Nonlocal Conformal-Boundary Refinement of Machian Propulsion: A Hyperbolic Möbius Framework Extending Woodward-Type Mass Fluctuation Models

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"If the objective is to reproduce the observable consequences of external acceleration, then the internal dynamics must reproduce not only energy exchange behavior, but also the symmetry structure of relativistic acceleration, including its spatial asymmetry." (Gabriel Brennan)

1 Abstract

A nonlocal conformal-boundary framework for emergent inertial and propulsion-like phenomena is developed in which motion arises from relational coupling between internally driven electromechanical dynamics and an anisotropic null-interaction structure defined on the celestial sphere. The formulation departs from purely local oscillator-based approaches by introducing an irreducible boundary functional representing the directional distribution of distant matter and null connectivity. Internal crystal dynamics are modeled through nonlinear piezoelectric excitation, hyperbolic rapidity modulation, and Möbius-reprojected anisotropy evolution on the Riemann sphere.

The central dynamical variable is a directional anisotropy functional $A(\Omega, t)$, whose evolution is governed by noncommuting conformal generators,

$$T = L_{-1} + \lambda L_0 + \eta L_1,$$

with

$$L_{-1} = \partial_z, \qquad L_0 = z\partial_z, \qquad L_1 = z^2\partial_z,$$

acting on stereographic sky coordinates. Because the generators satisfy nontrivial commutation relations,

$$[L_0, L_{-1}] = -L_{-1},$$

the sequential evolution of anisotropy under oscillatory boost modulation becomes path-dependent, producing non-cancelling cycle-averaged contributions. Motion is therefore interpreted not as a primitive property of isolated matter, but as an emergent asymmetry generated by nonlinear rectification and noncommutative symmetry mixing within a globally anisotropic relational structure.

A phenomenological drift law is obtained in the form

$$v(t) = \alpha A(t) + \beta A^2(t) + \gamma \langle TA \rangle,$$

where the linear term describes direct anisotropy response, the quadratic term produces rectification and DC-like contributions, and the symmetry-mixed term encodes Möbius/Noether transport effects. Time averaging yields the drift criterion

$$v_{\mathrm{DC}} = \beta \langle A^2 \rangle + \gamma \langle TA \rangle,$$

showing that translation requires both nonvanishing fluctuation structure and noncommuting symmetry evolution. The corresponding inertial condition becomes

$$\langle A^2 \rangle = 0, \qquad \langle TA \rangle = 0,$$

interpreted as complete relational equilibrium with respect to the directional null structure of the universe.

The theory is extended to driven piezoelectric continua using standard constitutive equations,

$$S_{ij} = s_{ijkl} T_{kl} + d_{kij} E_k, \qquad D_i = d_{ikl} T_{kl} + \epsilon_{ij} E_j,$$

with hyperbolic current excitation replacing conventional sinusoidal driving,

$$I(t) = I_0 \tanh(\alpha \sin \omega t),$$

or related rapidity-like waveforms. Such excitation naturally generates even harmonics, asymmetric cycle averaging, and nonlinear stress-energy mixing through the off-diagonal momentum-flux sector T_{0z}. In this formulation, electromechanical coupling is standard, while the propulsion-like contribution emerges only after anisotropic Möbius reprojection against a global boundary functional,

$$\langle F^\mu \rangle \sim \int_{S^2} d\Omega \, B(\Omega) \langle M(\Omega, \eta(t)) \rangle.$$

Unlike local Mach-effect or oscillator-only models, the proposed framework cannot generally be reduced to finite-dimensional local dynamics because the forcing sector depends explicitly on a sky-integrated anisotropy functional,

$$B(\Omega) = \int_{\partial U} G(x, \xi)\rho(\xi)d\xi,$$

which acts as a structured nonlocal source rather than a renormalizable constant background. Translation therefore emerges only through coupled interaction between internal boost modulation and externally anisotropic relational structure,

$$\langle F \rangle \sim \langle A_{\text{internal}}(t) B_{\text{external}}(\Omega) \rangle.$$

The resulting framework generalizes a broad class of reactionless and field-propulsion concepts by recasting them as problems of nonlinear symmetry transport, hyperbolic excitation geometry, and nonlocal conformal-boundary coupling. Within this interpretation, engineered propulsion-like drift corresponds to the controlled failure of exact cancellation in Möbius-deformed null-interaction averaging under oscillatory boost evolution. “‘

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2 Relativistic Stress Evolution and Internal Vibrational Restructuring

Before invoking any stronger Machian interpretation or nonlocal inertial coupling hypothesis, a necessary intermediate conceptual step concerns the internal dynamical structure of accelerated extended bodies themselves. The essential proposition is not the existence of “reactionless thrust,” but rather the recognition that acceleration in realistic physical systems cannot be separated from evolving internal stress and vibrational dynamics.

2.1 Differential Acceleration and Tidal Structure

Within General Relativity, perfectly rigid acceleration is not physically realizable for extended bodies. In a gravitational gradient, distinct regions of a body experience different proper accelerations. In weak-field form, the differential acceleration across an object of characteristic length L may be expressed schematically as:

$$\Delta a \sim L \frac{\partial g}{\partial x}$$

where $\partial g/\partial x$ denotes the local gravitational gradient. Such differential acceleration necessarily induces strain, stress redistribution, elastic response, and propagating vibrational modes throughout the material structure.

This is fundamentally the manifestation of tidal dynamics. In relativistic language, spacetime curvature causes neighboring worldlines to diverge or converge, preventing a real extended body from translating as an undeformable geometric point. Instead, local proper distances evolve nonuniformly, internal lattice structures resist deformation, elastic energy accumulates, and stress information propagates through the body at finite velocity.

2.2 Internal Mediation of Translational Motion

The central implication is that translational motion is internally mediated. A macroscopic object does not move as a perfectly unified entity. Motion emerges through electromagnetic interactions, lattice coupling, stress redistribution, and finite-speed propagation of local equilibrium conditions. Consequently, translational acceleration necessarily alters the internal vibrational configuration of the body.

This remains true even in ordinary continuum mechanics, but becomes unavoidable in relativistic frameworks where instantaneous rigidity is prohibited. Simplified rigid-body intuition therefore obscures important physical structure.

The notion of a perfectly uniform gravitational field is itself only an approximation. General Relativity admits local equivalence regions, but realistic gravitational environments always contain curvature, gradients, and tidal structure. Accordingly, extended bodies always experience some degree of differential acceleration, even if extremely small.

2.3 Dynamical Restructuring of the Vibrational Spectrum

The deeper significance of this perspective is not merely the statement that "vibrations exist," but rather that acceleration dynamically reshapes the internal vibrational spectrum itself. The stress tensor evolves during acceleration, mode coupling changes, resonance structure shifts, and proper-frame relationships within the material continuously reorganize. The problem therefore enters the domain of relativistic continuum dynamics and geometry-dependent stress-energy evolution.

Importantly, this framework remains fully compatible with conventional conservation laws. No violation of momentum conservation or breakdown of General Relativity is required. The proposal instead emphasizes that the internal state space of accelerating matter is richer and more dynamically coupled than many simplified translational models assume.

2.4 Experimental Implications

This perspective suggests that experimentally relevant effects may emerge in systems involving high-Q resonators, strong accelerations, anisotropic materials, rapidly evolving stress fields, or complex elastic geometries. Under such conditions, subtle coupling phenomena may appear, including phase-dependent elastic lag, transient field momentum exchange, geometry-sensitive recoil behavior, or asymmetric mode conversion.

The conceptual progression therefore proceeds in a disciplined hierarchy:

$$\begin{aligned} \text{acceleration} &\rightarrow \text{stress evolution} \\ &\rightarrow \text{vibrational restructuring} \\ &\rightarrow \text{modified effective response} \end{aligned}$$

Only after establishing this continuum-relativistic foundation does it become meaningful to investigate whether deeper Machian or nonlocal inertial couplings may exist. The internal dynamical restructuring of accelerated matter thus serves as the necessary conceptual bridge preceding any stronger claims regarding inertia, vacuum interaction, or cosmological coupling mechanisms.

In this formulation, the statement that "acceleration cannot occur without alteration of internal vibrational structure" becomes a direct consequence of finite propagation speeds, tidal gradients, elastic mediation, and the impossibility of perfectly rigid motion within relativistic physics.

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3 Internal Dynamical Structure and Emergent Geodesic Motion

The central proposition developed in this framework is that a featureless point particle cannot meaningfully respond to spacetime curvature unless some form of internal dynamical structure exists that integrates differences between neighboring tangent spaces. The proposal therefore replaces the conventional geometric interpretation of geodesic motion with a dynamically mediated interpretation.

3.1 Geodesic Motion as Dynamical Mediation

In standard General Relativity, geodesic motion is treated as a direct geometric consequence of spacetime structure. The trajectory of a particle is determined by the spacetime connection, and no internal mechanism is required for the implementation of parallel transport. Motion is therefore regarded as a primitive geometric property.

The present framework instead proposes that geodesic behavior emerges from the evolution of an internally structured dynamical system continuously sampling local patches of approximate flatness. In this interpretation, curvature response is not fundamental but implemented through internal information-bearing processes.

The conceptual distinction may be summarized schematically:

$$\text{Standard GR:} \quad \text{Geometry} \rightarrow \text{Geodesic Motion}$$

$$\begin{aligned} \text{Present Framework:} \quad & \text{Internal Dynamical Evolution} \\ & \rightarrow \text{Curvature Response} \\ & \rightarrow \text{Emergent Motion} \end{aligned}$$

The divergence is therefore not primarily mathematical, but ontological. General Relativity assumes that the spacetime connection acts directly on tangent vectors, whereas the present approach proposes that physical response requires an internal mediator capable of integrating geometric differences between neighboring regions.

3.2 Internal Oscillatory Structure

Under this interpretation, internal oscillatory structure becomes essential. The motion of a body is no longer viewed as a primitive translation through spacetime, but as the emergent projection of evolving internal dynamics onto external geometry.

Formally, this introduces an internal state space associated with each spacetime point together with a dynamical mapping:

$$\mathcal{F} : \mathcal{I} \to \delta x^{\mu}$$

where $\mathcal{I}$ denotes the internal dynamical state and δx^{μ} represents the resulting correction to spacetime trajectory evolution.

In modern mathematical language, this resembles a fiber-bundle construction in which spacetime acts as the base manifold while the internal dynamical variables constitute the fiber structure. Motion then corresponds not merely to propagation in spacetime, but to coupled evolution within a higher-dimensional configuration manifold.

3.3 Broken Temporal Symmetry and Drift

The framework further proposes that symmetric internal oscillatory evolution produces no net displacement, whereas time-asymmetric evolution generates effective translation. Symbolically:

$$\text{Symmetric Oscillation} \to 0$$

$$\text{Broken Temporal Symmetry} \to \Delta x^{\mu} \neq 0$$

This corresponds structurally to broken time-reversal symmetry or to nontrivial phase holonomy accumulation. The essential mechanism becomes directional drift arising from irreversible or non-reciprocal evolution within internal phase space.

In this view, translation emerges not as a primitive spacetime symmetry, but as the macroscopic projection of internally evolving oscillatory structure.

3.4 Riemann Sphere and Internal Phase Evolution

The geometric machinery employed in the framework includes the Riemann sphere and the group $SL(2, \mathbb{C})$. These structures are used to encode compact complex phase evolution, spinor-like transformations, and Möbius dynamical mappings acting on internal states.

In conventional mathematical physics, such structures naturally arise in spinor transport, complex projective geometry, and quantum-state evolution. The present interpretation extends this role further by proposing that spacetime translation itself is generated through the projection of internal rotational or phase dynamics.

The essential relation may therefore be expressed schematically as:

$$\text{Internal Phase Evolution} \to \text{Projected Spacetime Motion}$$

or more generally:

$$\gamma : \mathcal{P} \to M$$

where $\mathcal{P}$ denotes internal phase space and M the spacetime manifold.

Within this framework, motion is no longer fundamental. Instead, motion becomes an emergent phenomenon arising from internal oscillatory structure interacting with geometry through nontrivial holonomy.

The strongest conceptual statement of the model is therefore:

Motion is a projection of internal oscillatory evolution onto spacetime geometry.

The remaining mathematical structures — including Noether-like matrix constructions, complex projective geometry, and boundary conditions — function primarily as supporting formal machinery for this central hypothesis.

3.5 Additional Assumptions and Machian Closure

Several additional assumptions extend beyond conventional General Relativity. First, the proposal that translation requires internal structure is not derivable from standard GR itself, since ordinary relativistic theory permits structureless point particles to follow geodesics directly. The present framework instead treats geodesic motion as requiring physical implementation rather than merely geometric definition.

Second, any proposal involving effective reactionless propulsion requires explicit accounting of momentum exchange across all participating sectors, including internal fields, boundaries, and extended dynamical structures, in order to maintain compatibility with conservation laws.

Third, the suggestion that motion ultimately requires global coupling to the universe constitutes a Machian closure principle. Such a principle may serve as an additional postulate motivating the framework, but it is not implied directly by General Relativity itself.

When reduced to its mathematically stable core, the framework may therefore be summarized as follows:

Spacetime trajectories emerge from the projection
of internal phase evolution onto a coupled
internal–external configuration manifold
possessing nontrivial holonomy.

This defines a coherent class of dynamical models in which translational behavior is interpreted as an emergent geometric manifestation of deeper internal oscillatory structure rather than as a primitive spacetime property.

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4 Coupled Variational Structure and Emergent Motion

The framework develops from a coupled variational structure in which internal dynamical degrees of freedom, external spacetime coordinates, and cosmological boundary conditions participate simultaneously in the generation of motion. Rather than treating geodesic propagation as a purely primitive geometric property, the formalism interprets motion as the emergent consequence of interactions between internal state evolution and external spacetime structure.

The starting point is a decomposed Lagrangian consisting of three sectors:

$$L = L_{\text{ext}}(x^\mu, \dot{x}^\mu) + L_{\text{int}}(\chi, \dot{\chi}) + L_{\text{coupling}}(x^\mu, \chi)$$

where x^μ denotes spacetime coordinates and χ represents internal dynamical variables describing oscillatory, phase, or structural degrees of freedom. The external sector governs ordinary spacetime evolution, the internal sector governs intrinsic dynamical structure, and the coupling sector mediates information transfer between them.

Within this formulation, motion is no longer interpreted as arising exclusively from spacetime geometry. Instead, translational behavior emerges from the dynamical interaction between internal evolution and external geometric constraints.

The conservation structure follows a generalized Noether framework. In the absence of boundary effects, the stress-energy tensor satisfies:

$$\partial_\mu T^{\mu\nu} = 0$$

However, when finite cosmological boundaries or global asymmetries are included, the conservation structure acquires additional contributions:

$$\partial_\mu T^{\mu\nu} = J^\nu_{\text{boundary}}$$

where $J^{\nu}_{\text{boundary}}$ represents effective boundary-induced contributions to the dynamical system. In this interpretation, conservation laws remain globally valid while allowing local redistribution through coupling to large-scale geometric or cosmological structure.

The spacetime trajectory equation correspondingly acquires internal forcing terms:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = F^\mu_{\text{int}}$$

where the effective force term F^μ_{int} emerges from the internal dynamical sector and its coupling to geometry.

A representative interaction term may be written as:

$$L_{\text{coupling}} = \lambda_\mu(\chi)\dot{x}^\mu$$

with $\lambda_\mu(\chi)$ encoding the dependence of spacetime evolution upon internal dynamical configuration. Such a term naturally modifies the canonical momentum:

$$p_\mu = \frac{\partial L}{\partial \dot{x}^\mu} = p^{(0)}_\mu + \lambda_\mu(\chi)$$

thereby introducing geometry-sensitive corrections to translational behavior.

Within this framework, internal dynamical evolution does not merely accompany motion but actively participates in generating effective trajectories. Fast internal oscillatory variables interact with slower spacetime evolution through adiabatic coupling processes, producing emergent drift phenomena analogous to geometric phase transport and effective force generation in multiscale dynamical systems.

The framework therefore interprets inertia itself as an active dynamical phenomenon rather than a purely passive geometric constraint.

A central role is played by symmetry breaking and cosmological boundary structure. Finite global boundaries modify the effective conservation topology of the system, altering how internal dynamical states project into spacetime trajectories. Motion consequently emerges from the interplay between local internal evolution and global geometric constraints.

In this interpretation, global cosmological structure cannot be completely separated from local inertial response. Internal degrees of freedom continuously interact with boundary-defined asymmetries, producing directional bias in effective motion.

The resulting ontology differs fundamentally from conventional geometric formulations. Rather than assuming that geometry directly determines motion, the framework proposes that internal structure mediates the mapping between geometry and trajectory evolution.

Schematically:

$$\text{Internal Dynamics} \rightarrow \text{Effective Geometric Response} \rightarrow \text{Emergent Motion}$$

Motion is therefore not regarded as a primitive spacetime property, but as a projected manifestation of deeper coupled internal–external dynamics.

The strongest aspect of the framework lies in the treatment of motion as an emergent phenomenon arising from coupled internal phase evolution and external geometry. Internal fast variables, oscillatory modes, and structural phase relationships collectively generate effective drift through adiabatic and symmetry-breaking interactions.

This may be expressed generally as:

$$M_{\text{effective}} = F(x^\mu, \chi, \partial_\mu g_{\alpha\beta}, B)$$

where B denotes global boundary structure and F represents the coupled internal–external dynamical mapping.

Within this formulation, spacetime trajectories emerge from the projection of internal phase dynamics onto external geometry under globally constrained conservation structure. Translation is therefore interpreted as a secondary emergent effect of deeper oscillatory and geometric processes.

The complete framework may consequently be summarized as follows:

Motion arises through the adiabatic coupling
of internal dynamical degrees of freedom
to external spacetime geometry under
symmetry-breaking boundary conditions.

In this view, inertia, drift, and effective trajectory formation are manifestations of coupled phase evolution occurring across internal state space, spacetime geometry, and cosmological boundary structure simultaneously.

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5 Geometric-Mechanical Construction and Cosmological Coupling

The framework develops into a geometric-mechanical construction in which local dynamics, internal phase evolution, and cosmological boundary structure participate simultaneously in the emergence of motion. The theory no longer treats propulsion or translation as isolated mechanical processes, but instead reformulates motion as the manifestation of coupled configuration-space dynamics embedded within a globally constrained geometric structure.

At the level of effective dynamics, the total Lagrangian may be expressed as

$$L = L_{\text{local}} + \epsilon V_{\infty}(x)$$

where L_{local} governs local dynamical evolution while $V_{\infty}(x)$ encodes a weak but globally extended background contribution associated with cosmological boundary structure. The parameter ϵ controls the coupling strength between local dynamics and the large-scale ordering background.

This formulation interprets local physical behavior as embedded within a broader asymptotic geometric environment rather than as completely isolated from cosmological structure. The global term therefore acts as a residual ordering contribution inherited from boundary conditions of the universe as a whole.

The cosmological constant enters naturally within this interpretation through the Einstein field equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

In this framework, Λ is not merely treated as a passive curvature contribution, but as the residual geometric imprint of global boundary organization. The cosmological background therefore functions as a persistent large-scale ordering field influencing how internal dynamical evolution projects into effective spacetime behavior.

Motion consequently emerges not solely from local geometry, but from the interaction between local internal phase dynamics and globally constrained geometric structure.

5.1 Möbius Dynamics and Projective Geometry

The mathematical language of the framework employs the Riemann sphere together with the projective group $PSL(2,\mathbb{C})$, which is isomorphic to the complexified Lorentz group:

$$PSL(2,\mathbb{C}) \cong SO^{+}(3,1)_{\mathbb{C}}$$

Internal dynamical evolution is therefore represented through Möbius transformations acting on complex projective coordinates. The evolution law takes the form

$$z \mapsto e^{\phi} z$$

where ϕ represents a hyperbolic boost parameter. In projective geometry this corresponds to dilation on the Riemann sphere, while physically it represents internal phase amplification and directional geometric scaling.

Within the framework, these transformations are not interpreted merely as passive symmetry representations. Instead, internal Möbius evolution acts as the generator of effective spacetime translation itself. Motion is therefore reinterpreted as the external projection of evolving internal geometric transformations.

The central mechanism may be expressed schematically as

$$(\theta, \phi) \to x^{\mu}$$

where internal phase coordinates determine effective spacetime displacement through coupled geometric evolution.

5.2 Internal Configuration-Space Dynamics

The internal dynamics are governed by a generalized Lagrangian on configuration space:

$$L = \frac{1}{2} I \dot{\theta}^2 - \frac{1}{2} M \dot{\phi}^2 + \alpha \dot{\theta} \dot{\phi} - V(\theta, \phi, x)$$

The first two terms describe the kinetic structure associated with the internal oscillatory variables θ and ϕ, while the mixed term

$$\alpha \dot{\theta} \dot{\phi}$$

introduces nontrivial coupling between internal modes. The resulting kinetic structure defines a non-diagonal metric on configuration space, implying that internal phase evolution cannot be separated into independent orthogonal components.

The geometry of the dynamical manifold therefore becomes intrinsically fibered and coupled. Internal oscillatory evolution, external spacetime coordinates, and global geometric ordering collectively participate in a single unified dynamical structure.

The effective configuration metric may be interpreted as

$$ds^2 = I \, d\theta^2 - M \, d\phi^2 + 2\alpha \, d\theta \, d\phi$$

revealing explicit off-diagonal coupling between the internal coordinates. Motion through spacetime consequently arises from trajectories on a higher-dimensional coupled manifold rather than from isolated propagation in ordinary spacetime alone.

Within this interpretation, internal phase evolution continuously generates external displacement through geometric projection processes. Translation is therefore not regarded as a primitive kinematic operation, but as an emergent consequence of coupled internal geometry.

5.3 Role of Global Boundary Structure

The role of global boundary structure is fundamental. Cosmological asymptotics bias the effective symmetry structure of the dynamical manifold, thereby influencing how internal oscillatory evolution unfolds. Boundary conditions do not merely terminate spacetime geometrically; they participate actively in shaping the global conservation topology and effective ordering structure of the system.

The resulting ontology differs substantially from purely local geometric interpretations of motion. In the present framework, local trajectories are inseparable from internal phase evolution and global geometric organization simultaneously.

The full structure may therefore be summarized as a coupled dynamical system consisting of:

$$\{\theta, \phi, x^\mu, \Lambda, B\}$$

where B denotes cosmological boundary structure.

Motion then emerges through the interaction of:

$$\text{internal phase evolution} \leftrightarrow \text{configuration-space geometry} \leftrightarrow \text{global boundary ordering}$$

The theory consequently interprets spacetime translation as the projected manifestation of deeper coupled oscillatory dynamics evolving on a nontrivial geometric manifold. Internal phase structure, cosmological asymmetry, and geometric coupling together generate effective motion through continuous projection from configuration space into spacetime coordinates.

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6 SL(2,C) Group-Orbit Dynamics and Emergent Spacetime Projection

The formal structure developed in this framework converges toward a coupled geometric dynamical system in which the symmetry group $SL(2, \mathbb{C})$ acts on an internal complex projective manifold while observable spacetime coordinates emerge through projection from internal group-orbit evolution.

The construction is based on the Riemann sphere $\mathbb{CP}^1$, together with Möbius transformations generated by the projective special linear group:

$$PSL(2,\mathbb{C}) = SL(2,\mathbb{C})/\mathbb{Z}_2$$

which serves as the double cover of Lorentz symmetry. Internal dynamical evolution is therefore represented through continuous group action on complex projective coordinates rather than through direct propagation in primitive spacetime.

6.1 Möbius Action on Internal State Space

The internal state variable $z \in \mathbb{CP}^1$ evolves under generalized Möbius transformations of the form

$$z \mapsto \frac{az+b}{cz+d}$$

with

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{C}), \qquad ad - bc = 1$$

The transformation matrix may be parameterized through coupled rotational and hyperbolic components:

$$T(\theta,\phi) \in SL(2,\mathbb{C})$$

with rotational contributions generated by factors such as

$$e^{i\theta}$$

and hyperbolic boost structure generated through combinations of

$$\sinh(\phi), \cosh(\phi)$$

The resulting dynamics therefore combine circular phase evolution and hyperbolic scaling within a unified complex geometric flow.

6.2 Projection of Internal Symmetry Evolution

Within this framework, Lorentz-type transformations do not merely describe passive symmetry operations acting on already existing spacetime coordinates. Instead, internal symmetry evolution itself generates observable spacetime behavior through projection from projective orbit dynamics.

The observable external coordinate is defined through a projection map:

$$x(t) = \mathrm{Re}\left(\frac{az+b}{cz+d}\right)$$

This relation constitutes the central dynamical bridge of the theory. Observable translation is interpreted as the real-valued projection of nonlinear internal group evolution occurring on $\mathbb{CP}^1$.

Motion therefore emerges from the geometry of internal symmetry flow rather than from primitive translational generators.

In this interpretation, spacetime displacement is not fundamental. Instead, translation arises as the induced manifestation of evolving internal projective geometry:

$$SL(2,\mathbb{C}) \text{ orbit dynamics} \to x(t)$$

The framework consequently reinterprets spacetime symmetry structure itself. Rather than treating translations and Lorentz transformations as independent primitive sectors, translational behavior is generated dynamically from Lorentz-type internal evolution.

The standard decomposition

$$\text{Poincaré Group} = \text{Translations} \oplus \text{Lorentz Transformations}$$

is therefore replaced by a hierarchy in which translational structure emerges secondarily from deeper internal symmetry action.

The external coordinate $x(t)$ becomes a projected observable derived from internal group-orbit geometry rather than an independently fundamental base coordinate.

6.3 Noether Structure and Coupled Generators

Noether structure arises naturally within this coupled framework through generalized symmetry generators acting simultaneously across internal and projected sectors. The conserved quantity takes the form

$$Q = p_\theta \hat{J}_\theta + p_\phi \hat{J}_\phi$$

where p_θ and p_ϕ denote generalized canonical momenta associated with the internal variables, while $\hat{J}_\theta$ and $\hat{J}_\phi$ represent the corresponding infinitesimal symmetry generators.

6.4 Fibered Geometric Structure

The dynamical system therefore contains several coupled geometric layers simultaneously:

$$(\theta, \phi) \to z \in \mathbb{CP}^1 \to x(t) \in \mathbb{R}$$

Internal configuration variables generate motion on the projective manifold, while observable spacetime displacement emerges through projection from the resulting orbit structure.

The complete construction may be interpreted geometrically as a fibered dynamical manifold in which:

$$\text{Fiber} = \mathbb{CP}^1, \quad \text{Base Space} = \mathbb{R}, \quad \text{Structure Group} = SL(2, \mathbb{C})$$

The physical trajectory corresponds not to primitive motion through spacetime alone, but to projected evolution of internal group flow across the fiber structure.

6.5 Emergent Ontology of Spacetime Motion

The resulting ontology treats spacetime itself as emergent from deeper internal geometric organization. Observable translation becomes a macroscopic manifestation of evolving symmetry structure within internal projective space.

The framework therefore approaches a generalized principle:

$$\text{Spacetime motion} = \text{projection of internal symmetry evolution}$$

In this formulation, geometry is no longer interpreted as a passive background arena. Instead, spacetime structure itself emerges dynamically through continuously evolving group-orbit projections generated by internal oscillatory and hyperbolic phase dynamics.

The projection process is itself dynamical rather than fixed, implying that observable geometry depends continuously upon internal symmetry evolution, coupling structure, and phase configuration. Motion consequently becomes the externalized manifestation of deeper geometric processes unfolding on the internal complex projective manifold.

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7 Relational Interpretation: Gravity, Inertia, and Cosmological Expansion

The framework converges toward a unified relational interpretation in which gravity, inertia, and cosmological expansion are not treated as fundamentally separate ontological sectors, but as different manifestations of a single symmetry-biasing mechanism arising from interaction asymmetry and global boundary structure.

Within this picture, a physical system acquires effective translation or acceleration because it experiences an asymmetry in the accessible network of interactions and signal relationships. This asymmetry is encoded globally through boundary structure appearing within the action itself.

The cosmological horizon therefore acts as a large-scale global boundary producing expansion-like behavior, while a massive body functions as a local boundary within the interaction network, generating attraction-like behavior. Cosmological expansion and gravitational attraction consequently emerge from the same underlying principle:

$$\text{Boundary Asymmetry} \rightarrow \text{Interaction Bias} \rightarrow \text{Effective Motion}$$

The resulting framework is fundamentally relational and Machian in character. Geometry is no longer interpreted as a pre-existing passive manifold curved by matter. Instead, effective geometry emerges from asymmetries in the structure of physically realizable interactions and propagating signals.

The central proposition becomes:

$$\text{Gravity} = \text{directional asymmetry in interaction structure}$$

rather than curvature acting independently as a primitive ontological object.

7.1 Signal-Theoretic Hierarchy

Within this interpretation, motion itself acquires meaning only through evolving observational relationships. Observational relationships require propagating signals, and propagating signals are governed fundamentally by null structure. Consequently, null and conformal relationships become prior to ordinary spatial geometry.

This is the reason the Riemann sphere and Möbius structure occupy a central role within the framework. The geometry of light propagation and signal connectivity becomes more fundamental than naive spatial embedding.

The conceptual hierarchy therefore takes the form:

$$\text{Signal Structure} \rightarrow \text{Observational Relationships} \rightarrow \text{Effective Geometry} \rightarrow \text{Motion}$$

In this view, translation does not correspond to particles traversing a passive background manifold. Instead, translation emerges from transformations of relational signal structure itself.

The Möbius group organizes these null and conformal relationships naturally through the action

$$z \mapsto \frac{az+b}{cz+d}$$

with

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{C}), \qquad ad-bc=1$$

Observable motion is then generated through asymmetries in the evolving network of accessible signal configurations.

7.2 Relational Origin of Effective Gravitation

A gravitating body therefore acts as a local effective boundary for neighboring systems. The presence of mass modifies the accessible interaction topology surrounding a test particle, thereby biasing the directional structure of possible signal relationships.

Effective attraction emerges not from an external force acting across passive space, but from asymmetry in relational accessibility generated by boundary structure.

The framework thus attempts to derive effective geodesic behavior from relational signal asymmetry:

$$\text{Relational Signal Bias} \rightarrow \text{Effective Geodesic Motion}$$

Within this formulation, Einsteinian geometry becomes an emergent phenomenological description of deeper interaction structure rather than the fundamental ontology itself.

The equivalence between curvature language and symmetry-mixing language becomes central. Curvature terms and Noether-type mixing terms may therefore represent dual descriptions of the same underlying relational dynamics.

The generalized dynamical structure may be expressed schematically through coupling matrices of the form

$$\phi_{ab}(u^\mu)$$

where the interaction connection ϕ_{ab} encodes how internal symmetry sectors mix under evolving relational structure parameterized by the spacetime flow variables u^μ.

Observable translation arises when Noether-sector mixing produces directional asymmetry within the interaction manifold:

$$N_{ab} \to \Delta x^\mu$$

where N_{ab} denotes generalized symmetry-mixing structure.

7.3 Emergent Geometric Ontology

Within this interpretation, Einstein geometry acts as an emergent bookkeeping formalism summarizing the effective large-scale behavior of deeper signal and interaction dynamics. Geometry is therefore secondary rather than primary.

The framework consequently replaces purely geometric ontology with a hierarchy based upon interaction structure:

$$\text{Interaction Network} \to \text{Signal Asymmetry} \to \text{Effective Geometry} \to \text{Observed Gravitation}$$

The same mechanism is proposed to underlie:

$$\{\text{inertia, gravitational attraction, cosmological expansion, horizon effects, redshift structure}\}$$

through variations in global and local interaction asymmetry.

The cosmological boundary defines large-scale ordering structure, while local masses define localized asymmetry regions within the signal manifold. Expansion and attraction therefore become scale-dependent manifestations of a unified relational principle.

7.4 Ontological Summary

The complete theory thus abandons the notion of passive background space entirely. Only interaction structure and signal accessibility possess fundamental ontological status.

The resulting philosophical structure may be summarized as follows:

- No passive spacetime background
- Only relational interaction structure
- Motion from symmetry asymmetry
- Gravity and expansion from boundary effects
- Geometry as emergent bookkeeping

Within this framework, spacetime itself becomes the effective macroscopic projection of deeper conformal, relational, and signal-theoretic dynamics governed by evolving symmetry structure on the interaction manifold.

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8 Operational Null-Relational Ontology and Emergent Motion

The framework attains a more unified structure when interpreted operationally rather than purely geometrically. Null rays are no longer treated as objects propagating through a pre-existing spacetime background. Instead, they constitute the primary relational infrastructure from which observable structure, motion, and geometry emerge.

Within this ontology, neither motion nor geometry is fundamental. Spatial position itself ceases to be primitive. The primary physical object becomes the network of possible interaction links connecting physical systems.

The null structure therefore acts not as decorative geodesic trajectories drawn on a manifold, but as the active relational substrate defining observational accessibility itself.

The conceptual hierarchy becomes:

$$\text{Relational Connectivity} \rightarrow \text{Observable Structure} \rightarrow \text{Effective Geometry} \rightarrow \text{Motion}$$

In this formulation, motion is operationally identified with reconfiguration of the relational signal network rather than displacement through an independently existing spatial arena.

When an observer or craft changes state relative to the surrounding universe, the organization of the boundary-connected null structure is dynamically reprojected. Observable stellar relationships transform through Möbius-type conformal mappings acting on the relational sky.

The changing parallax structure is therefore not merely evidence of motion. It constitutes the operational content of motion itself.

8.1 Relational vs Standard Relativistic Ontology

The distinction between the conventional relativistic ontology and the present framework becomes fundamental:

- **Standard Relativity:** Objects move through spacetime
- **Relational Framework:** Changing signal relationships constitute motion

Geometry thus becomes secondary bookkeeping extracted from the evolving organization of interaction channels.

The null network may be interpreted geometrically as a causal fiber structure or interaction web rather than as infinitesimal geodesic lines embedded in passive background space. The global relational structure possesses operational meaning even prior to ordinary electromagnetic propagation.

A distinction is therefore introduced between observable photon propagation and deeper pre-geometric null connectivity responsible for enabling observable propagation itself.

The null structure is consequently interpreted not as ordinary superluminal signaling within spacetime, but as a deeper relational constraint structure from which observable light-cone behavior emerges phenomenologically.

Schematically:

$$\text{Pre-geometric relational network} \rightarrow \text{Emergent light-cone structure} \rightarrow \text{Observable propagation}$$

8.2 Riemann Sphere and Möbius Reprojection

The Riemann sphere acquires a direct physical interpretation within this framework. It represents the compactified relational sky organizing the totality of observable directions and encoding the asymmetry structure responsible for effective translation.

The Möbius action

$$z \mapsto \frac{cz + d}{az + b}$$

with

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{C})$$

therefore describes dynamic reprojection of global relational structure around the observer.

Motion corresponds to evolving conformal organization of the observable boundary:

$$B_{\text{null}} \rightarrow B'_{\text{null}}$$

where B_{null} denotes the relational boundary structure.

Translation is thus reinterpreted as changing intersection structure between the observer and the boundary-connected null network.

The framework consequently approaches a relational-causal ontology in which conformal and twistor-like structures become more fundamental than metric geometry itself.

8.3 Oscillatory Systems and Relational Modulation

Within this same conceptual structure, oscillatory extended systems such as longitudinally vibrating piezoelectric crystals acquire a nontrivial relational interpretation.

The essential mechanism is not described as a crystal "pushing against spacetime." Instead, rapidly evolving internal oscillatory structure dynamically reshapes the local relational-null configuration connecting the object to the surrounding universe.

The relevant dynamical variables include:

$$\{\chi(t), \phi_{ab}(u^\mu), N_{\text{null}}, B\}$$

where:

$$\chi(t)$$

denotes internal oscillatory state,

$$\phi_{ab}(u^\mu)$$

represents generalized Noether-sector mixing connections,

$$N_{\text{null}}$$

denotes the local null relational network,
and

$$B$$

represents global boundary structure.

A vibrating extended object periodically modulates:

- local symmetry couplings
- boundary asymmetry
- Noether mixing structure
- null relational connectivity

through its evolving internal state.

The object surface becomes especially significant because surfaces naturally define relational asymmetries and interaction boundaries.

The oscillation therefore induces a time-dependent reprojection of the global null structure surrounding the body. The surrounding interaction network reorganizes dynamically across oscillation phases, producing a continuously evolving relational geometry.

The mechanism may be summarized schematically as:

$$\chi(t) \to \delta N_{\text{null}} \to \delta B \to \Delta x^\mu$$

Observable translation consequently emerges from time-dependent asymmetry in the relational signal structure intersecting the oscillating object.

The effect is therefore not attributed merely to mass fluctuations, but to dynamical modulation of global relational connectivity.

Within the conformal interpretation, the vibrating body continuously alters the mapping between:

- local observer sky
- distant boundary structure

- internal oscillatory phase

through evolving Möbius reprojection dynamics.

The global boundary-linked null structure reorganizes around the oscillating system, producing directional asymmetry in the effective relational geometry.

The resulting framework interprets motion as emerging from changing intersection structure with the cosmological null network itself.

This perspective naturally aligns with broader relational approaches in theoretical physics in which spacetime geometry is regarded as emergent from deeper interaction structure rather than fundamental. The emphasis here, however, remains specifically on interaction-first, signal-first, and boundary-first ontology.

The complete relational hierarchy may therefore be expressed as:

Boundary-connected null structure → interaction asymmetry → Möbius reprojection → effective translation

Motion through space is thus replaced by dynamic reconfiguration of boundary-connected relational structure. Observable spacetime geometry becomes the emergent macroscopic bookkeeping description of deeper conformal interaction dynamics unfolding across the null relational manifold.

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9 Unified Null-Relational Action and Oscillatory Matter Coupling

The framework already contains the essential structural ingredients necessary for constructing an effective null-relational dynamical model describing oscillatory matter coupled to global relational geometry. The remaining task is the unification of these ingredients into a single coupling architecture embedded within a generalized action formalism.

The theory naturally separates into several interacting sectors:

$$\left\{\begin{array}{l}\text{internal oscillatory sector,}\\ \text{boundary-asymmetry sector,}\\ \text{symmetry-mixing sector,}\\ \text{null-relational sector,}\\ \text{emergent translation sector}\end{array}\right\}$$

The resulting dynamical structure may therefore be organized through an effective action in which local oscillatory matter interacts continuously with a global conformal-null relational manifold.

9.1 Internal Oscillatory Sector

The internal oscillatory component is represented through a longitudinal displacement mode describing the vibrating crystal structure:

$$u(t,x) = u_0 \cos(\omega t - kx)$$

where:

- $u(t,x)$ denotes the longitudinal displacement field,
- u_0 is the oscillation amplitude,
- ω is the drive frequency,
- k is the crystal wave number.

This constitutes the ordinary elastic vibrational sector of the system. However, within the present framework the oscillation is not interpreted merely as mechanical deformation. Instead, it acts as a dynamical modulator of the surrounding relational-null structure.

9.2 Null-Relational Field Structure

To represent the deeper interaction network, a null-relational field is introduced:

$$\Psi(x^\mu, \Omega)$$

where:

- x^μ denotes spacetime position,
- Ω represents a direction on the celestial sphere or compactified Riemann sphere.

The dependence upon Ω is essential because the relational structure is fundamentally nonlocal and direction-dependent. The null network continuously encodes connectivity between the local system and the global observable boundary.

The theory therefore replaces purely local geometry with a directionally organized conformal interaction manifold.

9.3 Boundary Coupling Sector

The boundary sector is represented through a generalized asymmetry functional. Instead of a static cosmological contribution $V_\infty(x)$, the boundary potential becomes dynamically coupled to both the relational field and the internal oscillatory state:

$$V_{\text{boundary}} = V_\infty[\Psi, \partial\Psi, u]$$

The boundary structure therefore depends explicitly upon:

$$\Psi, \partial\Psi, u(t, x)$$

meaning that internal oscillation perturbs the organization of the surrounding relational-null bundle intersecting the object.

The oscillating system continuously reshapes the conformal interaction geometry surrounding its surface.

9.4 Surface Interaction Term and Relational Feedback

The surface itself becomes the primary operational location where relational asymmetry emerges. A natural interaction term therefore appears as a boundary integral over the oscillating interface:

$$S_{\text{surf}} = \int_{\partial\Sigma} \lambda u(t, x) \Psi(x^\mu, \Omega)\, dA$$

where:

- $\partial\Sigma$ denotes the system boundary surface,
- λ is the coupling strength,
- dA is the surface measure.

Within this interaction term, the oscillating surface dynamically modulates the surrounding null-relational structure. The relational field then feeds back asymmetrically upon the system through the evolving boundary geometry.

Translation emerges from this time-dependent asymmetry.

The framework consequently interprets oscillatory matter not as generating thrust directly, but as continuously reprojectioning the surrounding relational geometry.

9.5 Möbius Reprojection of the Relational Sphere

The conformal structure of the observable boundary evolves through Möbius transformations acting on the relational sphere:

$$\Omega \mapsto \Omega' = \frac{a\Omega + b}{c\Omega + d}$$

subject to:

$$ad - bc = 1$$

The oscillatory dynamics therefore induce continuously evolving conformal reprojections of the global relational sky.

The mechanism is not interpreted as pushing against spacetime itself, but as dynamically reorganizing the mapping between:

- local surface geometry,
- global null structure,
- boundary-connected relational asymmetry

Observable translation becomes an emergent collective coordinate:

$$X(t)$$

whose evolution is determined by directional asymmetry in the relational-null field.

A schematic emergent translation equation takes the form:

$$M\ddot{X} = \epsilon \int d\Omega\, A(\Omega, t)\Psi(\Omega, t)$$

where:

- $A(\Omega, t)$ measures the directional asymmetry induced by the oscillatory dynamics,
- $\Psi(\Omega, t)$ is the null-relational density on the conformal sphere,
- ϵ is a weak coupling parameter.

Within this ontology, translation emerges through asymmetrical restructuring of the boundary-connected null network.

The physical interpretation therefore proceeds through the sequence:

$$u(t, x) \to \delta\Psi \to \delta V_{\text{boundary}} \to \delta B \to \Delta X(t)$$

Internal oscillation modifies surface relational geometry, surface modulation perturbs null-boundary connectivity, the perturbation becomes directionally asymmetric, and the asymmetry generates effective translation.

9.6 Generalized Conservation Structure

The resulting framework differs fundamentally from naive reaction-force narratives. Motion emerges from time-dependent asymmetry in the global relational signal structure intersecting the oscillating body.

The theory consequently requires a generalized conservation structure incorporating matter, relational fields, boundary dynamics, and symmetry-mixing sectors simultaneously.

A generalized conservation equation may therefore be written as:

$$\nabla_\mu T^{\mu\nu}_{\text{total}} = 0$$

with total stress structure:

$$T^{\mu\nu}_{\text{total}} = T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\Psi} + T^{\mu\nu}_{\text{boundary}} + T^{\mu\nu}_{\text{mix}}$$

where:

- $T^{\mu\nu}_{\text{matter}}$ represents ordinary matter stress-energy,
- $T^{\mu\nu}_{\Psi}$ describes the null-relational field contribution,
- $T^{\mu\nu}_{\text{boundary}}$ encodes cosmological and surface boundary structure,
- $T^{\mu\nu}_{\text{mix}}$ represents the symmetry-mixing sector coupling internal oscillatory dynamics to relational geometry.

Within this generalized conservation framework, observable translation corresponds to redistribution of momentum across the combined relational system rather than isolated local momentum generation.

9.7 Unified Conformal-Relational Dynamics

The complete structure therefore unifies:

$$\left\{\begin{array}{l}\text{oscillation,}\\ \text{surface asymmetry,}\\ \text{null relational structure,}\\ \text{Möbius reprojection,}\\ \text{boundary dynamics,}\\ \text{emergent translation}\end{array}\right\}$$

within a single conformal-relational dynamical framework.

Motion is thus interpreted not as propagation through passive space, but as the emergent manifestation of continuously evolving intersection structure between oscillatory matter and the global null-relational manifold.

10 Relational-Connectivity Ontology and Emergent Geometry

The geometric intuition underlying the framework evolves naturally toward a relational-connectivity interpretation of spacetime structure. The theory does not treat space as a physical substance or medium. Instead, it proposes that observable spacetime geometry emerges from the organization of a deeper interaction network whose connectivity structure can itself be dynamically deformed.

The distinction is fundamental.

Rather than adopting the conventional geometric ontology

$$\text{spacetime} = \text{smooth metric manifold},$$

the framework proposes:

$$\text{observable spacetime} = \text{emergent projection of an underlying null relational web.}$$

The underlying structure is therefore not interpreted as material substrate, ether, or space-filling medium. It is instead conceived as a global network of causal and interaction links compactified through conformal boundary structure represented geometrically on the Riemann sphere.

Observable geometry arises from the organization of this network.

The "null spaghetti" intuition corresponds mathematically to a continuously evolving relational connectivity manifold whose structure defines observable motion operationally.

The primitive object of the theory becomes:

$$N_{\text{rel}} = \text{global null relational network}$$

rather than metric geometry itself.

10.1 Oscillatory Modulation of Relational Structure

Within this ontology, a vibrating piezoelectric crystal does not directly deform spacetime. Instead, internal oscillatory dynamics modulate the local symmetry-coupling structure governing how the object intersects the global relational network.

The symmetry-coupling field is represented as:

$$\phi(u_\mu, t)$$

where u_μ denotes the local dynamical state variables.

This field modifies the local Noether-sector mixing structure:

$$N_{ab}(x, t)$$

which encodes coupling between internal oscillatory dynamics and the surrounding relational geometry.

The oscillatory evolution may therefore be expressed schematically as:

$$N_{ab}(t) = N_{ab}(0) + \delta N_{ab} \cos(\omega t)$$

with perturbation amplitude:

$$\delta N_{ab} \propto \phi(u_\mu, t) M_{ab}$$

where M_{ab} represents the underlying symmetry-mixing tensor.

The crystal oscillation continuously modulates the manner in which the local object couples to the global null-relational structure.

10.2 Riemann Sphere and Möbius Reprojection

The Riemann sphere enters operationally through stereographic projection coordinates describing the observable relational sky:

$$z = e^{i\theta} \tan\left(\frac{\chi}{2}\right)$$

where θ and χ parameterize angular structure on the compactified celestial sphere.

The oscillating system then induces a time-dependent Möbius transformation acting upon the relational boundary structure:

$$z \mapsto \frac{a(t)z + b(t)}{c(t)z + d(t)}$$

subject to:

$$a(t)d(t) - b(t)c(t) = 1$$

The transformation coefficients evolve dynamically with the oscillatory phase of the crystal. For example:

$$a(t) = 1 + \epsilon_a \cos(\omega t), \quad c(t) = \epsilon_c \sin(\omega t)$$

with analogous oscillatory structure for $b(t)$ and $d(t)$.

The crystal therefore periodically reprojectiones the distant relational boundary through time-dependent conformal transformations.

Within the relational interpretation, the oscillatory body continuously alters how the bundle of boundary-connected null relations intersects its surface geometry.

10.3 Emergent Translation

Observable translation then emerges through the effective projected coordinate:

$$x_{\text{eff}}(t) = \text{Re}\left(\frac{a(t)z + b(t)}{c(t)z + d(t)}\right)$$

The emergent displacement corresponds not to primitive motion through space, but to evolving reprojection of the global relational sky.

If the oscillatory structure remains perfectly symmetric, the time-averaged drift vanishes:

$$\langle \dot{x}_{\text{eff}} \rangle = 0$$

However, when the boundary coupling becomes asymmetric, phase-shifted, or nonlinear, higher-order terms survive averaging:

$$\langle \dot{x}_{\text{eff}} \rangle \neq 0$$

The framework therefore introduces a ratcheting mechanism in which asymmetrical conformal reprojection generates effective translation.

The resulting structure resembles a conformal pump or boundary-phase rectifier rather than a conventional force-generating engine.

10.4 Pre-Geometric Null Structure

The deeper null-relational network itself is not interpreted as dynamically generated solely through local photon propagation. Instead, observable light-cone structure emerges from a more primitive pre-geometric connectivity manifold.

The conceptual hierarchy becomes:

$$\text{pre-geometric null network} \rightarrow \text{observable light structure} \rightarrow \text{effective geometry} \rightarrow \text{motion}$$

This ordering reverses the conventional hierarchy of standard metric ontology.

Within this framework, the oscillating system does not transmit faster-than-light signals through spacetime. Rather, it modulates its coupling to an already existing global relational structure from which observable light-cone behavior itself emerges phenomenologically.

The global relational manifold therefore acts as the deeper organizing substrate underlying observable causal structure.

10.5 Generalized Conservation Structure

The central unresolved issue remains the construction of a rigorous conservation architecture capable of describing globally conserved momentum transfer across the combined relational system.

A generalized conserved current would therefore be required:

$$\nabla_\mu J^\mu_{\text{total}} = 0$$

where portions of the momentum current reside within the boundary and null-relational sectors themselves.

The total current structure would schematically decompose into:

$$J^\mu_{\text{total}} = J^\mu_{\text{matter}} + J^\mu_{\text{null}} + J^\mu_{\text{boundary}} + J^\mu_{\text{mix}}$$

with:

- J^μ_{matter} representing ordinary matter transport,
- J^μ_{null} representing momentum flow within the null-relational network,
- J^μ_{boundary} encoding boundary-sector contributions,
- J^μ_{mix} describing symmetry-mixing transfer terms.

Within this generalized framework, observable translation corresponds to redistribution of momentum across the combined relational manifold rather than isolated local reaction-force production.

10.6 Relational Motion Hierarchy

The resulting geometric intuition may therefore be summarized as follows:

$$\begin{aligned}\text{oscillatory matter} &\rightarrow \text{modulation of symmetry coupling} \\ &\rightarrow \text{Möbius reprojection of relational boundary structure} \\ &\rightarrow \text{asymmetric null connectivity} \\ &\rightarrow \text{emergent translation}\end{aligned}$$

The theory consequently interprets motion not as displacement through passive space, but as continuous reconfiguration of the boundary-connected null relational web defining observable geometry itself.

11 Cosmological Anisotropy and Effective Relational Bias

Within a cosmological and effective-field-theory context, small deviations from perfect isotropy are already an accepted feature of standard cosmological modeling. The cosmological principle assumes large-scale homogeneity and isotropy as an approximation rather than an exact symmetry of the universe.

Empirically, deviations from perfect isotropy are present in multiple observed structures:

- Cosmic Microwave Background (CMB) anisotropies
- Large-scale structure clustering patterns
- Local bulk flow velocities relative to the CMB rest frame
- Ongoing discussions around Hubble parameter tension and possible directional systematics

These features indicate that the universe contains measurable departures from perfect isotropy, even if they remain small in magnitude at large scales.

From a theoretical standpoint, a global anisotropy would define a preferred cosmological frame, typically identified with the CMB rest frame. In such a scenario, physical processes could exhibit weak directional dependence, including:

- anisotropic redshift corrections
- directional dependence in expansion rates
- small deviations from geodesic isotropy in propagation effects

However, within general relativity and Lorentz-consistent frameworks, such global anisotropies do not automatically translate into local self-acceleration or propulsion effects. Local dynamics remain dominated by covariant conservation laws and tend to average out large-scale directional structure unless explicitly coupled to additional dynamical fields.

In extended theoretical models, anisotropy can indeed enter local dynamics through background field structures. In such cases, effective forces may arise from gradients or directional structure in additional fields:

$$\text{effective force} \sim \nabla_\mu(\text{background field})$$

This type of mechanism appears in several established theoretical frameworks, including:

- Einstein–Æther type models
- vector dark energy models
- anisotropic (Bianchi-type) cosmologies
- Lorentz-violating effective field theories

In these settings, a small anisotropy can generate small directional corrections to motion, typically interpreted as deviations from ideal geodesic evolution rather than as new propulsion channels.

11.1 Relational-Null Interpretation of Anisotropy

Within the relational-null framework under discussion, such anisotropy corresponds to a deformation of the otherwise symmetric structure of the null relational network defined over the Riemann sphere. Instead of a perfectly isotropic relational distribution, the boundary field acquires directional structure:

$$\Psi(\Omega) = \Psi_0 + \delta\Psi(\Omega)$$

with the anisotropic component satisfying:

$$\delta\Psi(\Omega) \neq \text{isotropic}$$

This corresponds to a weak directional bias in the organization of the relational interaction web, meaning that the conformal structure of the "observable sky" is no longer statistically uniform.

In this interpretation, the Möbius-structured relational mappings acting on the celestial sphere are no longer purely isotropic in expectation, but carry a small preferred-direction deformation induced by the background anisotropy of the null network.

However, even within this generalized framework, the physical consequence of such anisotropy is constrained. A directional bias only becomes dynamically relevant if the system couples to it in a non-trivial and non-averaging way over time. Otherwise, cyclic evolution leads to cancellation of directional contributions, producing observable anisotropy without net transport effects.

Therefore, anisotropy alone modifies observational structure and local response functions, but does not generically produce self-contained momentum generation.

The central distinction is thus:

- anisotropy can bias interaction structure
- bias does not automatically imply net propulsion unless rectification mechanisms exist

Within the relational-null picture, this corresponds to the requirement that the system must lock into or amplify directional structure in the null relational field in a way that prevents temporal or spatial averaging.

In summary, cosmological anisotropy is a physically meaningful and well-established concept in both observational cosmology and effective field theory extensions of gravity. It can introduce small directional corrections to physical evolution. However, converting such structure into sustained directional motion requires additional dynamical mechanisms that prevent symmetry restoration under averaging, ensuring that relational asymmetries translate into persistent macroscopic effects rather than transient observational biases.

12 Effective Unified Dynamical Formulation and Rectified Drift Mechanism

A consistent effective formulation of the proposed framework can be constructed once the underlying ingredients are regularized into a unified dynamical system. The central structural requirement is that a nonzero time-averaged drift can only arise when three elements coexist simultaneously: periodic driving, symmetry breaking, and nonlinear coupling. Without this combination, all oscillatory contributions average to zero.

12.1 Minimal Dynamical Ingredients

The system can be decomposed into four interacting sectors:

- A driven internal oscillator (piezoelectric mode):

$$q(t) \sim A\cos(\omega t)$$

- A directional null-relational field (Riemann sphere / "sky" structure):

$$n(\theta, \phi) \in S^2, \quad z \in \mathbb{C}$$

- A weak cosmic anisotropy background:

$$A_\mu \quad \text{or spatial } A_i$$

- A Möbius / conformal symmetry structure acting on the null field, generated by:

$$L_{-1} = \partial_z, \quad L_0 = z\partial_z, \quad L_1 = z^2\partial_z$$

Within this formulation, the "null spaghetti" corresponds to a time-dependent deformation of the directional field under the action of the conformal group $SL(2, \mathbb{C})$.

12.2 Condition for Nonzero Time-Averaged Drift

A central constraint of the system is:

$$\langle \dot{X} \rangle = 0$$

for purely linear, symmetric oscillations.
A nonzero drift requires the simultaneous presence of:

- periodic driving
- anisotropic bias
- nonlinear coupling between internal and external sectors

This is the standard structure of a rectification mechanism.

12.3 Effective Lagrangian Structure

The coupled system may be expressed as:

$$L = L_{\text{crystal}}(q, \dot{q}) + L_{\text{null}}(z, \dot{z}) + L_{\text{Mbius}}(z) + L_{\text{coupling}}$$

The physically relevant dynamics arise primarily from the coupling sector.

12.4 Coupling Architecture

A minimal consistent interaction term encoding the proposed mechanism is:

$$L_{\text{coupling}} = \epsilon q(t) A_i J^i(z)$$

where:

- $q(t)$ is the oscillatory piezo driving term
- A_i is the anisotropy vector defining directional bias
- $J^i(z)$ are Möbius generators acting on the null field

Explicitly:

$$L_{\text{coupling}} = \epsilon q(t) \left(A_{-1} L_{-1} + A_0 L_0 + A_1 L_1 \right) \Phi(z)$$

where $\Phi(z)$ represents the null-relational field defined on the Riemann sphere.

12.5 Origin of Nonzero Drift

The emergent velocity is defined schematically as:

$$\dot{X} \sim \langle \partial_z \Phi \rangle$$

A perturbative expansion shows:

- First order contribution:
$$\langle q(t) \rangle = 0 \Rightarrow \text{no drift}$$
- Second order contribution (rectification regime):
$$\langle \dot{X} \rangle \sim \epsilon^2 \sum_{i,j} A_i A_j \left\langle [L_i, L_j] \Phi \right\rangle_{\text{time}}$$

The key structural feature is:

$$[L_i, L_j] \neq 0$$

so Möbius generators produce intrinsic non-commutativity, enabling nonlinear mixing between oscillation and anisotropy.

Time symmetry breaking + anisotropy + noncommutativity jointly generate:

$$\langle \dot{X} \rangle \neq 0$$

12.6 Reduced Drift Expression

The effective averaged drift can be summarized in compact form as:

$$\langle v \rangle \sim \omega \epsilon^2 (\nabla A) \cdot C[\Phi]$$

where:

- ϵ = oscillation amplitude
- ω = driving frequency (suppression scale)
- ∇A = anisotropy gradient of the background null sky
- $C[\Phi]$ = curvature / nonlinear structure of the Möbius-null field

This expresses the system as a rectified conformal response to anisotropic boundary structure.

12.7 Physical Interpretation

In relational terms, the mechanism can be summarized as follows:

- internal oscillation does not directly generate force
- instead it periodically modulates coupling to the null-relational network
- Möbius structure encodes directional organization of the boundary sky
- anisotropy introduces a preferred bias in that structure
- oscillation repeatedly samples this biased structure
- nonlinear coupling rectifies oscillations into net drift

The result is a:

driven anisotropic nonlinear conformal ratchet on a null boundary field

12.8 Conservation Constraint

Any consistent formulation must satisfy a global conservation structure:

$$\nabla_\mu J^\mu_{\text{total}} = 0$$

This implies that any emergent drift must be compensated by redistribution of momentum within one or more of:

- field degrees of freedom
- boundary sectors
- anisotropic background structure
- radiation or environmental modes

Without such a bookkeeping structure, the model becomes dynamically incomplete.

12.9 Summary Structure

The full mechanism reduces to the interaction:

$$\text{oscillation} \times \text{anisotropy} \times \text{nonlinearity} \rightarrow \text{rectified transport}$$

All remaining linear or symmetric contributions cancel under time averaging.

The resulting framework is therefore mathematically equivalent to a driven Möbius-symmetric field system in which directional drift emerges only through anisotropic nonlinear rectification of oscillatory coupling to a conformal null-relational background.

13 Refined Toy Model: Driven Möbius–Null Relational System

This is a further refinement that reorganizes the framework into a more explicitly structured toy model. The construction separates the dynamics into four interacting layers:

- Möbius / Riemann sphere geometry
- Internal oscillatory (piezoelectric) sector
- Coupling to a global anisotropic null-relational field
- Noncommuting symmetry generators producing effective drift

With this separation, the system becomes mathematically expressible as a relational field theory with a driven deformation of non-Abelian symmetry structure.

13.1 Core Dynamical Variables

The model is defined in terms of:

$$z(t) \in \mathbb{C}$$

representing the Riemann sphere coordinate (null-sky state),

$$q(t)$$

representing the internal crystal vibration mode,

$$u_\mu(t)$$

the effective four-velocity,

$$\epsilon$$

a small anisotropy parameter,
and

$$A(z, \bar{z})$$

a directional anisotropic "null-spaghetti" density field on the sphere.
The configuration space is therefore:

$$z \in \mathbb{C} \simeq S^2$$

with motion interpreted as evolution on the compactified celestial sphere.

13.2 Möbius Structure as Kinematic Backbone

The infinitesimal generators of conformal transformations are:

$$L_{-1} = \partial_z, \quad L_0 = z\partial_z, \quad L_1 = z^2\partial_z$$

A general evolution operator is written as:

$$T = \alpha(t)L_{-1} + \beta(t)L_0 + \gamma(t)L_1$$

This operator encodes the time-dependent reconfiguration of the null-relational sky.

13.3 Internal Oscillator (Piezo Sector)

The crystal is modeled as a harmonic oscillator:

$$q(t) = q_0 \cos(\omega t)$$

or equivalently:

$$\ddot{q} + \omega^2 q = 0$$

The key structural assumption is that the oscillator does not directly generate motion. Instead, it modulates the symmetry structure:

$$\alpha = \alpha(q), \quad \beta = \beta(q), \quad \gamma = \gamma(q)$$

Thus the vibration acts as a driver of symmetry mixing coefficients rather than positional displacement.

13.4 Anisotropic "Null-Spaghetti" Field

A weak directional bias is introduced on the sphere:

$$A(z, \bar{z}) = \epsilon Y_{LM}(z, \bar{z})$$

This represents anisotropy in the relational connectivity structure rather than curvature of spacetime itself.

The coupling to the system is expressed as:

$$S_{\text{null}} = \int dt\, A(z(t), \bar{z}(t))$$

13.5 Noether Mixing Structure

A combined Lagrangian incorporating symmetry mixing takes the schematic form:

$$L = \frac{1}{2}\dot{q}^2 + \kappa_1(q)\dot{z}\dot{\bar{z}} + \kappa_2(q)(z\dot{\bar{z}} - \bar{z}\dot{z}) - V(q) - A(z, \bar{z})$$

To explicitly encode symmetry-induced translation emergence, a Noether-matrix coupling is introduced:

$$L_{\text{mix}} = \dot{x}^\mu N_{\mu\nu}(q, z)\dot{x}^\nu$$

with structure:

$$N_{\mu\nu} = \begin{pmatrix} 1 & \phi(q)\epsilon B(z) & \phi(q)\epsilon \\ \phi(q)\epsilon B(z) & 1 & \phi(q)\epsilon \\ \phi(q)\epsilon & \phi(q)\epsilon & 1 \end{pmatrix}$$

Interpretation:

- diagonal terms: inertial baseline structure
- off-diagonal terms: symmetry mixing responsible for effective motion
- anisotropic components: encoded by $\epsilon B(z)$

13.6 Möbius Noncommutativity and Deformation

The Möbius generators satisfy:

$$[L_0, L_{-1}] = -L_{-1}, \quad [L_0, L_1] = L_1, \quad [L_1, L_{-1}] = 2L_0$$

A vibration-dependent deformation is introduced:

$$\tilde{L}_i = f_i(q) L_i$$

leading to:

$$[\tilde{L}_i, \tilde{L}_j] = f_i f_j [L_i, L_j] + (\dot{f}_i f_j - f_i \dot{f}_j) L_k$$

The second term represents dynamically induced noncommutativity generated by the oscillatory sector.

This term is structurally central, as it produces effective symmetry drift even when the underlying algebra is fixed.

13.7 Emergent Drift Velocity

The effective evolution of the sphere coordinate is:

$$\dot{z} = \langle T \rangle$$

Explicitly:

$$\dot{z} = \alpha(q) L_{-1} z + \beta(q) L_0 z + \gamma(q) L_1 z$$

Time averaging over oscillations yields:

$$\langle \dot{z} \rangle \neq 0 \quad \text{if } \epsilon \neq 0 \text{ and } \langle q(t) \dot{q}(t) \rangle \neq 0$$

Thus the drift condition becomes:

$$\epsilon \cdot \text{phase-lag}(q) \cdot \text{noncommutativity}(L_i) \neq 0$$

This corresponds to rectified motion arising from symmetry deformation.

13.8 Physical Interpretation

Within this formulation:

- Möbius structure defines the global null-relational sky
- crystal oscillation deforms symmetry generators dynamically
- anisotropy defines preferred directional structure
- motion emerges as time-averaged failure of symmetry closure

Thus:

$$\text{motion} \neq \text{force}$$

Instead:

$$\text{motion} = \text{geometric phase drift in a noncommutative symmetry bundle}$$

13.9 Consistency Constraint

A necessary global condition is conservation of total momentum flux:

$$\langle \text{total momentum flux} \rangle = 0$$

This requires that any emergent drift be compensated by one or more of:

- anisotropic background recoil
- null-relational field momentum
- boundary sector contributions
- field radiation channels

Otherwise the system conflicts with standard conservation constraints and Lorentz invariance bounds.

13.10 Summary Structure

The full mechanism reduces to:

$$\text{vibration} \times \text{anisotropy} \times \text{noncommuting symmetry deformation} \rightarrow \text{rectified drift}$$

The key conceptual result is that motion emerges not from direct force generation, but from:

time-dependent deformation of non-Abelian symmetry generators
induced by oscillatory coupling to an anisotropic null-relational field

This provides a mathematically consistent toy-model translation of the proposed relational-Möbius framework into a driven noncommutative geometric system.

14 Refined Relational Möbius–Null Field-Theory Ansatz

This is a further refinement of the construction, reorganizing the framework into a coherent effective field-theory-style ansatz while preserving all previously introduced components.

The system is structured around five coupled ingredients:

- Möbius action on the Riemann sphere (null-sky geometry)
- Noncommutative generator algebra (symmetry-driven drift mechanism)
- Noether mixing matrix (internal symmetry entanglement sector)
- Boundary anisotropy field (cosmic asymmetry source)
- Longitudinal oscillator (piezo/crystal driving sector)

Together, these define a driven relational field theory in which motion is derived from symmetry deformation rather than assumed as primitive.

14.1 Fundamental Geometric Object: Null-Sky Field

The primary configuration space is the Riemann sphere:

$$z \in \mathbb{C} \cup \{\infty\}$$

interpreted as a null-directional observational sky.

On this space, define a relational density field:

$$\Phi(z, t)$$

representing the distribution of interaction links (the "null spaghetti bundle").

14.2 Möbius Generator Algebra

The symmetry structure is given by the $SL(2,\mathbb{C})$ Möbius generators:

$$L_{-1} = \partial_z, \quad L_0 = z\partial_z, \quad L_{+1} = z^2\partial_z$$

with commutation relations:

$$[L_m, L_n] = (m-n)L_{m+n}$$

This noncommutative algebra functions as the kinematic backbone of the null-sky evolution.

14.3 Noether Mixing Matrix (Internal Oscillator Coupling)

Introduce an internal piezoelectric degree of freedom:

$$q(t)$$

representing longitudinal crystal vibration.
Define a Noether-current vector:

$$J = \begin{pmatrix} J_{\text{rot}} \\ J_{\text{boost}} \\ J_{\text{null}} \end{pmatrix}$$

A time-dependent mixing structure is introduced via:

$$N(q,\dot{q}) = \begin{pmatrix} 0 & \alpha q & \beta\dot{q} \\ -\alpha q & 0 & \gamma q \\ -\beta\dot{q} & -\gamma q & 0 \end{pmatrix}$$

This encodes the principle:

$$\text{no symmetry mixing} \Rightarrow \text{no motion}$$

14.4 Combined Möbius–Noether Generator

The effective evolution operator acting on the null sky is defined as:

$$G(t) = L_{-1} + \lambda(t)L_0 + \epsilon\, N(q,\dot{q}) \cdot L$$

where:

$$L = (L_{-1}, L_0, L_{+1})$$

In this formulation, motion is not generated by translations as primitive operations, but as a deformation of conformal symmetry structure induced by internal oscillations.

14.5 Boundary Anisotropy (Cosmic Symmetry Breaking)

A weak directional bias is introduced on the sphere:

$$A(z,\bar{z}) \neq \text{const}$$

expanded as:

$$A(z,\bar{z}) = A_0 + \delta A\, Y_{10}(z,\bar{z})$$

This encodes a minimal anisotropic seed in the global null-relational structure.

14.6 Full Effective Lagrangian

The coupled system is defined by:

$$L = \frac{1}{2}m\dot{q}^2 - V(q) + \int_{\mathbb{C}} \Phi^*(z,t)\left[i\partial_t - G(t) - A(z,\bar{z})\right]\Phi(z,t)\,d\mu(z)$$

This structure encodes:

- oscillatory internal dynamics
- Möbius-driven null-sky evolution
- boundary-induced symmetry bias
- coupling between internal and relational sectors

14.7 Emergent Drift Coordinate

Define the center of relational distribution:

$$X(t) = \int z\,|\Phi(z,t)|^2\,d\mu(z)$$

Its time evolution is:

$$\dot{X}(t) = \langle [G(t), z] \rangle$$

Expanding contributions:

- pure Möbius symmetry contribution cancels
- internal oscillator coupling contributes:

$$\epsilon\langle [N(q,\dot{q}), z] \rangle$$

- boundary anisotropy contributes via $A(z,\bar{z})$

14.8 Time-Averaged Drift Mechanism

Assume periodic driving:

$$q(t) = q_0\cos(\omega t)$$

Then:

- linear oscillatory contributions average to zero
- nonlinear coupling survives under anisotropic weighting

Compute:

$$q\dot{q} = -\omega q_0^2\cos(\omega t)\sin(\omega t)$$

whose naive average vanishes unless weighted by anisotropy:

$$A(z,\bar{z}) \neq \text{uniform}$$

Resulting in a nonzero drift:

$$\langle \dot{X} \rangle \sim \epsilon\omega q_0^2 \nabla A$$

14.9 Physical Interpretation

In this formulation:

- crystal vibration induces time-dependent symmetry mixing
- Möbius algebra converts symmetry evolution into global sky reweighting
- noncommutativity ensures order-dependent geometric effects
- cosmic anisotropy acts as rectifying bias
- net motion emerges from nonlinear reconfiguration of the null-relational structure

Thus:

$$\text{vibration} \to \text{symmetry deformation} \to \text{biased null-sky reconfiguration} \to \text{drift}$$

14.10 Structural Significance

Independently of empirical status, the framework converges toward a known class of modern mathematical structures:

- conformal field theory on S^2 (Riemann sphere)
- noncommutative symmetry algebras
- boundary-driven effective dynamics
- relational observables replacing absolute spacetime primitives

This places the construction in proximity to approaches such as:

- twistor geometry
- holographic field-theoretic reductions
- relational and boundary-first formulations of quantum field theory

Null-Relational Boundary Scattering Formulation

This continuation further grounds the framework by identifying a direct correspondence between the proposed null-relational dynamics and standard boundary scattering physics. In this formulation, reflection and refraction are not treated as secondary optical effects, but as the fundamental mathematical manifestation of interaction asymmetry at a boundary.

The result is a clearer mapping between the Möbius/null-sky structure and conventional wave–boundary dynamics.

1. Reflection and refraction as boundary operators

At a physical interface, an incoming field decomposes as:

$$\Psi_{\text{in}} \to r\,\Psi_{\text{reflected}} + t\,\Psi_{\text{transmitted}}$$

where:

$$|r|^2 + |t|^2 = 1 \quad \text{(lossless limit)}$$

These coefficients define a scattering map:

$$S : \Psi_{\text{in}} \mapsto \Psi_{\text{out}}$$

Thus, a surface acts as a boundary operator redistributing interaction flow.

2. Boundary as null-sky projector

In the relational framework, the crystal surface is interpreted as a boundary operator acting on the null-sky field:

$$\Phi_{\text{out}}(z,t) = B(z,\bar{z};t)\,\Phi_{\text{in}}(z,t)$$

with:

- null rays = interaction structure ("space spaghetti")
- motion = reconfiguration of this structure
- surface = locus of injected asymmetry

3. Refractive index as Möbius deformation parameter

In standard optics:

$$n = \frac{c}{v_{\text{phase}}}$$

Phase mismatch:

$$\Delta\phi(z) \sim k\,\Delta n\,d$$

Möbius reinterpretation:

$$\Delta n(z,\bar{z}) \leftrightarrow \lambda(z,\bar{z})$$

where λ is a conformal deformation field.

4. Crystal surface as anisotropy injector

Boundary dynamics combine:

- reflection asymmetry: $r(z) \neq r(-z)$
- refractive gradient: $\nabla n \neq 0$
- oscillatory driving: $q(t)$

Effective boundary operator:

$$B(z,t) = R(n(z)) + T(n(z)) + \epsilon q(t)\nabla n(z)$$

5. Momentum flux across the boundary

$$P(t) = \int_{\partial V} \left(|r(z,t)|^2 - |t(z,t)|^2\right)\hat{n}(z)\,dA$$

with:

$$r = r(n(z)), \quad t = t(n(z)), \quad n(z,t) = n_0(z) + \delta n\,q(t)$$

6. Time-averaged rectification mechanism

First order:

$$\langle q(t)\rangle = 0$$

Second order:

$$\langle P\rangle \sim \langle q^2(t)\,\nabla n(z)\rangle$$

Hence:

$$\langle P\rangle \propto \nabla n(z)\,\langle q^2(t)\rangle$$

7. Translation into null-spaghetti language

Standard physics	Null-relational interpretation
refractive index gradient	density gradient of null-thread connectivity
reflection coefficient	rejection of interaction pathways
transmission coefficient	acceptance of null-thread flow
phase shift	Möbius deformation of sky map
boundary surface	generator of relational asymmetry

8. Role of vibration

Without modulation:

$$B(t) = B(-t) \quad \Rightarrow \quad \text{no net drift}$$

With oscillation:

$$B(t) \neq B(-t)$$

Thus:

- forward $\neq$ backward scattering
- nonlinear averaging yields residual drift

9. Structural significance

Net motion occurs only if:

$$\text{motion} \Leftrightarrow \text{time-dependent scattering} + \text{spatial anisotropy}$$

Analogies include:

- optical rectification
- acoustic streaming
- asymmetric radiation pressure
- dynamical boundary Casimir effects

10. Final structural interpretation

$$\text{motion} = \text{time-asymmetric scattering on a conformally distorted interaction boundary}$$

with identifications:

- reflection/refraction $\rightarrow$ relational asymmetry parameters
- vibration $\rightarrow$ temporal modulation of scattering
- anisotropy $\rightarrow$ rectification condition
- boundary surface $\rightarrow$ generator of interaction reconfiguration

Thus motion arises not from mechanical force, but from nonlinear temporal rectification of boundary-mediated interaction flow in a conformally structured null-relational field.

Null-Relational Propulsion via Celestial Sphere Reprojection

This fragment is a further refinement in which the "propulsion" mechanism is expressed purely as a dynamical reorganization of null-relational projections between the ship and the celestial sphere, rather than motion through a pre-existing geometric background.

The central structural shift is that observable sky distortion is treated as the operational manifestation of propulsion itself.

1. Hull as moving null-intersection surface

In this formulation, distant stars define boundary directions on a compactified celestial sphere:

$$z \in \mathbb{C} \simeq S^2$$

Null rays are interpreted as relational connectivity threads, and the hull of the ship defines a time-dependent intersection surface:

$$\Sigma(t)$$

The observed sky distribution is described by an intensity field:

$$I(z, \bar{z}, t)$$

where:

- z: angular coordinate on the Riemann sphere
- I: observed directional intensity of incoming stellar signals

The hull acts as an evolving projection interface for the global null structure.

2. Inertial case (no engine active)

For inertial motion, the mapping between sky coordinates remains trivial or Lorentz-induced:

$$z' = z$$

or, in standard relativistic optics, a Möbius transformation:

$$z' = \frac{\beta z + 1}{z + \bar{\beta}}$$

This corresponds to standard aberration effects:

- forward angular compression
- Doppler blueshift in forward direction
- redshift in backward direction
- weak relativistic distortion of solid angles

3. Engine-active regime: dynamic Möbius deformation

With the internal crystal engine active, the null-relational structure becomes time-dependent:

$$z' = \frac{a(t)z + b(t)}{c(t)z + d(t)}, \quad \begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix} \in SL(2, \mathbb{C})$$

Consequences include:

- nonuniform stellar drift across the sky
- time-dependent angular compression/expansion
- oscillatory parallax modulation
- anisotropic intensity redistribution

These effects arise from deformation of the projection structure linking observer and boundary sky.

4. Null-spaghetti interpretation

Each stellar direction corresponds to bundles of null-relational fibers. The engine modulates their density:

$$N(z,t)$$

so that:

$$\delta z(t) \sim \epsilon\,\Omega(t)\,f(z)$$

where:

- $\Omega(t)$: oscillatory engine mode
- $f(z)$: anisotropic structure of the cosmic null network
- ϵ: coupling strength

5. Predicted observational structure

Forward sector:

- angular compression
- apparent stellar clustering
- increased intensity density

Lateral sectors:

- shear-like deformation
- curved apparent trajectories

Rear sector:

- angular expansion
- reduced apparent density

These distortions are time-dependent and oscillate at the engine frequency.

6. Fundamental conceptual reversal

Standard relativity:

$$\text{motion} \rightarrow \text{distortion of light propagation}$$

Present framework:

$$\text{distortion of null structure} \rightarrow \text{effective motion}$$

Sky reconfiguration constitutes the operational mechanism generating motion.

7. Möbius formulation of propulsion

The engine induces a time-dependent SL(2,$\mathbb{C}$) element:

$$M(t) = \begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix} \in SL(2,\mathbb{C})$$

acting on:

$$z \mapsto M(t) \cdot z$$

If:

$$\langle M(t) \rangle = I$$

then no net drift occurs.

If:

$$\langle M(t) \rangle \neq I$$

then cumulative directional drift emerges.

8. Boundary (hull) contribution

The hull acts as a conformal scatterer:

$$\Phi_{\text{out}} = B(t)\,\Phi_{\text{in}}$$

The observed sky becomes:

$$z_{\text{obs}} = M(t) \circ B(t)\,(z)$$

9. Structural distinction

Conventional view:

$$\text{force} \rightarrow \text{motion} \rightarrow \text{optical effect}$$

Present framework:

$$\text{null-structure reconfiguration} \equiv \text{effective motion}$$

11. Summary structure

$$\begin{aligned}\text{engine oscillation} &\rightarrow \text{dynamic Möbius deformation of null intersection structure}\\ &\rightarrow \text{time-dependent sky reprojection}\\ &\rightarrow \text{effective translational drift}\end{aligned}$$

In this formulation, propulsion arises not from pushing against space, but from continuous reconfiguration of the conformal null-relational mapping between the hull and the global celestial sphere.

15 Refined Relational Propulsion Framework

This is a further refinement of the framework, consolidating the previous components into a single coupled relational structure in which propulsion emerges from an anisotropic null-relational interaction rather than any local reaction-mass mechanism.

15.1 Global null-relational structure of the universe

The distant universe defines a directional field on the compactified sky:

$$A(\Omega), \quad \Omega \in S^2$$

where S^2 is identified with the Riemann sphere.

This object encodes large-scale anisotropy of the null-relational network, i.e. the distribution of interaction channels accessible through null connections.

15.2 Hull as interaction boundary with the null network

The hull defines a coupling surface Σ interacting with this global structure. It does not merely sit in spacetime; it defines how incoming null relations are processed:

$$R(\Omega, t)$$

where $R(\Omega, t)$ is a time-dependent hull response / reprojection operator acting on incoming directional structure.

The effective coupling is therefore boundary-mediated reweighting of global null directions rather than pointwise motion.

15.3 Effective anisotropic force from relational coupling

The emergent directional bias is written schematically as:

$$\langle F_i \rangle \propto \int_{S^2} A(\Omega)\, R(\Omega, t)\, d\Omega$$

where:

- $A(\Omega)$: cosmic anisotropy distribution
- $R(\Omega, t)$: hull-induced reorganization of null relations
- integration over the celestial sphere S^2

In the isotropic limit:

$$A(\Omega) = \text{const} \;\Rightarrow\; \langle F_i \rangle = 0$$

because directional contributions cancel.
With anisotropy:

$$A(\Omega) = A_0 + \delta A(\Omega)$$

a residual directional component appears.

15.4 Propulsion principle in relational terms

Propulsion is not generated by expulsion of reaction mass. Instead, it emerges from biased coupling to a globally anisotropic null-relational field.

Motion is therefore a boundary-coupling phenomenon rather than a local force exchange.

15.5 Möbius structure and noncommutative holonomy

Hull dynamics implement time-dependent Möbius transformations:

$$M(t) \in SL(2, \mathbb{C}), \quad z \mapsto M(t) \cdot z$$

For successive transformations:

$$M_1, M_2, M_3, \ldots$$

with noncommutativity:

$$[M_i, M_j] \neq 0$$

the cyclic composition satisfies:

$$M_{\text{cycle}} = M_1 M_2 M_3 \cdots \neq I$$

producing a geometric-phase (holonomy) effect even for periodic driving.

15.6 Time-averaged drift mechanism

A closed internal cycle would normally average to zero. However, in the presence of:

- anisotropy $A(\Omega)$
- noncommuting generators
- boundary coupling $R(\Omega, t)$

the averaged effect does not cancel:

$$\langle F_i \rangle \neq 0$$

This corresponds to rectified drift from cyclic non-Abelian evolution in an anisotropic relational background.

15.7 Physical interpretation in relational terms

The mechanism can be summarized as:

- the universe provides a directional field $A(\Omega)$
- the hull defines a time-dependent Möbius reprojector $R(\Omega, t)$
- internal cycles generate noncommuting transformations $M(t)$
- anisotropy prevents cancellation of cyclic evolution
- resulting holonomy produces effective translation

15.8 Structural consistency condition

In isotropic conditions:

$$A(\Omega) = \text{const} \Rightarrow \text{no net drift}$$

In anisotropic conditions:

$$\delta A(\Omega) \neq 0 \Rightarrow \langle F_i \rangle \neq 0$$

Anisotropy is therefore the symmetry-breaking ingredient enabling non-cancellation.

15.9 Conceptual status of the framework

At this stage, the structure functions as a relational dynamical model in which:

- motion is emergent from boundary-coupled null geometry
- Möbius transformations act as the symmetry engine
- noncommutativity generates geometric-phase drift
- cosmic anisotropy provides rectification

The central mechanism is:

$$\text{cyclic noncommutative Möbius dynamics} + \text{anisotropic null background} \rightarrow \text{time-averaged translation}$$

The remaining open issue is embedding this structure in a fully Lorentz-consistent, globally conserved momentum framework.

16 Structural Comparison with Woodward-Style Mach Effect Frameworks

The difference between the present framework and Woodward-style Mach effect formulations is primarily not in the presence of oscillatory physics, but in the level of ontology at which "inertia" and "translation" are defined, and in the role assigned to the rest of the universe.

The distinction can be made precise by separating the two approaches structurally.

1. Woodward-type framework: local inertial modulation

In Woodward's formulation, the structure is fundamentally local in spacetime. The core idea is that transient variations in internal energy density produce small fluctuations in effective inertial mass.

A schematic form is:

$$\delta m \sim \frac{1}{c^2} \frac{d^2 E}{dt^2}$$

The dynamics are therefore built from:

- local energy density $E(t)$
- local oscillatory power $P(t)$
- proper-time derivatives
- material stress-energy variations inside the device

The universe appears only indirectly, as:

- a background inertial reference frame
- a Machian justification for the existence of inertia

but it does not explicitly enter the dynamical operator structure.
The effective schematic chain is:

$$\text{local oscillation} \rightarrow \text{local mass fluctuation} \rightarrow \text{effective thrust}$$

This remains within standard spacetime as the fundamental stage.

2. Present framework: nonlocal relational dynamics on the celestial sphere

In this formulation, the structure is nonlocal from the outset. The fundamental object is not local inertial mass modulation, but coupling to a global null-relational field defined on the celestial sphere S^2.

The universe enters explicitly through:

- boundary integrals over S^2
- anisotropy functions $A(\Omega)$
- Möbius transformations on the Riemann sphere
- null-relational coupling operators $M(\Omega, t)$

A characteristic structure is:

$$\int_{S^2} A(\Omega)\, M(\Omega, t)\, d\Omega$$

where:

- S^2 is the global observational sky
- $A(\Omega)$ encodes cosmic anisotropy of the null network
- $M(\Omega, t)$ is a Möbius/null interaction operator acting on directional structure

The effective causal chain is:

local oscillation $\rightarrow$ modulation of global null coupling $\rightarrow$ anisotropic boundary response $\rightarrow$ emergent translation

3. Structural difference: locality vs relational nonlocality

Woodward-type models assume:

- spacetime geometry already exists as the primary stage
- inertia is locally modified within that stage

The present framework assumes:

- translation and inertia are not primitive
- spacetime behavior emerges from a global relational structure

Motion depends on relations between the subsystem and the full null-relational configuration of the universe, not only on internal energy cycling.

4. Role of the universe: implicit vs dynamical

In Woodward-style approaches:

- the universe provides an inertial background
- Machian ideas remain largely interpretive

In the present framework:

- the universe enters directly in the dynamical equations
- through $A(\Omega)$, $M(\Omega, t)$, and integrals over S^2
- as an active component of evolution

This makes the structure:

- boundary-dependent
- globally coupled
- conformally structured
- observationally defined via S^2

5. Conceptual shift in what is generated

Woodward's formulation asks whether local energy oscillations induce transient mass fluctuations that produce thrust.

The present framework instead addresses whether translation itself can emerge from symmetry-breaking in a global null-relational field.

In this formulation, thrust is not primary; translation is the emergent object.

6. Summary of the distinction

Woodward framework:

- local inertial engineering
- spacetime as fixed background
- mass fluctuation produces force-like effects

Present framework:

- global relational dynamics on S^2
- spacetime motion as emergent
- anisotropic Möbius/null structure produces translation as a collective effect

Formally, structures such as:

$$\int_{S^2} A(\Omega)\, M(\Omega, t)\, d\Omega$$

have no analogue in standard Woodward formulations.

7. Final structural characterization

The present framework is not a refinement of Mach-effect mechanics in the Woodward sense. It is a different class of model:

a relational-causal system in which translation emerges from global conformal and null-boundary dynamics rather than from local inertial mass modulation within pre-existing spacetime

17 From Local Propulsion Models to Relational Kinematic Ontology

What is being approached here is no longer a propulsion hypothesis in the usual sense, but a candidate kinematic ontology in which standard relativistic motion would appear as an emergent effective description.

The conceptual shift is that motion is no longer treated as a primitive evolution of objects through a pre-given spacetime, but as a derived structure generated by transformations of a more relational field.

This is why the framework does not reduce to a modification of local mechanical equations such as capacitor-driven force models or Newtonian reaction bookkeeping. Those descriptions presuppose that inertia, translation, and boundary conditions are separable ingredients. In the present construction, they are instead mutually entangled aspects of a single underlying structure.

The implied requirement is therefore a dynamics that is

nonlocal, conformal, relational, and geometrically constrained by boundary structure.

18 Lorentz Structure as Emergent from Möbius Geometry

A central structural observation is that Lorentz symmetry is not introduced as an external axiom but arises naturally from conformal geometry.

The Möbius transformation

$$z \mapsto \frac{az+b}{cz+d}$$

acts on the Riemann sphere and is mathematically isomorphic (via well-known correspondence) to the action of

$$SL(2,\mathbb{C}),$$

which is the double cover of the Lorentz group in 3+1 dimensions.

Thus the following structures

- celestial sphere structure,
- null directions,
- conformal maps,
- boundary transformations

already reside within a framework tightly connected to relativistic kinematics.

This makes it structurally consistent that boosts, rotations, and light-cone geometry can be expressed in terms of Möbius-type transformations rather than fundamental motion through a background manifold.

19 Reinterpretation of Motion as Conformal Action on Null Structure

Instead of treating particles as entities moving through spacetime, the alternative structure treats observed kinematics as the result of transformations acting on null relational data defined on the celestial sphere:

$$z \mapsto \frac{az+b}{cz+d}.$$

In this interpretation:

- boosts correspond to conformal reweightings of null directions,
- translations correspond to emergent projections of evolving conformal structure,
- inertia corresponds to stability of relational configurations under these transformations.

Thus

$$\text{motion} \equiv \text{evolution of conformal structure on null boundary}$$

rather than trajectory through a fixed metric space.

20 Geometric Reinterpretation of Oscillatory Driving

The oscillatory element is recast in relativistic terms using rapidity rather than linear displacement.

Relativistic velocity is expressed as

$$v = c \tanh \eta,$$

where η is rapidity.

A periodic driving of boost-like character is written as

$$\eta(t) = \eta_0 \sin(\omega t),$$

leading to

$$v(t) = c \tanh\big(\eta_0 \sin(\omega t)\big).$$

This encodes a nonlinear, bounded, Lorentz-compatible oscillation in velocity space.

More geometrically, the corresponding transformation on null coordinates is written in Möbius form:

$$z(t) = \frac{z_0 \cosh \eta(t) + \sinh \eta(t)}{z_0 \sinh \eta(t) + \cosh \eta(t)}.$$

This represents a time-dependent conformal boost acting directly on the null relational structure.

21 Physical Interpretation of the Oscillator

In this formulation, the crystal or internal oscillator is not a mechanical source of displacement. Instead, it functions as a periodic modulator of geometric structure:

- null-relational coupling,
- conformal boundary mapping,
- effective boost parameter $\eta(t)$,
- Noether symmetry mixing coefficients.

The oscillation is therefore fundamentally geometric rather than elastic in nature.

22 Noncommutativity and Geometric Phase Accumulation

A key structural element is that successive Möbius transformations generally do not commute:

$$[M_1, M_2] \neq 0.$$

This allows a closed cycle in parameter space to fail to return exactly to identity:

$$M_{\text{cycle}} = M_1 M_2 M_3 \cdots \neq I.$$

This is analogous to geometric phase or holonomy accumulation.

Cyclic modulation of conformal transformations embedded in a noncommutative structure can therefore produce residual effects after averaging, provided that additional symmetry breaking is present.

23 Structural Mechanism for Emergent Drift

The combined structure is summarized as:

- time-dependent conformal (Möbius) transformations on null space,
- oscillatory modulation of boost/rapidity parameters,
- noncommutative composition of transformations,
- anisotropic boundary structure as symmetry-breaking background.

This combination allows a non-vanishing averaged effect of the form

$$\langle \Delta z \rangle \neq 0,$$

arising from holonomy-like accumulation in conformal parameter space.

24 Conceptual Status

The resulting structure is internally consistent as a geometric-relational model in which:

- Lorentz kinematics emerges from Möbius structure,
- motion corresponds to evolution of conformal transformations on null space,
- oscillatory dynamics act on geometric degrees of freedom rather than mechanical displacement,
- noncommutativity enables geometric phase accumulation.

Whether such a structure corresponds to physical reality remains separate from its internal mathematical coherence.

The framework is therefore not reducible to a reactionless propulsion model, but instead describes motion as a noncommutative, time-dependent conformal evolution of null-relational structure with potential holonomy-induced drift terms.

25 Null–Möbius Coupling Model with Finite Boundary Anisotropy and Planetary Contribution

A consistent toy formulation of a "null–Möbius coupling model with finite boundary anisotropy" that incorporates the planetary boundary effect can be organized as follows.

25.1 Core relational state

The system is described by an internal complex phase variable:

$$\psi(t) = \theta(t) + i\phi(t)$$

The external universe is not treated as a geometric manifold but as a directional distribution of interaction sources on the celestial sphere:

$$\rho(n), \quad n \in S^2$$

where:

- n labels directions on the Riemann sphere ("null sky")
- $\rho(n)$ encodes directional intensity of interaction sources

This establishes a boundary-defined relational representation of the environment.

25.2 Finite null interaction kernel ("relational network structure")

A directional coupling kernel is introduced:

$$K(n,t) = K_0 \exp(i\Phi(n,t))$$

where:

- K_0 sets coupling strength
- $\Phi(n,t)$ is accumulated phase from propagation, reflection, and Möbius-type transformations

The system therefore couples not to spatial geometry directly, but to a weighted field of directional null connectivity.

25.3 Effective anisotropy functional

The global anisotropy experienced by the system is defined as:

$$A(t) = \int_{S^2} \rho(n)\, K(n,t)\, d\Omega$$

This object determines whether directional cancellation occurs:

$$A(t) = 0 \quad \text{isotropic cancellation, no net effect}$$

$$A(t) \neq 0 \quad \text{symmetry breaking in relational coupling}$$

Thus $A(t)$ functions as the effective Mach-type source term in a relational formulation.

25.4 Möbius–Noether symmetry mixing operator

The directional dynamics are generated by a conformal algebra element:

$$T = L_{-1} + \lambda L_0 + \eta L_1$$

with generators:

$$L_{-1} = \partial_n, \quad L_0 = n\partial_n, \quad L_1 = n^2\partial_n$$

These correspond respectively to:

- translation-like action
- dilation
- inversion / special conformal transformation

In this structure, translation is not fundamental but arises as a linear combination of conformal generators.

The anisotropy functional is deformed via:

$$\delta A = \epsilon T A$$

25.5 Nonlinear response and emergent drift

The system's velocity is modeled as a nonlinear functional of anisotropy:

$$v(t) = \alpha A(t) + \beta A^2(t) + \gamma \langle TA \rangle$$

Time averaging over oscillatory cycles yields:

- linear contributions cancel under symmetric oscillations
- nonlinear and mixed Möbius terms survive under asymmetry

This defines a rectified (DC) drift component:

$$v_{\text{DC}} = \langle v(t) \rangle \neq 0$$

The existence of nonzero drift requires simultaneous:

- anisotropy in $A(t)$
- nonlinear response terms
- noncommuting Möbius mixing contributions

25.6 Planet as a localized boundary contribution

A planet is represented as a localized perturbation of the directional source distribution:

$$\rho(n) = \rho_{\mathrm{bg}}(n) + \rho_{\mathrm{planet}}(n)$$

This induces a corresponding decomposition:

$$A = A_{\mathrm{bg}} + A_{\mathrm{planet}}$$

If the background contribution cancels:

$$A_{\mathrm{bg}} \approx 0$$

but the planetary term does not:

$$A_{\mathrm{planet}} \neq 0$$

then the rectified drift becomes:

$$v_{\mathrm{DC}} \neq 0$$

with a preferred direction induced by the anisotropic weighting of the planetary contribution.

25.7 Relational interpretation of the mechanism

Within this formulation:

- A planet is not interpreted as a source of curvature acting on trajectories
- Instead, it is a localized enhancement in the boundary structure of the null interaction network

Motion is therefore not described as motion through space, but as:

- a biased reconfiguration of coupling to the directional null field
- driven by asymmetric weighting of the celestial distribution

The effective directional preference emerges because the planet dominates the anisotropic component of the relational kernel.

25.8 Compact formulation of the mechanism

The overall structure can be summarized as:

$$\text{motion} \sim \langle \text{nonlinear Möbius-deformed coupling} \times \text{finite anisotropic null distribution} \rangle$$

where the planet acts as a localized boundary term that biases the symmetry-breaking contribution, producing a nonzero time-averaged deformation of the conformal Noether structure and therefore an emergent drift velocity.

26 Zero-Force Condition in the Anisotropic Möbius–Noether–Null Coupling Framework

A zero-force (inertial) condition can be formulated directly within the anisotropic Möbius–Noether–null coupling framework by expressing dynamics entirely in terms of directional relational balance on the celestial sphere.

26.1 Drift as functional of anisotropy field

The effective velocity is defined as a functional of the directional null anisotropy distribution:

$$v(t) = F[A(n,t)]$$

where:

- $A(n,t)$ is the directional anisotropy field on S^2
- $n \in S^2$ denotes Riemann-sphere directions
- F encodes Möbius action, Noether mixing, and nonlinear response

26.2 Relational momentum and force definition

An effective relational momentum is defined as:

$$P(t) = \int_{S^2} \rho(n)\, \Phi(n,t)\, n\, d\Omega$$

The corresponding effective force is:

$$F_{\text{eff}} = \frac{dP}{dt}$$

This expresses force as time variation of weighted directional coupling to the null-relational field.

26.3 Möbius–Noether evolution of anisotropy

The anisotropy field evolves under a combined linear–nonlinear operator:

$$\partial_t A = TA + N(A)$$

with Möbius–Noether generator:

$$T = L_{-1} + \lambda L_0 + \eta L_1$$

where:

$$L_{-1} = \partial_n, \quad L_0 = n\partial_n, \quad L_1 = n^2\partial_n$$

and:

- $N(A)$ represents nonlinear rectification (DC-generating contribution)
- noncommutativity of T encodes symmetry mixing effects

26.4 Substitution into force functional

The force becomes:

$$F_{\text{eff}} = \int_{S^2} \rho(n)\, \partial_t \Phi(n,t)\, n\, d\Omega$$

Using the evolution equation:

$$\partial_t \Phi \sim T\Phi + N(\Phi)$$

the force decomposes into:

- linear Möbius transport term
- nonlinear rectified contribution

Thus:

$$F_{\text{eff}} = \int_{S^2} \rho(n)\, (T\Phi(n) + N(\Phi(n)))\, n\, d\Omega$$

26.5 Zero-force (inertial) condition

The inertial frame condition is defined by:

$$F_{\text{eff}} = 0$$

which yields the constraint:

$$\int_{S^2} \rho(n)\, (T\Phi(n) + N(\Phi(n)))\, n\, d\Omega = 0$$

This is the fundamental equilibrium condition of the relational system.

26.6 Interpretation of inertial condition

The condition separates into two independent balance requirements:

26.6.1 (A) Linear Möbius balance

The conformal transport contribution must cancel globally:

- no net directional bias from Möbius evolution on the null sphere

26.6.2 (B) Nonlinear rectification balance

The nonlinear term must also vanish under averaging:

- no residual DC component generated from oscillatory anisotropy mixing

26.7 Compact form of inertial equilibrium

The condition can be rewritten as:

$$\langle \rho(n)\, \delta_T \Phi(n)\, n \rangle_{S^2} = 0$$

where:

- δ_T includes Möbius + Noether mixing contributions
- angular brackets denote full-sphere averaging

26.8 Relational interpretation of inertia

In this formulation, inertia corresponds to a global relational equilibrium state:

$$\text{inertial frame} \equiv \text{balanced Möbius-deformed null coupling}$$

A system is inertial if internal oscillatory dynamics, conformal transformations, and null-relational coupling collectively produce zero net directional bias when integrated over the full celestial sphere.

26.9 Effect of a localized boundary source

A planet introduces a perturbation in the directional distribution:

$$\rho(n) = \rho_{\text{bg}}(n) + \delta\rho_{\text{planet}}(n)$$

This generically breaks the balance condition unless compensated by internal phase adjustment or symmetry cancellation within the Möbius–Noether dynamics.

If compensation fails:

$$F_{\text{eff}} \neq 0$$

resulting in a directed drift toward the anisotropy centroid defined by the modified null distribution.

26.10 Final structural statement

The inertial condition is therefore expressed as:

$$\text{No net Möbius-deformed null anisotropy flux through } S^2$$

and the emergence of motion or gravitational-like effects corresponds to:

- failure of this cancellation due to nonlinear rectification of anisotropic relational coupling

This identifies inertia as a special case of global symmetry balance in a Möbius-deformed null-relational field, rather than a primitive property of isolated local dynamics.

27 Symmetry-Response Structure of the Null–Möbius–Noether Velocity Law

A further refinement of the formal structure can be given by isolating the dynamical meaning of each contribution and embedding it into a single symmetry-response framework. The central object remains the anisotropy functional $A(t)$, and the velocity law

$$v(t) = \alpha A(t) + \beta A^2(t) + \gamma \langle TA \rangle$$

is interpreted as a graded expansion of response to a null–Möbius–Noether coupling environment.

27.1 Interpretation of the anisotropy functional $A(t)$

The quantity $A(t)$ is not a force field in spacetime but a directional projection of the global null relational structure onto the system's effective observational frame:

$$A(t) = \int_{S^2} \rho(n)\, K(n,t)\, d\Omega$$

with:

- $n \in S^2$ as Riemann-sphere directions (null sky)
- $\rho(n)$ as directional density of external sources
- $K(n,t)$ as the null interaction kernel encoding phase-weighted connectivity

Thus:

$A(t) = 0$ corresponds to perfect directional cancellation of the null sky

$A(t) \neq 0$ corresponds to effective asymmetry in relational connectivity

27.2 Linear response term

$$\alpha A(t)$$

This term represents first-order sensitivity of motion to directional imbalance in the null structure. It corresponds to:

- proportional response to anisotropic weighting
- direct projection of null-sky asymmetry into motion space

For purely oscillatory $A(t)$ with zero mean:

$$\langle A(t) \rangle = 0$$

this term alone yields no net drift over a cycle.

27.3 Nonlinear rectification term

$$\beta A^2(t)$$

This term introduces symmetry breaking via quadratic response.
Key property:

$$\langle A(t) \rangle = 0 \quad \text{but} \quad \langle A^2(t) \rangle \neq 0$$

Hence it produces a rectified (DC-like) contribution even under symmetric oscillations.
In the null–Möbius interpretation:

- sign information in directional fluctuations is lost under squaring
- oscillatory sampling of the null sky produces a net bias

This term functions as a geometric ratchet mechanism:

$$\text{oscillation} \times \text{nonlinearity} \rightarrow \text{time-averaged drift channel}$$

27.4 Möbius–Noether mixing term

$$\gamma\langle TA\rangle$$

The operator T is defined as a noncommuting conformal generator on the Riemann sphere:

$$T = L_{-1} + \lambda L_0 + \eta L_{+1}$$

with Möbius generators:

$$L_{-1} = \partial_z, \quad L_0 = z\partial_z, \quad L_{+1} = z^2\partial_z$$

Thus TA represents:

- conformal evolution of anisotropy on the null sphere
- symmetry-mixed transport of directional structure
- noncommutative deformation of observational sky geometry

The time-average $\langle TA\rangle$ represents accumulated holonomy-like effects of repeatedly applying noncommuting Möbius transformations during oscillation cycles.

Thus this term encodes:

$$\text{anisotropy} + \text{conformal reprojection} + \text{noncommuting evolution} \rightarrow \text{net transport bias}$$

27.5 Combined dynamical interpretation

The full structure

$$v(t) = \alpha A(t) + \beta A^2(t) + \gamma\langle TA\rangle$$

defines a hierarchy of motion generation mechanisms:

Term	Mechanism	Role
αA	linear anisotropic response	direct coupling
βA^2	nonlinear rectification	DC generation via symmetry breaking
$\gamma\langle TA\rangle$	Möbius–Noether transport	geometric phase accumulation

All three act on the same underlying object: the null relational structure projected onto S^2.

27.6 Emergent physical meaning

The equation encodes the following unified principle:

motion arises from time-averaged deformation of anisotropic null coupling under nonlinear and noncommutative conformal evolution.

More explicitly:

- $A(t)$ defines how the universe is "seen" as a directional interaction network
- $\beta A^2(t)$ ensures oscillations do not cancel trivially
- T ensures that the observational structure is dynamically reprojected in a noncommuting way
- $\langle\cdot\rangle$ enforces that only cycle-accumulated effects survive

27.7 Structural significance

The expression is mathematically analogous to known classes of systems:

- nonlinear response theory (rectification phenomena)
- geometric phase transport (holonomy accumulation)
- driven noncommutative dynamical systems
- ratchet-type symmetry breaking mechanisms

but reinterpreted in a conformal null-sphere ontology rather than a local spacetime ontology.

27.8 Compact interpretation

The velocity law can be summarized as:

motion emerges when anisotropic null coupling is simultaneously nonlinear, cyclically averaged, and subjected to noncon

28 Piezo-Crystal Vibrations Mapped to Conformal Geometry on the Riemann Sphere

A consistent formulation of the vibrating-crystal picture can be constructed by embedding the idea into a coupled oscillator–field system in which Möbius generators act as symmetry operators on an effective null-relational state space, and momentum emerges as a Noether charge only in the presence of symmetry mixing and rectification.

28.1 Dynamical state space

The full system is described by a combined state:

$$X(t) = \{q(t), \phi(t), A(n,t)\}$$

where:

- $q(t)$ is the internal longitudinal vibration mode (piezo oscillator)
- $\phi(t)$ is an internal phase variable
- $A(n,t)$ is the directional anisotropy field on the sphere
- $n \in S^2$ represents directions on the Riemann sphere

28.2 Internal oscillatory sector

The crystal vibration is modeled as:

$$q(t) = q_0 \cos(\omega t + \phi(t))$$

This represents the internal carrier oscillation that drives coupling to external structure.

28.3 Coupling to anisotropy field

Interaction with the external null-relational structure is introduced through:

$$L_{\text{int}} = g\, q(t)\, A(n,t) \cos\theta$$

This term encodes the fact that the internal oscillation samples directional anisotropy in the external field.

28.4 Möbius symmetry structure

The conformal generators acting on the Riemann sphere are:

$$L_{-1} = \partial_z, \quad L_0 = z\partial_z, \quad L_1 = z^2\partial_z$$

with mixed symmetry generator:

$$T = L_{-1} + \lambda L_0 + \eta L_1$$

In this formulation, T defines the baseline conformal deformation of the null structure.

28.5 Time-dependent noncommutative deformation

The key structural ingredient is that symmetry evolution is modulated by vibration:

$$T(t) = T_0 + \epsilon q(t)[L_0, L_{-1}] + \delta \dot{q}(t) L_1$$

This implies:

- the symmetry algebra is dynamically deformed by the oscillator
- the effective transformation does not remain in a fixed Lie subalgebra during evolution
- time evolution becomes noncommutative across oscillation cycles

28.6 Noether momentum structure

The canonical Noether momentum is defined as:

$$P = \frac{\partial L}{\partial \dot{x}}$$

In the coupled system, the effective momentum becomes:

$$P_{\text{eff}} = P_0 + g\langle Aq(t)\rangle + \gamma\langle [T(t), A]\rangle$$

This separates three contributions:

- intrinsic mechanical momentum P_0
- linear anisotropy coupling $g\langle Aq(t)\rangle$
- noncommutative symmetry injection $\gamma\langle [T(t), A]\rangle$

28.7 Condition for DC momentum generation

A nonzero time-averaged momentum requires:

$$\langle P_{\text{eff}}\rangle \neq 0$$

which is equivalent to:

$$g\langle Aq(t)\rangle + \gamma\langle [T(t), A]\rangle \neq 0$$

This defines the propulsion condition in the model.

28.8 Mechanisms of rectification

The nonzero contribution arises only if all of the following are satisfied:

1. **Anisotropic sampling of the null field**

   $$A(n) \neq 0$$

   meaning the directional structure of the external field is not perfectly isotropic.

2. **Oscillatory rectification channel**

   $$\langle q(t)A(t)\rangle \neq 0$$

   indicating that time-dependent sampling produces a non-cancelling correlation between internal vibration and external anisotropy.

3. **Noncommutative symmetry evolution**

   $$\langle [T(t), T(t')]\rangle \neq 0$$

   meaning the conformal transformations applied during different phases of oscillation do not commute, producing geometric phase accumulation.

28.9 Interpretation of mixed Noether symmetry

Within this structure, mixed Noether symmetry corresponds to the coupling of:

- internal vibrational mode $q(t)$
- external anisotropy field $A(n, t)$
- conformal Möbius transport generator T

Such that the conserved current is no longer diagonal in the combined configuration space:

$$(q, A, \text{conformal structure})$$

This produces off-diagonal mixing terms in the effective symmetry representation.

28.10 Emergent physical picture

The system can be summarized as:

a driven oscillator that continuously redefines its conformal mapping of an external anisotropic null field, producing a non-cancelling cycle-averaged Noether current when symmetry mixing and noncommutative evolution are present.

28.11 Structural characterization

Mathematically, the construction belongs to the class of:

- driven nonlinear dynamical systems
- geometric phase accumulation mechanisms
- noncommutative symmetry flows
- ratchet-type rectification processes
- field–oscillator coupled response theories

but expressed in a Möbius-conformal null-sphere representation rather than a standard spacetime field formulation.

29 DC Thrust Scaling Law in the Null–Möbius–Noether Framework

A further refinement of the previous construction yields a compact DC thrust scaling law within the same null–Möbius–Noether framework, now expressed in a form suitable for comparison between linear, nonlinear, and symmetry-mixed contributions.

The starting point is the effective DC momentum definition:

$$F_{\text{DC}} \sim \langle P_{\text{eff}} \rangle$$

with the effective momentum structure:

$$P_{\text{eff}} \sim g\langle A(t)q(t)\rangle + \gamma\langle [T(t), A(t)]\rangle$$

29.1 Crystal vibration model

The dominant longitudinal mode is taken as:

$$q(t) = q_0 \cos(\omega t)$$

The anisotropy field is decomposed into harmonic components:

$$A(t) = A_0 + A_1 \cos(\omega t + \phi) + A_2 \cos(2\omega t + \psi)$$

A key structural requirement follows: DC response requires even-harmonic structure or phase asymmetry, since purely odd symmetric components cancel under time averaging.

29.2 Linear coupling contribution

The linear term is:

$$\langle A(t)q(t)\rangle$$

Time averaging yields a nonzero contribution only under DC bias or phase locking:

$$\langle A(t)q(t)\rangle \sim A_0 q_0 + \frac{1}{2}A_1 q_0 \cos\phi$$

Thus the linear thrust component scales as:

$$F_{\text{lin}} \sim g q_0 A_0$$

with the implication:
linear thrust requires a pre-existing anisotropic bias $A_0 \neq 0$; otherwise the contribution vanishes under symmetric oscillation.

29.3 Nonlinear rectification contribution

The quadratic time average of the oscillator is:

$$\langle q^2(t)\rangle = \frac{1}{2}q_0^2$$

Assume anisotropy responds nonlinearly to oscillation:

$$A(q) \approx A_0 + \eta q(t) + \xi q^2(t)$$

The dominant rectified contribution becomes:

$$F_{\text{rect}} \sim \beta\langle A^2(t)\rangle \sim \beta\left(A_0^2 + \frac{1}{2}A_1^2\right)$$

Hence the scaling form:

$$F_{\text{rect}} \propto \beta q_0^2 A_{\text{rms}}^2$$

This corresponds to a quadratic rectification channel, structurally analogous to nonlinear response systems.

29.4 Möbius–Noether symmetry mixing contribution

The symmetry generator is time-modulated as:

$$T(t) = T_0 + \epsilon q(t) L_0 + \delta\dot{q}(t) L_1$$

The commutator-driven contribution scales as:

$$\langle [T, A]\rangle \sim \epsilon\delta\omega q_0^2 A_1^2 \sin\phi$$

This term is nonzero only if:

- phase asymmetry exists ($\sin\phi \neq 0$)
- symmetry operations do not commute under oscillation
- anisotropy has directional structure ($A_1 \neq 0$)

Thus the Möbius channel scales as:

$$F_{\text{Mbius}} \propto \gamma\epsilon\delta\omega q_0^2 A_1^2 \sin\phi$$

29.5 Full DC thrust scaling law

Combining all contributions:

$$F_{\text{DC}} \sim g q_0 A_0 + \beta q_0^2 A_{\text{rms}}^2 + \gamma\epsilon\delta\omega q_0^2 A_1^2 \sin\phi$$

29.6 Structural interpretation

Linear channel

$$gq_0A_0$$

requires explicit anisotropic bias and behaves like a Mach-type coupling to background structure.

Nonlinear rectification channel

$$\beta q_0^2 A_{\text{rms}}^2$$

survives even when $\langle A \rangle = 0$, representing cycle-averaged rectification structurally analogous to nonlinear inertial response mechanisms.

Möbius symmetry-mixing channel

$$\gamma\epsilon\delta\omega q_0^2 A_1^2 \sin\phi$$

is a frequency-enhanced contribution requiring noncommuting symmetry evolution and phase-sensitive geometric mixing, representing conformal/non-Abelian rectification of the null structure.

29.7 Reduced experimental scaling form

Neglecting internal decomposition details:

$$F_{\text{DC}} \sim aA_0 + bA_{\text{rms}}^2 + c\omega A_{\text{ac}}^2 \sin\phi$$

29.8 Observable predictions

The model implies:

$$F \propto q_0^2$$

quadratic amplitude scaling in the dominant rectification regime,

$$F \propto \omega$$

frequency enhancement in the symmetry-mixing channel,

$$F \sim \sin\phi$$

phase-dependent sign reversal of thrust.

29.9 Consolidated physical statement

DC thrust arises only when oscillatory crystal motion rectifies anisotropic null structure through nonlinear amplitude response and noncommuting Möbius symmetry mixing, producing a nonzero time-averaged Noether current.

30 Reduced Effective Dynamical Formulation of DC Drift

A further reduction of the framework can be expressed as a purely effective dynamical system capturing the same structural ingredients: oscillatory internal dynamics, nonlinear rectification, anisotropic coupling, and symmetry-mixed operators. The central assumption remains:

AC internal dynamics + nonlinearity + anisotropy + symmetry mixing ⇒ nonzero time-averaged momentum (DC drift)

30.1 Starting response law

The effective velocity is given by:

$$v(t) = \alpha A(t) + \beta A^2(t) + \gamma\langle TA \rangle$$

with interpretation:

- $A(t)$: internal oscillatory field (crystal strain / mode amplitude)

- $A^2(t)$: nonlinear self-coupling (rectification channel)
- $\langle TA \rangle$: symmetry-mixed response under operator T

Assume a single-mode excitation:

$$A(t) = A_0 \cos(\omega t + \phi)$$

30.2 Linear contribution

$$\alpha A(t) = \alpha A_0 \cos(\omega t + \phi)$$

Time averaging yields:

$$\langle \alpha A(t) \rangle = 0$$

Hence no DC contribution arises from linear response alone.

30.3 Quadratic rectification channel

$$A^2(t) = A_0^2 \cos^2(\omega t + \phi)$$

Using:

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

one obtains:

$$A^2(t) = \frac{A_0^2}{2} + \frac{A_0^2}{2} \cos(2\omega t + 2\phi)$$

Time averaging gives:

$$\langle A^2(t) \rangle = \frac{A_0^2}{2}$$

Thus the rectified contribution is:

$$v_{\mathrm{DC}}^{(2)} = \frac{1}{2} \beta A_0^2$$

This constitutes the primary nonlinear DC generation channel.

30.4 Mixed symmetry (Noether operator contribution)

Define the weak anisotropic operator action:

$$TA(t) = \eta A_0 \cos(\omega t + \phi + \delta)$$

Direct averaging yields:

$$\langle TA \rangle = 0$$

Thus no DC contribution arises without nonlinear coupling.
To generate rectification, consider the mixed term:

$$\langle TA^2 \rangle$$

Compute:

$$TA^2 \sim \eta A_0^2 \cos(\omega t + \phi) \cos(\omega t + \phi + \delta)$$

Using:

$$\cos x \cos(x + \delta) = \frac{1}{2}(\cos \delta + \cos(2x + \delta))$$

Time averaging yields:

$$\langle TA^2 \rangle = \frac{1}{2} \eta A_0^2 \cos \delta$$

Thus the mixed contribution becomes:

$$v_{\mathrm{DC}}^{(\mathrm{mix})} = \frac{1}{2} \gamma \eta A_0^2 \cos \delta$$

30.5 Total DC drift expression

Combining all contributions:

$$v_{\mathrm{DC}} = \frac{1}{2} A_0^2 \left(\beta + \gamma\eta \cos\delta\right)$$

30.6 Zero-force (inertial) condition

The equilibrium condition is:

$$v_{\mathrm{DC}} = 0$$

which implies:

$$\beta + \gamma\eta \cos\delta = 0$$

This defines an effective inertial constraint in the toy model.

30.7 Inclusion of anisotropic background structure

Introduce directional anisotropy dependence:

$$\eta = \eta_0 + \Delta(\hat{n})$$

where:

- η_0: isotropic background coupling
- $\Delta(\hat{n})$: directional anisotropy field

Substitution yields:

$$v_{\mathrm{DC}} \propto A_0^2 \left[\beta + \gamma(\eta_0 + \Delta)\cos\delta\right]$$

Hence:

$$\Delta \neq 0 \Rightarrow v_{\mathrm{DC}} \neq 0$$

30.8 Hyperbolic (Lorentz-like) extension

Replace oscillatory dynamics with rapidity-like evolution:

$$A(t) = A_0 \sinh(\Omega t + \phi)$$

Then:

$$A^2(t) = A_0^2 \sinh^2(\Omega t + \phi)$$

Using:

$$\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$$

Time averaging over symmetric intervals yields boundary-dependent contributions:

$$\langle A^2 \rangle \sim -\frac{A_0^2}{2} + \text{boundary terms}$$

Thus the DC term becomes:

$$v_{\mathrm{DC}} \sim \frac{1}{2} A_0^2 \left[\beta + \gamma\eta(\text{boundary})\right]_{\text{hyperbolic-weighted}}$$

Key implication:

- rectification depends on boundary conditions of averaging domain
- hyperbolic evolution introduces asymmetry in time weighting
- DC emergence becomes history-dependent rather than purely periodic

30.9 Structural interpretation

The decomposition maps onto:

- linear term → symmetric response sector (no DC generation)
- quadratic term → nonlinear rectification channel
- mixed term → noncommutative symmetry coupling
- anisotropy → external directional bias field
- hyperbolic extension → Lorentz-like deformation of averaging measure

30.10 Consolidated statement

Translation emerges only when nonlinear rectification, symmetry mixing, and anisotropic boundary coupling fail to cancel under time averaging of oscillatory internal dynamics.

31 Irreducible Coupling to an External Anisotropic Boundary Field

A further clarification of the framework is that the central claim is not simply the existence of oscillatory dynamics producing rectified motion, but the stronger structural assertion that the system possesses an irreducible coupling to a global anisotropic boundary field. In this sense, the dynamics cannot be consistently reduced to a purely local momentum-conserving system of standard form.

This requires formulation not as a perturbation removable by coordinate transformation, but as an explicit external symmetry source term.

31.1 Minimal structured formulation

Let the dynamical variables be:

$$x(t), \quad q(t), \quad B(x,t), \quad A[q]$$

where:

- $x(t)$: center-of-mass coordinate
- $q(t)$: internal oscillatory (piezo) mode
- $B(x,t)$: external anisotropic boundary field
- $A[q]$: internal oscillation functional

A minimal Lagrangian encoding the structure is:

$$L = L_{\text{local}}(x,q) + \lambda M(q) B(x,t) + \eta N(q,\dot{q}) \nabla B(x,t)$$

where:

- $M(q)$: nonlinear mass-fluctuation / rectification channel
- $N(q,\dot{q})$: phase-sensitive symmetry-mixing functional
- $B(x,t) \neq 0$: externally fixed anisotropy field

The key structural assumption is that $B(x,t)$ is not dynamical.

31.2 Irreducibility condition

The irreducibility claim corresponds to:

$$\frac{\delta L}{\delta B} \neq 0$$

with B treated as externally prescribed and not eliminated by coordinate redefinition or coarse-graining.

This differs from general relativity-type formulations, where geometry is dynamical and all fields are ultimately encoded in $g_{\mu\nu}(x)$.

Here:

- B is non-dynamical
- B introduces a preferred asymmetry structure
- B cannot, in general, be absorbed into the metric sector

31.3 Equations of motion and origin of translation

From the Euler–Lagrange equation for $x(t)$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

one obtains:

$$m\ddot{x} = -\lambda M(q)\nabla B(x,t) - \eta N(q,\dot{q})\nabla^2 B(x,t)$$

Thus:

- translation arises only if spatial symmetry of $B(x,t)$ is broken
- the effective propulsion condition is:

$$\nabla B(x,t) \neq 0$$

or in averaged form:

$$\langle \nabla B(x,t) \rangle_T \neq 0$$

31.4 Mixed Noether structure

The symmetry structure corresponds to a non-diagonal conserved current:

$$J^{\mu}_{\text{eff}} = J^{\mu}_{x} + \alpha J^{\mu}_{x \leftrightarrow q}$$

with modified conservation law:

$$\partial_\mu J^{\mu}_{\text{eff}} = S[B]$$

where:

- standard theory: $S[B] = 0$
- present framework: $S[B] \neq 0$, representing boundary exchange

This encodes explicit exchange of momentum with the external anisotropic structure.

31.5 DC thrust condition

For oscillatory dynamics:

$$q(t) = A(t)\cos(\omega t + \phi) + \text{nonlinear corrections}$$

the time-averaged force is:

$$F_{\text{DC}} = \frac{1}{T}\int_0^T F(x,t)\,dt$$

with:

$$F(x,t) = -\lambda \langle M(q)\nabla B(x,t) \rangle_T$$

A nonzero DC contribution requires simultaneously:

$$\nabla B(x,t) \neq 0, \qquad \langle M(q) \rangle_T \neq 0$$

Thus motion arises only when both spatial anisotropy and nonlinear rectification coexist.

31.6 Non-reducibility to geometric form (structural claim)

A reduction to standard general relativity would require the existence of a metric $g_{\mu\nu}(x)$ such that:

$$m\ddot{x}^\mu = \Gamma^\mu_{\nu\rho}(g)u^\nu u^\rho$$

The irreducibility assertion is that:

$$\nexists\, g_{\mu\nu}(x) \;\text{ such that }\; \nabla B(x,t) \text{ is fully absorbable into } \Gamma^\mu_{\nu\rho}$$

i.e. the boundary field cannot be fully re-expressed as curvature of spacetime geometry.

31.7 Full force structure

The complete effective dynamics can be written as:

$$m\ddot{x} = -\nabla V_{\text{local}} + F_{\text{Mach}}[q, B]$$

where:

$$F_{\text{Mach}}[q, B] = \lambda M(q)\nabla B + \eta N(q, \dot{q})\nabla^2 B + \text{phase rectification terms}$$

31.8 Structural interpretation

The framework reduces to the statement:

motion arises only through non-vanishing interaction between internal oscillatory dynamics and a spatially structured, externally fixed anisotropic boundary field.

31.9 Critical structural fork

The interpretation depends on the ontological status of $B(x,t)$:

- if $B(x,t)$ can be absorbed into an effective metric or stress-energy tensor $\rightarrow$ equivalence with standard field theory remains possible
- if $B(x,t)$ is fundamentally non-geometric and externally fixed $\rightarrow$ the framework represents additional structure beyond general relativistic spacetime

This distinction determines whether the model is reinterpretation or genuinely extended dynamics.

32 Structural Difference Between Woodward-Type and Nonlocal Relational Frameworks

The structural difference between a Woodward-type formulation and the present framework can be stated precisely in terms of the dimensionality and locality of the forcing functional, and in terms of whether the external universe enters as a finite parameter set or as a spatially extended dynamical functional.

32.1 Woodward-type assumption (local closure structure)

In Woodward-style Mach-effect formulations, the effective force is reducible to a closed functional of internal degrees of freedom:

$$F(t) = F_{\text{local}}(q(t), \dot{q}(t), \ddot{q}(t))$$

The key structural property is locality in configuration space:

- all dynamics are encoded in $q(t)$ and its time derivatives
- any Machian content is implicitly absorbed into effective constants or parameters
- external structure does not appear as an explicit field degree of freedom

Even when interpreted philosophically as Machian, the operational model remains:

$$\text{external universe} \rightarrow \text{fixed effective background parameter}$$

Thus, the system is dynamically closed.

32.2 Present framework (nonlocal functional forcing)

In the present formulation, the force is explicitly nonlocal and field-functional:

$$F(t) = F_{\text{local}}(q, \dot{q}, \ddot{q}) + F_{\text{nonlocal}}[B(x,t), q(t)]$$

where:

- $B(x,t)$ is a spatially extended anisotropy field over cosmic directions and boundary structure
- $x(t)$ couples the system trajectory to a continuously sampled external distribution
- $B(x,t)$ is not reducible to finitely many constants or local parameters

A more explicit representation is:

$$F_{\text{nonlocal}} = F\left[q(t), \int_{\partial U} G(x,\xi)\rho(\xi)\, d\xi\right]$$

with:

- $G(x,\xi)$: relational propagation kernel ("null-spaghetti coupling")
- $\rho(\xi)$: directional cosmic anisotropy density on the boundary ∂U

This structure is intrinsically infinite-dimensional in its forcing term.

32.3 Mathematical distinction (functional class separation)

Woodward-type reduction:

$$F(t) \in \mathcal{F}(q, \dot{q}, \ddot{q})$$

Present framework:

$$F(t) \in \mathcal{F}\big(q, \dot{q}, \ddot{q}, B[x(t), \Omega_{\text{cosmic}}]\big)$$

with B defined over a continuous celestial-sphere domain:

$$B(x,t) = \int_{\partial U} G(x,\xi)\rho(\xi)\, d\xi$$

Hence:

- Woodward class: finite-dimensional dynamical system
- Present class: field-coupled dynamical system with external functional dependence

32.4 Failure of purely local oscillator reduction

A purely local oscillator model admits expansions of the form:

$$F(t) = \sum_n a_n \cos(n\omega t + \phi_n)$$

with coefficients:

$$a_n, \phi_n \in \mathbb{R}, \quad \text{constants or internal-state dependent only}$$

In the present framework, these coefficients become functionals of the external field:

$$a_n \to a_n[B(x)], \qquad \phi_n \to \phi_n[B(x), \text{null structure}]$$

$$a_n(t) \neq \text{constant}$$

Thus, harmonic closure is broken by external functional dependence.

32.5 Structural interpretation of the difference

The distinction can be summarized as:

Woodward-type structure:

$$\text{local oscillator} \rightarrow \text{local mass fluctuation} \rightarrow \text{force}$$

Present framework:

$$\text{oscillator} \rightarrow \text{modulation of coupling to global field functional} \rightarrow \text{nonlocal anisotropic forcing}$$

The oscillator does not generate force directly; it modulates the sampling of a global relational structure.

32.6 Null-spaghetti representation

The external field can be written in relational integral form:

$$B(x,t) = \int_{\partial U} G(x,\xi)\rho(\xi)\, d\xi$$

Interpretation:

- $\rho(\xi)$: cosmic directional density (boundary anisotropy)
- $G(x,\xi)$: propagation kernel of null relational links
- $B(x,t)$: effective anisotropy experienced by the local system

The system interacts with an extended relational network rather than a local field value.

32.7 Core formal distinction

The fundamental separation is:

Woodward-type:

$$F(t) \in \mathcal{F}_{\text{local}}(q, \dot{q}, \ddot{q})$$

Present framework:

$$F(t) \in \mathcal{F}_{\text{nonlocal}}(q, \dot{q}, \ddot{q}, B[x(t), \partial U])$$

where the second functional cannot be reduced to a finite-dimensional parameterization without loss of structure.

32.8 Final structural statement

The difference is therefore not primarily interpretational, but functional:

Woodward-type models assume a dynamically closed oscillator system with effective parameters encoding external influence, whereas the present framework defines a driven system whose force functional is intrinsically nonlocal, field-dependent, and irreducible to any finite set of internal state variables.

33 Irreducible Nonlocal Mach–Möbius Action and Boundary-Coupled Dynamics

A consistent way to enforce non-reducibility to a purely local oscillator is to embed the dynamics in an action functional whose force-generating sector is explicitly nonlocal over a boundary distribution and cannot be absorbed into renormalized local coefficients.

The central structural assumption is:

the local crystal degrees of freedom act only as modulators of a globally defined relational field on the celestial boundary.

This leads to an irreducible boundary-coupling formulation.

33.1 Minimal Nonlocal Mach–Möbius Action

Let:

- $x^\mu(\tau)$: worldline of the craft/crystal center of mass
- $q(\tau)$: internal oscillatory crystal mode
- $\eta(\tau)$: hyperbolic boost-like phase variable
- $B(\Omega)$: anisotropic cosmic boundary distribution on the compactified sky
- $\Omega \in S^2$: angular coordinate on the Riemann sphere
- $M(\Omega, \eta)$: Möbius response kernel

The action is defined as:

$$S = \int d\tau \, [L_{\text{local}} + L_{\text{mix}} + L_{\text{boundary}}]$$

33.2 1. Local oscillatory sector

$$L_{\text{local}} = \frac{1}{2} m \dot{x}^\mu \dot{x}_\mu + \frac{1}{2} I \dot{q}^{\,2} - V(q)$$

This sector alone describes a standard decoupled oscillator system with no translational generation mechanism.

No propulsion is possible from this sector alone.

33.3 2. Noether-mixing sector

$$L_{\text{mix}} = \alpha \dot{\eta} \, q \, \dot{x}^\mu n_\mu$$

where:

- n_μ is a preferred directional functional extracted from the boundary distribution
- α is a coupling constant

This term mixes:

- internal oscillation $q(\tau)$
- boost structure $\eta(\tau)$
- spacetime translation $x^\mu(\tau)$

This defines a mixed Noether-current structure.

In absence of this term, the Euler–Lagrange equations decouple:

$$\frac{\partial^2 S}{\partial x^\mu \partial q} = 0$$

and translational dynamics remain independent of internal oscillations.

33.4 3. Irreducible boundary functional

The key nonlocal sector is:

$$L_{\text{boundary}} = \lambda \int_{S^2} d\Omega \, B(\Omega) \, M(\Omega, \eta) \, \Phi(q, \dot{q})$$

where:

$$\Phi(q, \dot{q}) = q\dot{q} + \beta \dot{q}^{\,2} + \gamma q^2 \dot{\eta}$$

This term is structurally essential.

33.4.1 Nonlocal obstruction property

$$L_{\text{boundary}} \neq f(q, \dot{q}, \ddot{q})$$

because it depends explicitly on:

$$\int_{S^2} d\Omega\, B(\Omega)$$

a global functional over the observable sky.
No finite local expansion in $q, \dot{q}, \ddot{q}$ can absorb this dependence.
This constitutes the irreducibility condition.

33.4.2 Möbius kernel

The conformal response structure is defined as:

$$M(\Omega, \eta) = \frac{a(\eta)\Omega + b(\eta)}{c(\eta)\Omega + d(\eta)}$$

with constraint:

$$ad - bc = 1$$

This encodes conformal (Möbius) reparameterization of the celestial sphere induced by internal boost-like evolution.

33.4.3 Noncommuting generator structure

Define operators:

$$T = \partial_\Omega, \qquad D = \Omega\partial_\Omega$$

with commutation relation:

$$[T, D] = T$$

Hence:

$$TD \neq DT$$

This noncommutativity ensures that sequential operations of:

- internal oscillation evolution
- boundary reprojection

do not cancel under cyclic averaging.
This is the structural origin of directional drift.

33.5 Emergent momentum structure

Define averaged momentum:

$$\langle P_\mu \rangle = \frac{1}{T_1} \int_0^{T_1} d\tau\, \frac{\partial L}{\partial \dot{x}^\mu}$$

which yields:

$$\langle P_\mu \rangle = m\langle \dot{x}_\mu \rangle + \alpha \langle q\dot{\eta} \rangle n_\mu$$

with boundary-induced direction:

$$n_\mu = \int_{S^2} d\Omega\, B(\Omega)\, \hat{n}_\mu(\Omega)$$

33.5.1 Isotropy condition

If the boundary distribution is isotropic:

$$n_\mu = 0$$

then:

$$\langle P_\mu \rangle = 0$$

and no drift emerges.

33.5.2 Anisotropic drift condition

If:

$$n_\mu \neq 0$$

then a nonzero DC component becomes possible.

33.5.3 Minimal DC drift criterion

The system exhibits net translation iff:

$$\langle q\dot{\eta}\rangle \neq 0 \quad \text{and} \quad \int_{S^2} B(\Omega)\,\hat{n}_\mu\,d\Omega \neq 0$$

33.6 Structural interpretation

This implies:

- translation requires internal symmetry mixing
- translation requires global boundary anisotropy
- neither component alone is sufficient

33.7 Fundamental distinction from local oscillator theories

A local oscillator model reduces to:

$$L = L(q, \dot{q}, \ddot{q})$$

In contrast, the present framework is irreducibly nonlocal because:

- the force depends on a sky-integral over S^2
- the anisotropy field $B(\Omega)$ is not eliminable by redefinition
- Möbius kernels introduce noncommuting evolution on the boundary
- translation is generated relationally rather than locally

33.8 Final structural statement

The propulsion sector belongs to a class of:
nonlocal conformal-boundary dynamical functionals
rather than ordinary local mechanical systems.
This is the precise mathematical form of the non-reducibility condition.

34 Boost-Geometry Formulation and Emergent Mixing via Conformal Lorentz Flow

That is indeed the cleaner mathematical direction, because it removes the ad hoc nature of the mixing and replaces it with something already encoded in the symmetry structure of relativistic kinematics.

The central refinement is that "mixing" should not appear as an externally introduced interaction term, but instead emerge from the intrinsic geometry of Lorentz boosts and their action on conformal celestial data. In that sense, translation-like behavior is not generated by a separate mechanism, but by asymmetric accumulation of boost transformations in an anisotropic environment.

34.1 1. Boost structure as intrinsic mixing mechanism

In special relativity, boosts already implement a nontrivial mixing of temporal and spatial components:

$$ct' = ct\cosh\eta - x\sinh\eta$$
$$x' = x\cosh\eta - ct\sinh\eta$$

Thus, any time-dependent rapidity $\eta(t)$ automatically induces a continuous redistribution between temporal and spatial sectors.

If the rapidity becomes a functional of internal dynamics and external anisotropy:

$$\eta = \eta[q(t), B(\Omega)]$$

then translational drift is no longer an independent effect, but a geometric consequence of time-varying boosts.

34.2 2. Emergence of mixing from geometry rather than coupling constants

Instead of introducing a phenomenological mixing term such as:

$$L_{\text{mix}} \sim \alpha\dot{\eta}\, q\, \dot{x}^{\mu}$$

the structure can be embedded directly into the effective metric:

$$ds^2 = -c^2 d\tau^2 + e^{2\eta(q,t,\Omega)} dx^2$$

Here:

- the oscillator $q(t)$ modulates the local boost structure
- anisotropy enters through directional dependence in η
- translation emerges from accumulated geometric deformation rather than explicit force terms

Thus, "mixing" becomes a property of the geometry rather than an inserted interaction.

34.3 3. Möbius structure of boosts on the celestial sphere

Lorentz boosts act conformally on the celestial sphere via Möbius transformations:

$$z \to \frac{az+b}{cz+d}, \quad ad - bc = 1$$

Hence, time-dependent rapidity $\eta(t)$ induces a time-dependent Möbius flow on the Riemann sphere.

If anisotropy is introduced through a directional weighting:

$$B(\Omega) \neq \text{const}$$

then this Möbius evolution no longer preserves symmetry under cycle averaging.

34.4 4. Boost-driven boundary action (geometric formulation)

A more intrinsic action can be written as:

$$S = \int d\tau \left[\frac{1}{2} I \dot{q}^{\,2} - V(q) + L_{\text{boost}}\right]$$

with:

$$L_{\text{boost}} = \int_{S^2} d\Omega\, B(\Omega)\cosh(\eta(q, \dot{q}, \Omega))$$

and:

$$\eta(q, \dot{q}, \Omega) = \kappa(\Omega)\dot{q}$$

where $\kappa(\Omega)$ encodes directional coupling to the cosmic anisotropy distribution.

Expanding the hyperbolic structure:

$$\cosh(\eta) = 1 + \frac{1}{2}\eta^2 + O(\eta^4)$$

automatically generates even-order nonlinearities, producing rectification terms without explicit insertion.

34.5 5. Role of anisotropy in preventing cancellation

If the coupling kernel is isotropic:

$$\kappa(\Omega) = \text{const}$$

then symmetry implies:

$$\int_{S^2} d\Omega\, \kappa(\Omega) = 0$$

and no net drift survives cycle averaging.

However, for anisotropic distributions:

$$\kappa(\Omega_1) \neq \kappa(\Omega_2)$$

the boost accumulation becomes direction-dependent, yielding:

$$\langle \Delta x \rangle \neq 0$$

34.6 6. Structural replacement of the earlier model

The conceptual transition can be stated as:

- Old formulation: oscillator + ad hoc mixing term
- New formulation: oscillator → modulated rapidity → Möbius reprojection → anisotropic cancellation failure → translation

The mixing is no longer an independent ingredient; it is encoded in the geometry of the boost group acting on conformal sky data.

34.7 7. Noncommutativity as intrinsic source of rectification

Noncommutativity is already present in the underlying algebraic structure.

For Lorentz generators:

$$[K_i, P_j] = \delta_{ij} H$$

and for conformal generators on the sphere:

$$[L_0, L_{-1}] = L_{-1}$$

Thus:

$$AB \neq BA$$

for sequential operations involving boost evolution and conformal reprojection.

This implies that cyclic evolution of internal oscillation and boundary mapping does not cancel exactly under time averaging, allowing residual geometric phase accumulation.

34.8 8. Hyperbolic oscillatory structure

Standard harmonic motion:

$$q(t) = A\sin(\omega t)$$

is replaced by boost-compatible rapidity modulation:

$$\eta(t) = \eta_0 \sinh(\omega t)$$

or more generally:

$$\eta(t) = \eta_0 \tanh(\omega t)\sin(\omega t)$$

This reflects the fact that boost composition is hyperbolic rather than circular, aligning the internal dynamics with Lorentzian geometry.

34.9 9. Emergent conceptual structure

The resulting coherent structure becomes:

- internal oscillation modulates local rapidity (boost geometry)
- the celestial sphere undergoes time-dependent Möbius transformation
- anisotropic boundary structure breaks cancellation symmetry
- noncommutative boost composition produces residual geometric phase
- the residual phase manifests as net translation

34.10 10. Final structural interpretation

The refined framework is therefore not based on an externally imposed coupling mechanism, but on:

translation as emergent holonomy of time-dependent, anisotropy-weighted boost flows on the conformal boundary

In this form, the "mixing" is no longer a parameter of the model, but a consequence of the geometry of noncommuting Lorentz–Möbius evolution acting on an anisotropic sky distribution.

35 Piezoelectric System as a Geometric–Relational Actuator

If the goal is to express the mechanism in a way that is internally consistent and not reducible to a purely mechanical oscillator, then the essential shift is to treat the piezoelectric system as an intermediate actuator of a geometric–relational degree of freedom, rather than as the origin of force itself.

The starting point remains the standard electromechanical chain:

$$I(t) \to E(t) \to \epsilon(t) \to q(t)$$

with:

- $I(t)$: driving current
- $E(t)$: electric field
- $\epsilon(t)$: strain
- $q(t)$: longitudinal crystal mode

In conventional descriptions, this chain terminates in purely local vibration dynamics, and any net motion must arise from interaction with external mechanical constraints.

In the present framework, an additional mapping is introduced:

$$q(t) \to \eta(t)$$

where $\eta(t)$ is not merely a phase variable but an effective rapidity/boost parameter controlling a local conformal–hyperbolic deformation of the interaction structure.

This extends the causal chain to:

$$I(t) \to \epsilon(t) \to \eta(t) \to M(\Omega, \eta) \to \text{boundary reprojection}$$

where:

- $M(\Omega, \eta)$: Möbius response kernel
- $\Omega \in S^2$: celestial (Riemann sphere) direction space
- boundary reprojection: effective reweighting of the external anisotropic field

In this formulation, the piezo stack does not directly generate translation; it modulates the mapping between internal energy and the external null–relational structure.

35.1 Driving structure and harmonic content

For a purely sinusoidal drive:

$$I(t) = I_0 \sin(\omega t)$$

cycle-averaged effects cancel under symmetric response assumptions.

To obtain non-vanishing cycle-averaged contributions, the drive must support either harmonic asymmetry or phase-sensitive mixing:

$$I(t) = I_1 \sin(\omega t) + I_2 \sin(2\omega t + \phi)$$

This structure introduces:

- even-harmonic rectification channels
- phase asymmetry between coupled response modes
- non-cancellation under nonlinear boost composition

This is the same structural requirement appearing in nonlinear response systems, but here it is interpreted through a hyperbolic (rapidity-based) geometry rather than a purely mechanical nonlinearity.

35.2 Hyperbolic (boost) structure of the effective variable

The key nonlinearity is not additive in the Euclidean sense. The relevant composition law is boost-like:

$$\eta_{\text{tot}} \neq \eta_1 + \eta_2$$

while velocity composition follows:

$$v_{\text{tot}} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

This implies that even if the driving current satisfies:

$$\langle I(t) \rangle = 0$$

the induced rapidity-like variable may satisfy:

$$\langle \Delta\eta \rangle \neq 0$$

provided the mapping $I(t) \to \eta(t)$ is nonlinear and phase-asymmetric.

35.3 Relational coupling functional

The physically relevant object becomes the boundary-weighted coupling:

$$\langle B(\Omega)\, \eta(t)\, \dot{\eta}(t) \rangle$$

or more generally:

$$\langle B(\Omega)\, M(\Omega, \eta(t)) \rangle$$

where:

- $B(\Omega)$: anisotropic external field on S^2
- $M(\Omega, \eta)$: Möbius deformation kernel

Net drift requires failure of cancellation in this averaged functional, which occurs only if:

- $\eta(t)$ contains nonlinear harmonic structure,
- $B(\Omega)$ is anisotropic,
- Möbius composition is effectively noncommutative under time evolution.

35.4 Structural interpretation

In this formulation, the role of the piezo system is not that of a classical oscillator producing thrust.

Instead, the chain of causation is:

current drive $I(t) \to$ strain modulation $\epsilon(t) \to$ rapidity deformation $\eta(t) \to$ Möbius reweighting of $B(\Omega) \to$ cycle-average

The "output" is therefore not force in a local mechanical sense, but a non-vanishing averaged deformation of a global relational coupling functional.

In this ontology, the device functions as a current-driven modulator of a boost-structured null interaction network, rather than a conventional vibrating mechanical engine.

36 Hyperbolic Rapidity Drive and Möbius Boundary Coupling

In this formulation, the most consistent extension of the framework is to treat the piezo stack as being driven not by a purely Euclidean sinusoidal excitation, but by a hyperbolic rapidity-like input signal.

The motivation is structural:

- ordinary oscillators are naturally described in circular (trigonometric) geometry,
- Lorentz boosts and Möbius transformations are naturally described in hyperbolic geometry,
- the proposed mechanism depends on boost composition and null-structure reprojection.

Under this identification, the drive signal is better represented as a deformation of rapidity space rather than a simple harmonic displacement.

36.1 Hyperbolic drive structure

Instead of:

$$I(t) = I_0 \sin(\omega t)$$

the excitation can be generalized to hyperbolic or hybrid hyperbolic–periodic forms, for example:

$$I(t) = I_0 \sinh(\omega t)$$

or bounded periodic hyperbolic embeddings such as:

$$I(t) = I_0 \tanh(\alpha \sin(\omega t))$$

or mixed harmonic–hyperbolic coupling:

$$I(t) = I_1 \sin(\omega t) + I_2 \sinh(\omega t) \cos(\omega t)$$

The central assumption becomes:

$$\eta(t) \propto I(t)$$

so that the electrical drive directly modulates an effective rapidity parameter.

36.2 Nonlinear hyperbolic expansion structure

The relevant nonlinear functions introduce inherently asymmetric averaging properties:

$$\cosh(\eta) = 1 + \frac{\eta^2}{2} + \cdots$$

$$\sinh(\eta) = \eta + \frac{\eta^3}{6} + \cdots$$

These expansions generate:

- even-harmonic contributions (via cosh),
- odd-harmonic asymmetry (via sinh),
- non-vanishing cycle-averaged components under nonlinear mixing.

In contrast to purely sinusoidal driving, these structures do not generically cancel under symmetric time averaging.

36.3 Rapidity-driven coupling model

A minimal toy rapidity relation can be expressed as:

$$\eta(t) = \eta_0 \tanh\left(\frac{I(t)}{I_c}\right)$$

where:

- I_c: characteristic response scale of the crystal–electromechanical system

This introduces a saturation-limited hyperbolic response, ensuring bounded rapidity while preserving nonlinear sensitivity.

36.4 Möbius boundary coupling

The effective drift force is then expressed as a boundary-weighted average over the celestial sphere:

$$\langle F_\mu \rangle \sim \int_{S^2} d\Omega\, B(\Omega)\, \langle M(\Omega, \eta(t)) \rangle$$

where the Möbius kernel is:

$$M(\Omega, \eta) = \frac{a(\eta)\Omega + b(\eta)}{c(\eta)\Omega + d(\eta)}$$

with:

$$ad - bc = 1$$

This structure encodes the reparameterization of null directions under rapidity-dependent conformal transformations of the Riemann sphere.

36.5 Structural interpretation

Within this model, the chain of causation becomes:

$$I_{\text{hyperbolic}}(t) \to \epsilon(t) \to \eta(t) \to \text{Möbius reprojection} \to \text{anisotropic null asymmetry} \to \langle P_\mu \rangle$$

The drive current therefore functions as a direct control parameter for an effective boost geometry, rather than as a conventional actuator producing mechanical vibration.

36.6 Conceptual outcome

The resulting device can be interpreted as:

> a system that periodically modulates local rapidity structure through hyperbolic current excitation, producing asymmetric Möbius reprojection of an anisotropic boundary null field, leading to a non-vanishing cycle-averaged momentum functional.

37 Emergence of Stress–Energy Mixing from Hyperbolic Electromechanical Structure

The cleanest formulation of this idea avoids introducing any explicit "mixing term" at the level of the Lagrangian and instead allows the mixing structure to emerge from the intrinsic geometry of the electromechanical stress–energy tensor once the system is driven in a hyperbolic regime.

The key step is to treat Lorentz and Möbius structure as emergent from energy–momentum flux structure, rather than as externally imposed couplings.

37.1 Piezoelectric continuum structure

A piezoelectric medium already contains intrinsic electromechanical coupling through the constitutive relations:

$$S_{ij} = s_{ijkl}T_{kl} + d_{kij}E_k$$
$$D_i = d_{ikl}T_{kl} + \epsilon_{ij}E_j$$

where:

- S_{ij}: strain tensor
- T_{ij}: stress tensor
- D_i: electric displacement
- E_i: electric field
- d_{ijk}: piezoelectric coupling tensor

In this formulation, electromechanical mixing is not postulated; it is structurally embedded in the material response.

37.2 Hyperbolic current injection

A hyperbolic driving current is introduced as:

$$I(t) = I_0 \tanh(\omega t)$$

or in a bounded dissipative form:

$$I(t) = I_0 \sinh(\omega t)e^{-\alpha t^2}$$

The electric field generated along the polarization axis satisfies:

$$E_z(t) \propto \int I(t)\, dt$$

which yields a hyperbolic integral structure of the form:

$$E_z(t) \sim \ln\cosh(\omega t)$$

Thus the driving field is inherently non-sinusoidal and carries built-in even-harmonic structure.

37.3 Polarization and energy density

The polarization response becomes:

$$P_z(t) = \chi E_z(t)$$

so that:

$$P_z(t) \sim \chi \ln\cosh(\omega t)$$

The electromagnetic energy density:

$$u_E = \frac{1}{2}E \cdot D$$

therefore contains nonlinear combinations of hyperbolic functions, generating intrinsic contributions of the form:

- $\cosh^2(\cdot)$
- $\sinh^2(\cdot)$

These terms naturally produce:

- even harmonic components,
- non-zero cycle-averaged offsets,
- asymmetric temporal weighting over a period.

No additional rectification mechanism is required.

37.4 Elastic response of the crystal

The longitudinal displacement field $u_z(x,t)$ satisfies:

$$\rho \partial_t^2 u_z = c_L^2 \nabla^2 u_z + f_{\text{piezo}}$$

with forcing term:

$$f_{\text{piezo}} \sim d_{33} \partial_z E_z$$

Given the hyperbolic structure of $E_z(t)$, the displacement inherits a nontrivial temporal dependence:

$$u_z(x,t) \sim u_0(x) \tanh(\omega t)$$

rather than a purely harmonic oscillation.

37.5 Emergence of off-diagonal stress–energy structure

The energy–momentum tensor of the coupled medium contains off-diagonal flux components:

$$T_{0z}$$

which in electromechanical media takes the schematic form:

$$T_{0z} \sim u_E v_z + \sigma_{zz} \dot{u}_z + P_z \dot{u}_z$$

where:

- u_E: electromagnetic energy density
- σ_{zz}: mechanical stress
- $v_z = \dot{u}_z$: velocity field

Due to the hyperbolic structure of both $P_z(t)$ and $\dot{u}_z(t)$, the time-averaged product becomes generically non-zero:

$$\langle P_z \dot{u}_z \rangle \neq 0$$

This term is not introduced externally; it emerges from nonlinear stress–energy coupling under hyperbolic driving.

37.6 Lorentzian interpretation of mixing

In relativistic form, T_{0z} represents momentum flux density, and Lorentz transformations mix energy and momentum components:

- T_{00}: energy density
- T_{0i}: momentum density

Boost transformations intrinsically couple these sectors. Under hyperbolic excitation, the system naturally explores this mixing structure, producing effective transitions between:

$$T_{00} \leftrightarrow T_{0z}$$

without requiring any additional coupling terms.

37.7 Emergent Möbius / null coupling

The momentum flux is then coupled to an anisotropic boundary distribution:

$$B(\Omega)$$

yielding an effective averaged force:

$$\langle F_z \rangle \sim \int_{S^2} d\Omega \, B(\Omega) \, \langle T_{0z}(t, \Omega) \rangle$$

This explicitly introduces dependence on a nonlocal directional structure over the celestial sphere, preventing reduction to a purely local oscillator model.

37.8 Emergence of noncommutative structure

Möbius generators on the Riemann sphere are:

$$L_{-1} = \partial_z, \quad L_0 = z\partial_z, \quad L_1 = z^2\partial_z$$

with nontrivial commutation relations such as:

$$[L_0, L_{-1}] = -L_{-1}$$

This implies that sequential application of transformations does not commute:

$$M(\eta_1)M(\eta_2) \neq M(\eta_2)M(\eta_1)$$

Therefore, time-ordered hyperbolic modulation produces path-dependent accumulation effects.

37.9 Emergent dynamical chain

The full mechanism can be expressed as:

$$I(t) \to E(t) \to P(t) \to u_z(t) \to T_{0z}(t) \to \eta(t) \to M(\eta) \to B(\Omega) \to \langle F_z \rangle$$

In this structure:

- electromechanical coupling arises from standard piezo physics,
- hyperbolic behavior arises from nonlinear drive response,
- noncommutativity arises from Lorentz/Möbius geometry,
- net drift appears only after anisotropic boundary averaging.

37.10 Structural conclusion

The mixing responsible for momentum transfer is not an independently introduced interaction term but an emergent consequence of:

- nonlinear electromechanical constitutive relations,
- hyperbolic (boost-like) driving of the crystal,
- off-diagonal stress–energy generation,
- and noncommutative conformal structure on the boundary sphere.

In this formulation, the system cannot be reduced to a purely local oscillator because the effective force depends on a nonlocal, anisotropic boundary functional of the stress–energy tensor rather than on local state variables alone.

38 Geometric inversion, hyperbolic kinematics, and spatially asymmetric relativistic analogue structure

This formulation represents a reversal and subsequent refinement of the usual causal intuition found in propulsion and relativistic-analogue models.

Standard formulations assume a structure of the form:

$$\text{force} \rightarrow \text{acceleration} \rightarrow \text{Lorentzian kinematics}$$

where external interaction defines motion, and relativistic structure emerges as a consequence of that motion.

Mach- and Woodward-type approaches instead suggest a structure closer to:

$$\text{internal state modulation} \rightarrow \text{effective inertial response} \rightarrow \text{apparent translation}$$

in which internal energy exchange produces a time-dependent modification of inertial properties, interpreted as motion relative to an external frame.

The refinement introduced here is more restrictive and geometric:

if the objective is to reproduce the observable consequences of external acceleration, then the internal dynamics must reproduce not only energy exchange behavior, but also the symmetry structure of relativistic acceleration, including its spatial asymmetry.

38.1 1. Kinematic inversion principle

Rather than treating the crystal as an arbitrary nonlinear oscillator, it is modeled as a dynamical system whose internal state evolution is constrained to emulate relativistic kinematics.

This shifts the design constraint from:

$$q(t) \quad \text{arbitrary nonlinear dynamics}$$

to:

$$\text{internal evolution} \sim \text{representation of Lorentzian accelerated motion}$$

38.2 2. Emergent hyperbolic structure

Externally accelerated motion is naturally expressed in rapidity space:

$$\gamma = \cosh \eta, \qquad \gamma\beta = \sinh \eta$$

with composition law:

$$\eta_{\text{tot}} = \eta_1 + \eta_2$$

This shows that relativistic acceleration is inherently hyperbolic rather than circular.

Therefore, a Euclidean harmonic structure of the form:

$$x(t) \sim \sin(\omega t)$$

is replaced by hyperbolic rapidity evolution:

$$\eta(t) \sim \sinh(\omega t), \quad \eta(t) \sim \tanh(\omega t), \quad \text{or bounded hyperbolic flows}$$

38.3 3. Lorentzian stress–energy consistency

In relativistic field theory, acceleration is encoded through the mixing of energy and momentum in the stress–energy tensor:

$$T_{00} \text{ (energy density)}, \qquad T_{0i} \text{ (momentum flux)}$$

Lorentz boosts continuously mix these components:

$$T_{00} \leftrightarrow T_{0i}$$

A system that emulates relativistic acceleration must therefore reproduce this effective mixing structure dynamically.

38.4 4. Spatial asymmetry and relativistic potential structure

The refinement further requires reproduction of the spatial asymmetry characteristic of a relativistic potential field.

This shifts the target from symmetric oscillatory behavior toward a non-uniform acceleration environment.

Core structural condition:

$$\Delta\Phi \neq 0$$

across the spatial extent of the device.

38.5 5. Contrast with homogeneous driving

A uniform piezoelectric excitation is typically written as:

$$V(z,t) = V_0 \sin(\omega t)$$

In the present framework, this generalizes to:

$$V(z,t) = V_0(z)\, f(t)$$

with:

$$\partial_z V \neq 0$$

This introduces an intrinsic spatial gradient across the device.

38.6 6. Weak-field relativistic analogy

This spatial asymmetry maps onto the weak-field metric approximation:

$$g_{00}(z) \sim 1 + \frac{2\Phi(z)}{c^2}$$

which encodes position-dependent time dilation.

Consequences include:

- spatial variation of effective temporal rates
- non-uniform phase evolution across the device
- gradient-dependent energy storage and release

38.7 7. Emergent distributed field structure

Once spatial asymmetry is introduced, all internal quantities become position-dependent:

$$\eta(z,t), \quad P(z,t), \quad T_{\mu\nu}(z,t)$$

This implies:

- spatial variation of rapidity-like variables
- loss of global coherence in polarization
- emergence of stress–energy flux gradients

The system is therefore better described as a distributed relativistic analogue medium rather than a single oscillator.

38.8 8. Noncommutativity in spatially structured evolution

In this regime, Möbius-type transformations become spatially dependent:

$$M(\eta_1(z_1,t))\, M(\eta_2(z_2,t)) \neq M(\eta_2(z_2,t))\, M(\eta_1(z_1,t))$$

Noncommutativity arises because the system is no longer homogeneous.

The ordering of transformations depends on spatial location, producing path-dependent accumulation in both space and time.

38.9 9. Emergent geometric interpretation

The structure can be summarized as:

- internal oscillation → hyperbolic rapidity modulation
- spatial structure → relativistic potential gradient
- boundary mapping → Möbius reweighting of anisotropic sky data
- evolution → noncommutative boost composition
- outcome → residual geometric drift in averaged momentum

38.10 10. Structural conclusion

The formulation replaces a homogeneous resonant actuator with a distributed system in which:

$$\text{internal fields} \rightarrow \text{spatially graded relativistic analogue medium}$$

Any motion-like effect arises only from the interaction between:

- spatial potential asymmetry
- time-dependent internal phase evolution
- noncommutative transformation of local boost-like variables

This places the device conceptually closer to a synthetic analogue of an accelerating spacetime region than to a conventional vibrating mechanical resonator.