A_{\mathrm{K}} = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta. \tag{17}$$

Let r_+ denote the outer horizon radius. For spin below the extremal value, the domain used here is $|a| < r_g$ and $r > r_+ = r_g + \sqrt{r_g^2 - a^2}$. Within this exterior, f remains the clock factor and ω_{d} denotes the dragging angular rate. The exact clock, angular shift, and diagonal spatial ruler coefficients are [4,11]

$$f^2 = \frac{\Sigma\Delta}{A_{\mathrm{K}}}, \qquad \omega_{\mathrm{d}} = \frac{2r_g a r c_0}{A_{\mathrm{K}}},$$
$$\gamma_{rr} = \frac{\Sigma}{\Delta}, \qquad \gamma_{\theta\theta} = \Sigma, \qquad \gamma_{\phi\phi} = \frac{A_{\mathrm{K}}\sin^2\theta}{\Sigma}. \tag{18}$$

The inverse spatial coefficients are the reciprocals of these diagonal entries. A local observer normal to the constant-time slices turns at angular rate $d\phi/dt = \omega_{\mathrm{d}}$, with clock rate f. Such an observer generally needs support to remain at fixed r and θ. In the ergosphere, where holding a fixed azimuth is impossible for a timelike observer, this rotating comparison remains the relevant one.

Let $L_z = p_\phi$ denote the receiver's canonical angular momentum about the source axis. Substituting the azimuthal shift into the Hamiltonian gives

$$H = \omega_{\mathrm{d}} L_z + f\sqrt{m_0^2 c_0^4 + c_0^2 \gamma^{ij} p_i p_j}. \tag{19}$$

The shift changes frequency comparisons as well. Let ϖ be a mode's coordinate angular frequency, and let the integer n_ϕ be its azimuthal phase index. Such a mode has phase factor $\exp(-i\varpi t + i n_\phi \phi)$. Define ϖ_{loc} as the frequency measured by the rotating normal observer. Following the phase along that observer's motion and converting to its clock gives

$$\varpi_{\mathrm{loc}} = \frac{\varpi - n_\phi \omega_{\mathrm{d}}}{f}. \tag{20}$$

This is the frequency counterpart of the shift term in the receiver's energy. It is already present in the OQG rotational correspondence [4].

Let γ denote the determinant of the spatial ruler matrix. Here $f\sqrt{\gamma} = \Sigma\sin\theta$, away from the polar-coordinate axis. The propagation equation below has three lines. The first combines time evolution with the

angular shift. The second retains the varying radial and polar coefficients. The third contains the remaining azimuthal derivative and rest-mass term. Using the determinant and inverse coefficients gives

$$
\begin{aligned}
0 = &-\frac{A_{\mathrm{K}}}{\Sigma \Delta c_0^2}\left(\partial_t + \omega_{\mathrm{d}}\,\partial_\phi\right)^2 \Psi \\
&+\frac{1}{\Sigma}\partial_r(\Delta\,\partial_r \Psi) + \frac{1}{\Sigma \sin\theta}\partial_\theta(\sin\theta\,\partial_\theta \Psi) \\
&+\left[\frac{\Sigma}{A_{\mathrm{K}}\sin^2\theta}\partial_\phi^2 - \frac{m_0^2 c_0^2}{\hbar^2}\right]\Psi.
\end{aligned}
\tag{21}
$$

The squared derivative differentiates in t and ϕ; its coefficient ω_{d} depends only on r and θ. Expanding it reproduces the mixed time–azimuth derivative of the inverse Kerr metric. No expansion in source spin has been made.

We can now obtain a direct guidance result. Define the coordinate phase energy by $\mathcal{E} = -\partial_t S$, and the azimuthal phase momentum by $\ell = \partial_\phi S$. The symbols J^t and J^ϕ are the time and azimuthal components of the wave current in the (t, r, θ, ϕ) chart. Their ratio is the angular velocity of a current flow line whenever $J^t \neq 0$. The inverse spatial azimuthal coefficient is $\gamma^{\phi\phi} = \Sigma/(A_{\mathrm{K}}\sin^2\theta)$. Direct substitution in the current definition yields

$$
\frac{J^\phi}{J^t} = \omega_{\mathrm{d}} + \frac{f^2 c_0^2 \gamma^{\phi\phi} \ell}{\mathcal{E} - \omega_{\mathrm{d}} \ell}.
\tag{22}
$$

In particular, when the wave has zero azimuthal phase momentum, the second term vanishes. The angular velocity along a current flow line, denoted by $d\phi/dt$, then equals the exact dragging rate:

$$
\frac{d\phi}{dt} = \frac{J^\phi}{J^t} = \omega_{\mathrm{d}}.
\tag{23}
$$

This is the exact dragging rate, even if the phase also varies radially or with polar angle. The amplitude cancels from the ratio. The result follows directly from the conserved wave current and remains exact beyond the ray approximation. Appendix E gives the corresponding Dirac current relation.

The classical ray calculation gives the same result. Differentiating the Hamiltonian with respect to L_z gives angular velocity. At $L_z = 0$, the energy term $\omega_{\mathrm{d}} L_z$ is zero but its momentum derivative remains ω_{d}. Zero azimuthal canonical momentum therefore gives the coordinate rotation set by the dragging environment.

Reversing the source angular momentum reverses the angular shift and leaves the clock and spatial factors unchanged at fixed mass and coordinates. With zero source spin, the operator reduces to the Schwarzschild

equation. Far from a slowly rotating source, its leading angular shift is $2G_0J_s/(c_0^2r^3)$, matching the earlier OQG rotational result [4].

8 What the correspondence establishes

The equivalence compares two representations of the same specified scalar wave problem. Keeping the same initial data, boundary conditions, and phase conditions gives the same complex wave whether it is evolved directly or reconstructed from its amplitude and phase. The gravitational coefficients can be described through OQG's material response or through GR's metric notation. The exact classical correspondence makes those coefficients agree; the polar transformation then acts on that common wave equation.

The physical interpretations remain distinguishable. OQG assigns a material origin to the change in propagation: the source alters the supporting environment and the receiving matter responds. The Bohm–Madelung formulation expresses the resulting wave evolution in terms of phase momentum, amplitude curvature, and conserved current. These statements connect OQG's environmental mechanism to the phase flow and amplitude dynamics of the receiving wave.

An individual matter wave responds to the source environment through the gravitational propagation coefficients. This response does not require entanglement with another receiving wave. Several separately prepared waves can encounter the same environment, with each wave evolving from its own initial data under the common material coefficients.

The exact correspondence preserves the wave's initial data, boundary conditions, and phase circulation. It therefore connects OQG's gravitational material response to a reversible Bohm–Madelung description of the same scalar wave, including its rotational current.

9 Numerical checks and reproducibility

Online Resource 1 contains the Python 3 standard-library verification program and saved results for the scalar/exterior identities in Table 2. The program independently checks the full strong-field algebra. Appendix E's Dirac results are derived analytically.

The Kerr calculations use dimensionless variables: lengths are divided by r_g, time by r_g/c_0, and energy by $m_0c_0^2$. The corresponding radial momentum scale is m_0c_0, and the angular momentum scale is $m_0c_0r_g$. The dimensional equations in the paper retain $\hbar$ and c_0.

At 256 deterministic sample points, the program builds the standard Boyer–Lindquist metric independently and inverts its full matrix by elimination. It compares that inverse with the material Hamiltonian, the

current angular rate, and the expanded wave-operator coefficients. The mass-shell check tests the relativistic energy–momentum relation. Samples include both spin signs, $|a|/r_g < 0.95$, polar angles from 0.25 to $\pi - 0.25$ radians, and radii at least $0.2r_g$ beyond the outer horizon.

A separate check uses an exact superposition of two free massive Klein–Gordon modes. Let k_{scale} be the reference inverse-length scale for this calculation. The dimensionless mass is $m_0 c_0/(\hbar k_{\mathrm{scale}}) = 1$; the wave numbers divided by this scale are 0.7 and −1.2, and the mode amplitudes are 1 and 0.35. The unequal amplitudes keep the total amplitude away from zero. At 200 events, analytical derivatives of the complex field independently give the amplitude and phase derivatives needed to check the polar equations.

Table 2 reports the maximum discrepancies. A residual is the amount by which a computed expression differs from its required zero or matching value. The wave, phase, and mass-shell residuals are normalized by the sum of the magnitudes of their terms plus one. Angular-rate differences are measured in units of c_0/r_g. Every check passes the declared dimensionless tolerance of 10^{-10}.

Table 2 Maximum numerical discrepancies

Check	Maximum discrepancy
Kerr normalized mass-shell residual	5.23×10^{-16}
Kerr angular velocity from current and Hamiltonian	5.55×10^{-16}
Zero azimuthal momentum current and angular shift	8.33×10^{-17}
Kerr wave coefficients and independent inverse metric	2.71×10^{-15}
Schwarzschild profile identities	6.66×10^{-16}
Two-mode normalized wave residual	9.72×10^{-17}
Two-mode normalized polar phase residual	2.94×10^{-16}
Two-mode normalized current divergence	2.67×10^{-16}

These results check the consistency of the formulas and their implementation. The saved residuals and declared tolerance make the mathematical correspondence independently reproducible.

10 Conclusions

This paper establishes the exact connection between OQG's strong-field material response and Bohm–Madelung scalar wave dynamics. The full Schwarzschild and Kerr coefficients enter the wave equation and its equivalent amplitude-and-phase equations, without a weak-field or small-spin expansion. Appendix E

supplies the Standard Model electron field's Dirac equation and conserved-current guidance in the same coefficients, with Kerr dragging separated exactly from local azimuthal flow.

The two potentials have precise, different roles. The OQG storage potential records the energy change associated with the supporting environment. Bohm's quantum potential records the amplitude curvature of the particular receiving wave. The two terms participate in the same dynamics through these distinct physical roles. Keeping that distinction exposes the physical connection: the environment determines propagation, and the evolving wave determines its phase flow and amplitude-dependent quantum term.

Rotation supplies a direct strong-field consequence. A wave with zero azimuthal phase momentum has a current that turns at the exact Kerr dragging rate. This agrees with the existing material Hamiltonian in its ray regime and remains an exact wave-current identity beyond that approximation. The independent numerical checks verify the coefficients and the polar identities to the stated precision.

The correspondence is explicit in the material propagation coefficients, the reversible amplitude-and-phase equations, and the dragging current. The exact Schwarzschild and Kerr calculations place these results in a single consistent description of scalar matter-wave dynamics. **Strong gravity enters through the exact material response, and the resulting matter-wave evolution retains its Bohm–Madelung form.**

Appendix A Classical correspondence and supporting power

The main argument uses an existing classical result. This appendix states its mathematical basis so that the strong-field profiles can be distinguished from an extrapolated Newtonian potential.

The OQG companion construction uses a Weyl and transverse-diffeomorphism invariant completion, abbreviated WTDiff [3]. Let $R_{\mu\nu}$ be the Ricci curvature tensor and $\mathcal{R} = g^{\mu\nu}R_{\mu\nu}$ its scalar trace. Let $T_{\mu\nu}$ be the complete stress-energy tensor, including the contributions required by the matter model, and $T = g^{\mu\nu}T_{\mu\nu}$ its trace. Define the invariant gravitational normalization by $\kappa = G_0/c_0^4$. The trace-free field equation and stress conservation are

$$R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}\mathcal{R} = 8\pi\kappa\left(T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T\right), \qquad \nabla^{\mu}T_{\mu\nu} = 0. \tag{24}$$

The contracted Bianchi identity is the geometric conservation identity for curvature. Combined with the displayed stress conservation, it makes $\mathcal{R} + 8\pi\kappa T$ a constant. Write this constant as 4Λ, where Λ is the cosmological integration constant. Let $G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu}\mathcal{R}/2$ denote the Einstein tensor. Restoring the trace gives

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi\kappa T_{\mu\nu}, \qquad \Lambda = 0 \tag{25}$$

for the branch used in this paper. These nonlinear equations supply the exact Schwarzschild and selected Kerr vacuum exteriors under their stated symmetry and boundary conditions [1,3,4].

The corresponding-state theorem supplies the material Hamiltonian comparison. Let ψ be a normalized reference state in d spatial dimensions, with spatial position vector $\mathbf{x}$. Let U_b denote the unitary dilation that rescales its spatial profile by b; its normalization factor preserves the integral of the squared amplitude. Let H_0 be the complete reference Hamiltonian and $H_{f,b}$ the corresponding Hamiltonian after the clock and ruler scaling. The dagger denotes the adjoint operator. The transformation is [1,2]

$$(U_b\psi)(\mathbf{x}) = b^{-d/2}\psi(\mathbf{x}/b), \qquad H_{f,b} = fU_bH_0U_b^{\dagger}. \tag{26}$$

The operator domain, observables, and environmental response transform with the state. The energy factor f and dilation factor b describe the clock and ruler responses, respectively.

For the independent GR comparison, let N be the lapse or local stationary clock factor. Let A_{sp} be the isotropic spatial line-element factor, so local spatial distance is this factor times coordinate distance. Subscripts 0 and 1 label the reference and compared stationary laboratories. Then $f = N_1/N_0$ and $b = A_{\mathrm{sp},0}/A_{\mathrm{sp},1}$. Denote the two Hamiltonians expressed in the same comparison standards by $H_{\mathrm{GR,cmp}}$ and

$H_{\mathrm{OQG,cmp}}$, where cmp means comparison. Time conversion and normalized spatial dilation give [2, Eq. (16)]

$$H_{\mathrm{GR,cmp}} = f U_b H_0 U_b^{\dagger} = H_{\mathrm{OQG,cmp}}. \tag{27}$$

The same stationary comparison gives the squared-clock-factor power law. Let dE_{loc} and $d\tau_{\mathrm{loc}}$ be a local energy increment and local proper-time interval. Let dE_{ref} and dt_{ref} be that same process expressed in the reference energy and time standards. Define P_{loc} and P_{ref} as the respective energy-transfer rates. Since the energy conversion is $dE_{\mathrm{ref}} = f\, dE_{\mathrm{loc}}$ and the time conversion is $d\tau_{\mathrm{loc}} = f\, dt_{\mathrm{ref}}$,

$$P_{\mathrm{ref}} = f^2 P_{\mathrm{loc}}. \tag{28}$$

This compares the same specified stationary exchange. Propagation losses and collection geometry are separate observables. GR and OQG give the same comparison law here; OQG assigns the material response a causal role.

Appendix B The static operator and its ray limit

Use static isotropic Cartesian coordinates, zero shift, and the ordinary spatial gradient ∇ and Laplacian ∇^2. The real dimensionless constant ξ specifies scalar-curvature coupling; $\xi = 0$ is minimal coupling. The Ricci scalar $\mathcal{R}$, defined in Appendix A, is zero in the vacuum exteriors. The exact static equation is

$$\left[-\frac{1}{f^2 c_0^2}\partial_t^2 + \frac{b^3}{f}\nabla\cdot\left(\frac{f}{b}\nabla\right) - \frac{m_0^2 c_0^2}{\hbar^2} - \xi\mathcal{R}\right]\Psi = 0. \tag{29}$$

The spatial derivative acts on f/b as well as on the wave. The operator $b^2\nabla^2$ is recovered when the material factors are spatially constant. Their spatial variation supplies the gradient terms retained in the full operator.

For a short-wavelength wave, let E be coordinate phase energy and $\mathbf{p}$ its Cartesian momentum vector. Insert the local phase $\exp[i(\mathbf{p}\cdot\mathbf{x} - Et)/\hbar]$ and retain its leading derivatives:

$$\frac{E^2}{f^2 c_0^2} - b^2|\mathbf{p}|^2 - m_0^2 c_0^2 = 0, \qquad E = f\sqrt{m_0^2 c_0^4 + c_0^2 b^2 |\mathbf{p}|^2}. \tag{30}$$

The positive-energy result is the static Hamiltonian of Section 3. This check connects the wave operator to the established material dispersion.

To check the static energy operator without a curvature term, let $\gamma = \det\gamma_{ij}$ be the spatial determinant. Define K_{st} as the energy-squared operator obtained by isolating the second time derivative. The subscript st means static and distinguishes it from material stress stiffness:

$$\partial_t^2\Psi + \hbar^{-2}K_{\mathrm{st}}\Psi = 0,$$
$$K_{\mathrm{st}} = -\hbar^2 c_0^2 \frac{f}{\sqrt{\gamma}}\partial_i\left(f\sqrt{\gamma}\,\gamma^{ij}\,\partial_j\right) + f^2 m_0^2 c_0^4. \tag{31}$$

Let χ_1 and χ_2 be smooth test functions with boundary conditions removing surface flux. A star denotes complex conjugation. The weighted inner product, denoted by brackets with subscript w, uses the spatial measure $\sqrt{\gamma}/f$:

$$\langle\chi_1,\chi_2\rangle_{\mathrm{w}} = \int d^3x\,\frac{\sqrt{\gamma}}{f}\,\chi_1^*\chi_2. \tag{32}$$

Writing χ for either such function, integration by parts yields

$$\langle\chi, K_{\mathrm{st}}\chi\rangle_{\mathrm{w}} = \hbar^2 c_0^2\int d^3x\, f\sqrt{\gamma}\,\gamma^{ij}(\partial_i\chi)^*\left(\partial_j\chi\right)$$
$$+m_0^2 c_0^4\int d^3x\, f\sqrt{\gamma}\,|\chi|^2 \geq 0. \tag{33}$$

For $f > 0$ and a positive spatial matrix, both terms are nonnegative. A positive self-adjoint realization of K_{st} defines the static generator $i\hbar\,\partial_t\Psi = \sqrt{K_{\mathrm{st}}}\,\Psi$. This defines the positive static energy evolution with the coefficient gradients retained.

Appendix C Proof of the polar equivalence

Keep the real scalar-curvature coupling ξ and Ricci scalar $\mathcal{R}$ defined in Appendix B. Denote the full operator, including its rest-mass and curvature terms, by $\mathcal{D}_\xi$:

$$\mathcal{D}_\xi\Psi = 0, \qquad \mathcal{D}_\xi = \Box_g - \frac{m_0^2 c_0^2}{\hbar^2} - \xi\mathcal{R}. \tag{34}$$

Use the amplitude, phase, momentum, and quantum-term definitions of Section 4. Applying the derivative operator and removing the phase factor gives

$$e^{-iS/\hbar}\Box_g\Psi = \Box_g\mathcal{A} - \frac{\mathcal{A}}{\hbar^2}\nabla_\mu S\nabla^\mu S$$
$$+\frac{i}{\hbar}\left(2\nabla_\mu\mathcal{A}\nabla^\mu S + \mathcal{A}\Box_g S\right). \tag{35}$$

The first line is real and the second is imaginary. Multiplying the real part of the full wave equation by $-\hbar^2/\mathcal{A}$ gives the curvature-coupled quantum Hamilton–Jacobi equation:

$$g^{\mu\nu}p_\mu p_\nu + m_0^2c_0^2 + \hbar^2\xi\mathcal{R} + \mathcal{Q} = 0. \tag{36}$$

The imaginary part, multiplied by $\mathcal{A}$, is current conservation: $\nabla_\mu(\mathcal{A}^2\nabla^\mu S) = 0$. Conversely, a smooth positive amplitude and locally real phase satisfying these equations make both parts of $\mathcal{D}_\xi\Psi$ zero by the same identity. Their reconstructed wave therefore solves the original complex equation. The transformation is two-way. In the vacuum exteriors, $\mathcal{R} = 0$ gives the main-text equations for every finite ξ.

Differentiating the phase equation gives a momentum identity. Define the combined momentum-squared correction $\mathcal{W} = \mathcal{Q} + \hbar^2\xi\mathcal{R}$ and raised momentum $p^\nu = g^{\nu\alpha}p_\alpha$. Symmetry of the covariant derivatives of the scalar phase gives

$$p^\nu\nabla_\nu p_\mu = -\frac{1}{2}\partial_\mu\mathcal{W}. \tag{37}$$

The left side is the covariant rate of change of phase momentum along its own flow. Relating it to proper-time acceleration uses the normalization of the corresponding particle tangent. When the correction is negligible, the classical energy–momentum relation and geodesic rays follow. For $m_0 > 0$, $Q_{\mathrm{rel}} = \mathcal{Q}/(2m_0)$ is an energy convention with the limit in Section 5. The term $\mathcal{Q}$ also covers a massless scalar.

For a closed loop in a nonzero-wave region, let n count the integer phase winding, with $\mathbb{Z}$ the set of integers. The displacement along the loop is dx^μ. A single-valued reconstructed wave requires

$$\oint p_\mu\, dx^\mu = 2\pi n\hbar, \qquad n \in \mathbb{Z}. \tag{38}$$

Together with the amplitude-and-phase equations, this circulation condition and the original initial and boundary data reconstruct the same single-valued wave across compatible phase patches.

Appendix D A direct counterexample to equal potentials

Take a free one-dimensional wave in a uniform reference environment. Then the clock and ruler factors are both one and the environmental storage potential is zero. Let x be its Cartesian coordinate and $\sigma > 0$ the width parameter of a Gaussian amplitude. The proportionality sign below leaves the overall normalization unspecified; that constant cancels from the quantum potential. At a chosen initial time, specify

$$\mathcal{A}(x) \propto \exp[-x^2/(4\sigma^2)], \qquad f = b = 1, \qquad V_{\mathrm{OQG}} = 0. \tag{39}$$

Taking the second derivative of this amplitude gives

$$Q_{\mathrm{NR}}(x) = \frac{\hbar^2}{4m_0\sigma^2} - \frac{\hbar^2 x^2}{8m_0\sigma^4}. \tag{40}$$

This amplitude is allowed initial data for a free Schrödinger packet. Its generally nonzero quantum potential coexists with a zero environmental storage potential. This explicit solution demonstrates the distinct roles of the environmental storage potential and the receiving wave's quantum potential within the same dynamics.

Appendix E Dirac matter and its exact gravitational current

The scalar probe makes the amplitude-and-phase proof transparent. We now use the massive electron field, a Standard Model fermion. After electroweak symmetry breaking, its left- and right-handed components form a four-component Dirac spinor, denoted by ψ_{D}. Its reference mass is m_0. We use the free electron test-field limit, neglecting imposed gauge fields, electron self-interaction, radiative corrections, and backreaction on the prescribed gravitational background. The gravitational propagation equation retains the exact clock, ruler, and rotational coefficients already used in the paper [15,16].

E1 The local frame and Dirac equation

A spinor records spin relative to a local orthonormal frame. A tetrad is the conversion between that frame and the coordinates. Denote its local length and time one-forms by $\vartheta^{\hat{a}}$; a hat labels a frame direction. In the Kerr coordinates of Section 7, a frame carried by the rotating normal observers is

$$\begin{aligned} \vartheta^{\hat{0}} &= f c_0\, dt, & \vartheta^{\hat{r}} &= \sqrt{\gamma_{rr}}\, dr, \\ \vartheta^{\hat{\theta}} &= \sqrt{\gamma_{\theta\theta}}\, d\theta, & \vartheta^{\hat{\phi}} &= \sqrt{\gamma_{\phi\phi}}(d\phi - \omega_{\mathrm{d}} dt). \end{aligned} \tag{41}$$

Their interval is $ds^2 = -\left(\vartheta^{\hat{0}}\right)^2 + \sum_{\hat{\imath}} \left(\vartheta^{\hat{\imath}}\right)^2$. The dual frame vectors $e^{\mu}_{\hat{a}}$ convert local vector components to coordinate components. For the static isotropic case the same construction uses $\vartheta^{\hat{0}} = f c_0 dt$ and $\vartheta^{\hat{\imath}} = b^{-1} dx^i$. Thus both exteriors supply an explicit material frame.

The constant Dirac matrices $\gamma^{\hat{a}}$ act on the four spinor components. They are distinct from the spatial ruler coefficients γ_{ij}. Let I_4 be the four-dimensional identity matrix and $\{A, B\} = AB + BA$ the anticommutator. With $\eta^{\hat{a}\hat{b}} = \mathrm{diag}(-1,1,1,1)$, we use the conventional matrices whose time matrix squares to I_4:

$$\{\gamma^{\hat{a}}, \gamma^{\hat{b}}\} = -2\eta^{\hat{a}\hat{b}} I_4, \qquad \gamma^{\mu} = e^{\mu}_{\hat{a}} \gamma^{\hat{a}}, \qquad \{\gamma^{\mu}, \gamma^{\nu}\} = -2g^{\mu\nu} I_4. \tag{42}$$

The minus sign matches these matrices to the paper's $(-+++)$ interval convention [16]. The frame changes between nearby events through its torsion-free rotation coefficients $\omega_{\hat{a}\hat{b}\mu}$, antisymmetric in the hatted indices. They are fixed by $d\vartheta^{\hat{a}} + \omega^{\hat{a}}_{\hat{b}} \wedge \vartheta^{\hat{b}} = 0$, where d is the exterior derivative and $\wedge$ is the antisymmetric product of one-forms. Writing $\omega_{\hat{a}\hat{b}} = \omega_{\hat{a}\hat{b}\mu} dx^{\mu}$ defines their coordinate components. The corresponding spin-connection matrices Ω_{μ} and spinor derivative D_{μ} are

$$D_{\mu}\psi_{\mathrm{D}} = \left(\partial_{\mu} + \Omega_{\mu}\right)\psi_{\mathrm{D}}, \qquad \Omega_{\mu} = -\frac{1}{4}\omega_{\hat{a}\hat{b}\mu}\gamma^{\hat{a}}\gamma^{\hat{b}}. \tag{43}$$

All coordinate derivatives here use $x^0 = c_0 t$. The minimally coupled Dirac equation for the gravitational propagation of the electron is

$$\left(i\hbar c_0 \gamma^{\mu} D_{\mu} - m_0 c_0^2 I_4\right)\psi_{\mathrm{D}} = 0. \tag{44}$$

The tetrad converts the exact material comparisons; the spin connection transports the spinor as that frame varies. Both are determined by the displayed exterior coefficients and their derivatives. This gives the spin-one-half propagation equation directly in the same OQG gravitational environment.

E2 Conserved current and guidance

Define the Dirac adjoint by $\overline{\psi}_{\mathrm{D}} = \psi_{\mathrm{D}}^{\dagger}\gamma^{\hat{0}}$. The dagger denotes Hermitian conjugation. Let j_{D}^{μ} denote its coordinate current, $j_{\mathrm{D}}^{\hat{0}}$ its local time component, and n_{D} the local density. The compatible spin connection gives

$$\begin{aligned} j_{\mathrm{D}}^{\mu} &= c_0 \overline{\psi}_{\mathrm{D}} \gamma^{\mu} \psi_{\mathrm{D}}, \qquad \nabla_{\mu} j_{\mathrm{D}}^{\mu} = 0, \\ n_{\mathrm{D}} &= j_{\mathrm{D}}^{\hat{0}}/c_0 = \psi_{\mathrm{D}}^{\dagger}\psi_{\mathrm{D}} \geq 0. \end{aligned} \tag{45}$$

Conservation follows by multiplying the Dirac equation by its adjoint and combining it with the adjoint equation; the two mass terms cancel. Compatibility preserves the Dirac matrices under covariant differentiation. The local density is positive wherever the spinor is nonzero, and the local current is future directed and causal [16].

In the t, x^i chart, define v_{D}^i as the velocity along a current line. The spatial determinant is $\gamma = \det(\gamma_{ij})$, and $\mathcal{P}_{\mathrm{D}}$ is the density per coordinate spatial volume. Since $j_{\mathrm{D}}^t = n_{\mathrm{D}}/f$, conservation becomes

$$v_{\mathrm{D}}^i = \frac{j_{\mathrm{D}}^i}{j_{\mathrm{D}}^t}, \qquad \mathcal{P}_{\mathrm{D}} = \sqrt{\gamma}\, n_{\mathrm{D}}, \qquad \partial_t \mathcal{P}_{\mathrm{D}} + \partial_i\left(\mathcal{P}_{\mathrm{D}} v_{\mathrm{D}}^i\right) = 0. \tag{46}$$

For a normalized one-electron state, the integral of this density is preserved with its boundary flux accounted for. The guidance velocity transports that same density. This is the conserved-current formulation of fermionic wave guidance in the exact material frame.

E3 Rotation and the material dispersion

The time vector dual to the Kerr coframe is $\left(\partial_t + \omega_{\mathrm{d}}\,\partial_\phi\right)/(f c_0)$. Its azimuthal vector is $\partial_\phi/\sqrt{\gamma_{\phi\phi}}$. Therefore the local time and azimuthal currents give

$$j_{\mathrm{D}}^{t} = \frac{j_{\mathrm{D}}^{\hat{0}}}{f c_0}, \qquad j_{\mathrm{D}}^{\phi} = \omega_{\mathrm{d}} j_{\mathrm{D}}^{t} + \frac{j_{\mathrm{D}}^{\hat{\phi}}}{\sqrt{\gamma_{\phi\phi}}}. \tag{47}$$

Define the local azimuthal velocity by $v_{\mathrm{D}}^{\hat{\phi}} = c_0 j_{\mathrm{D}}^{\hat{\phi}}/j_{\mathrm{D}}^{\hat{0}}$. It includes the complete spinor current. The exact coordinate angular velocity is then

$$\begin{aligned} \frac{d\phi}{dt} &= \omega_{\mathrm{d}} + \frac{f}{\sqrt{\gamma_{\phi\phi}}} v_{\mathrm{D}}^{\hat{\phi}}, \\ j_{\mathrm{D}}^{\hat{\phi}} = 0 \quad &\Rightarrow \quad \frac{d\phi}{dt} = \omega_{\mathrm{d}}. \end{aligned} \tag{48}$$

The first term is gravitational dragging; the second is the receiver's local azimuthal flow converted by its clock and ruler factors. The Dirac current carries spin dependence, expressed by the convection and spin contributions in its Gordon decomposition [15]. For this field the exact dragging condition is zero total local azimuthal current. It retains all spin contributions and gives the same dragging rate as the scalar result under its corresponding condition.

Finally, the material dispersion follows from the leading phase of the Dirac equation. Write $\psi_{\mathrm{D}} = a_{\mathrm{D}} e^{iS/\hbar}$, with slowly varying spinor amplitude a_{D} and phase momentum $p_\mu = \partial_\mu S$. At leading ray order the Dirac matrix equation has a nonzero solution only on the mass shell. With coordinate energy $H = -\partial_t S$, its positive local-energy branch is

$$\begin{aligned} g^{\mu\nu} p_\mu p_\nu + m_0^2 c_0^2 &= 0, \\ H &= \omega_{\mathrm{d}} p_\phi + f\sqrt{m_0^2 c_0^4 + c_0^2 \gamma^{ij} p_i p_j}. \end{aligned} \tag{49}$$

This ray limit is an expansion in wavelength, with the gravitational coefficients retained exactly. The full Dirac equation retains spin transport and amplitude evolution. The scalar proof and the fermionic current therefore connect the same OQG material response to both transparent amplitude-and-phase dynamics and the gravitational propagation of Standard Model matter.

Statements and Declarations

Funding

This work was self-funded by the author. No funds, grants, or other external support were received during the preparation of this manuscript.

Conflict of Interest

Todd J. Desiato is the founding editor and publisher of the Journal of Alternative Theoretical Physics. This role is disclosed as an editorial conflict. Independent assessment and the handling of the acceptance decision will be documented separately.

Author contributions

Todd J. Desiato conceived the OQG programme and the present research question and directed the theoretical development. The author is responsible for the scientific arguments and final manuscript.

Data availability

No new experimental or observational datasets were generated. The analytical equations are contained in this article. Online Resource 1 contains the numerical verification program and its saved outputs.

Code availability

Online Resource 1 supplies the Python 3 standard-library scalar/exterior verification program, its saved results, and the complete version 0.6 construction source. The source contains all 49 numbered equations and Appendix E; the README explains reproduction and numerical coverage.

Ethics statement

Not applicable. This theoretical study involved no human participants, animals, or identifiable human data.

Use of generative artificial intelligence

OpenAI ChatGPT and Codex were used for literature discovery, mathematical cross-checking, numerical implementation, organization, drafting, and document preparation. The author is responsible for reviewing and approving the scientific content before release.

Supplementary information

Online Resource 1 accompanies this manuscript as OQG_Strong_Field_Bohm_V0_6_Supplement.zip. A versioned controlled-access copy is retained in the author's supporting archive. For editorial inspection or a verification copy, contact the corresponding author at mainengineering@gmail.com.

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