# The Brennan–Boboc Effect: Orbital Deviations from Nested Gravitational Spacetimes

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**Abstract**

We propose the **Brennan–Boboc effect: the change in the orbital trajectory of a system arising from the gradual activation and transport of an internally defined, correlation-based spacetime geometry within an external gravitational spacetime**. The approach considers the two-point electric-field correlators of dielectric systems, including contributions from applied fields, electromagnetic modes, quantum fluctuations, and thermal fluctuations, as the physical basis for reconstructing an effective geometry. This geometry may influence the affine transport and dynamical behaviour of an extended system, potentially producing deviations from trajectories predicted solely by the external gravitational metric.

We investigate the asymmetric capacitor associated with the Biefeld–Brown effect as a possible experimental setting for this mechanism. We hypothesize that electrode asymmetry and the resulting nonuniform electromagnetic correlation structure may generate an internal geometry whose transport properties contribute to the device's mechanical behaviour. We further extend the hypothesis to asteroids and comets, proposing that Solar System tidal interactions may induce stretching and compression within their mineral constituents, modifying local electrical states and producing sparse, statistically distributed electromagnetic correlations. These correlations could give rise to transient internal metrics whose affine transport properties differ from those of the external spacetime, potentially contributing to deviations from conventionally predicted gravitational trajectories.

The proposed framework establishes a common theoretical programme for investigating controlled electromagnetic systems and naturally occurring celestial bodies through correlation-defined geometry. It identifies the reconstruction of internal metrics, the comparison of internal and external affine connections, and the derivation of observable trajectory modifications as central steps toward testing the Brennan–Boboc hypothesis. The proposed effects remain hypotheses requiring quantitative development and experimental or observational verification, with conventional electromagnetic, mechanical, and gravitational explanations taken into account.

**B**rennan–Boboc effect; correlation-defined geometry; electromagnetic field correlations; dielectric media; Biefeld–Brown effect; asymmetric capacitors; affine transport; quantum vacuum fluctuations; tidal deformation; asteroid and comet dynamics.

# 1 Introduction

The present work develops a correlation-based framework in which electromagnetic field correlations are not regarded merely as physical quantities defined on a pre-existing spacetime, but as structures capable, under suitable physical conditions, of defining an

effective geometry of their own. The central hypothesis is that spatially and temporally organized electromagnetic correlations can determine an internal metric and its associated affine connection, thereby establishing geometric relationships that need not coincide with those of the surrounding gravitational spacetime. In this formulation, the geometry of a physical system is not introduced independently of its electromagnetic state: it is reconstructed from the correlations themselves. The resulting framework provides a possible basis for investigating how electromagnetic organization may influence the effective transport, deformation, and motion of extended physical systems.

We refer to the proposed mechanism as the Brennan–Boboc effect. Its formulation begins with the observation that a physical system containing correlated electromagnetic fields may possess an effective internal geometric structure, reconstructed from the two-point correlations of its electric field. For a dielectric system, these correlations include contributions associated with the applied electric field, electromagnetic modes, material response, quantum fluctuations, and thermal fluctuations. Their spatial and temporal organization determines the correlation structure from which the internal metric is reconstructed. In the particular capacitor model considered here, the proposed correlation strength is expressed as

$$\mathcal{C}(x,x') = \left(\frac{V(x,x')}{L(x,x')}\right)^2 + \frac{\hbar c}{\pi^2 \epsilon_0 R(x,x')^4}. \tag{1.1}$$

The first contribution represents the applied-field component, while the second introduces a quantum vacuum correlation term with the characteristic inverse-fourth-power dependence on separation. The precise form of the correlation function in a real dielectric capacitor must ultimately account for its electromagnetic modes, dielectric response, boundary conditions, and material properties. Nevertheless, this expression provides the working correlation structure from which the proposed internal geometry can be investigated.

The essential step is to reconstruct the metric from the two-point field correlator and then determine the affine connection associated with that metric. This creates a distinction between the internal geometry defined by the electromagnetic correlations and the external geometry defined by the gravitational spacetime. If the two connections differ, their difference can enter the transport law of the system. Consequently, an extended electromagnetic structure need not behave, in this proposed description, as a structureless point particle whose motion is determined exclusively by the external gravitational connection. Its internal correlation geometry may instead contribute to the transport of its center and to the relative motion of its constituent portions. The Brennan–Boboc formulation therefore proposes a mechanism through which electromagnetic correlations could produce effective geometric and dynamical effects.

One physical configuration of particular interest is the asymmetric capacitor associated with the Biefeld–Brown effect. This configuration has historically been associated with reports of force or motion when an electrical potential is applied across electrodes of unequal geometry. The present framework proposes a geometric interpretation of such behaviour: rather than treating the system solely as an arrangement of electrodes producing an electric field and an associated mechanical force, it considers whether the electromagnetic correlations established within the dielectric and between the electrodes could define an internal metric whose affine transport differs from that of the surrounding spacetime. In this interpretation, the asymmetry of the capacitor may be relevant because it produces a spatially nonuniform electromagnetic environment and therefore potentially a nonuniform correlation structure. The resulting internal geometry could, in principle, affect the transport properties of the complete device.

This proposal is intended as a candidate physical explanation to be developed and tested, rather than as an established resolution of the Biefeld–Brown effect. In particular, electrohydrodynamic forces, ion and neutral-gas momentum transfer, electrode interactions, thermal effects, and experimental artefacts must be distinguished from any additional contribution predicted by correlation-defined geometry. A successful account would therefore have to derive a measurable force or trajectory modification from the reconstructed metric and its transport law, and demonstrate that the result cannot be explained by conventional electromagnetic or mechanical mechanisms alone. The purpose of the present framework is to formulate such a possibility in geometric terms and identify the conditions under which it could become experimentally distinguishable.

We further extend the Brennan–Boboc hypothesis to the possible anomalous motion of certain small Solar System bodies, including asteroids and comets. We propose that, in some circumstances, their internal electromagnetic correlation structures may be affected by the tidal environment of the Solar System. Tidal stretching and compression can impose changing stresses on heterogeneous mineral structures, potentially modifying local charge distributions, polarization states, dielectric responses, and electromagnetic correlations within the material. Where these processes generate sufficiently organized, although possibly sparse and statistically distributed, electrical correlations, they may give rise to transient internal correlation geometries. Such geometries would not necessarily be spatially uniform or persistent: they could arise intermittently, vary between mineral regions, and evolve as the body moves through a changing tidal environment.

The proposed sequence is therefore:

Solar System tidal interactions

↓

Stretching and compression of mineral structures

↓

Modification of local electrical states and polarization

↓

Formation of sparse, statistical electromagnetic correlators

↓

Transient reconstruction of internal correlation metrics

↓

Possible conflict between internal affine transport

and the external gravitational-geodesic description

↓

Potential deviation from the expected trajectory. (1.2)

Within this hypothesis, the relevant effect would not require an asteroid or comet to possess a permanent, globally defined alternative spacetime. Rather, localized and transient correlation structures could produce an effective internal geometry over particular regions and time intervals. If those structures are sufficiently coherent to define a nondegenerate metric and an associated connection, their transport properties could, in principle, contribute to the motion of the body as a whole. The magnitude and direction of any resulting deviation would depend on the spatial distribution, strength, lifetime, and transport of the correlations, as well as on the mechanical coupling between the correlated regions and the rest of the body.

This extension is particularly speculative. Tidal deformation does not by itself establish the existence of the required electromagnetic correlations, and reported or inferred orbital deviations do not by themselves demonstrate a Brennan–Boboc contribution. The hypothesis requires a quantitative model connecting tidal stress to mineral electrical response, that response to two-point correlators, the correlators to a regularized internal metric, and the resulting connection to a calculable perturbation of the body's trajectory. It must also be tested against conventional explanations, including gravitational perturbations, outgassing, radiation pressure, thermal recoil, and uncertainties in the body's mass distribution.

The common principle connecting the capacitor and small-body applications is that the

electromagnetic correlation structure of a physical system may contribute to its effective geometry and therefore to its transport. In the capacitor, the correlations are externally controlled through an applied voltage and the electrode–dielectric configuration. In an asteroid or comet, they are hypothesized to arise dynamically and intermittently through the interaction of tidal deformation with the electrical and dielectric properties of its constituent minerals. These are very different physical environments, but they can be examined within the same mathematical programme: construct the two-point electromagnetic correlator, reconstruct the corresponding internal metric, determine its affine connection, and investigate how that connection relates to transport in the external gravitational spacetime.

The objective of this work is thus to establish a unified theoretical formulation of the Brennan–Boboc effect, beginning with controlled dielectric electromagnetic systems and extending, as a testable hypothesis, to naturally occurring bodies subject to tidal deformation. By treating electromagnetic correlations as potential generators of effective geometry, the framework seeks to investigate whether certain electromagnetic and gravitational phenomena conventionally described separately might instead admit a common geometric description. Its central predictions must ultimately be expressed in terms of measurable correlation functions, reconstructed metric components, connection differences, and observable deviations from conventional dynamical trajectories.

# 2 Further important considerations for nonclassical space propulsion

We propose that a new area of investigation, situated at the intersection of electromagnetism and general relativity, must be acknowledged and explored in order to investigate the possibility of novel forms of nonclassical space propulsion. We argue that a conceptual and theoretical grey area exists between these disciplines, within which previously unconsidered effects may emerge from the interaction of electromagnetic fields, solid-state constitutive physics, general relativity, and the geometry of dielectric media.

Our proposal is that, if nonclassical propulsion can ultimately be realized, its underlying physical mechanisms may be found in the combined application of electromagnetism and constitutive solid-state physics, together with effects that couple electromagnetic correlations to geometry and gravitational spacetime. Such mechanisms may provide an alternative route to investigating metric engineering without necessarily following the conventional general-relativistic approach of attempting to generate sufficiently large stress-energy tensors to produce spacetime curvature through the standard Einstein field equations.

Instead, we propose investigating whether the spacetime metric can be engineered locally, point by point, through electromagnetic field correlators and an expanded physical definition of space and distance. This approach considers the possibility that the relationship between electromagnetic energy and effective geometry may be more flexible than is ordinarily assumed, allowing the conversion coefficient between electromagnetic phenomena and geometrical structure to be modified and potentially giving rise to entirely new physical effects.

This proposal does not reject general relativity or question its established success in describing astronomical observations, GPS, gravitational-wave detection, and cosmology. Rather, it seeks to investigate whether the conventional general-relativistic framework, while highly successful in these domains, is sufficient by itself as a practical theoretical basis for nonclassical space propulsion. We propose that alternative approaches to the relationship between electromagnetic correlations and geometry may be necessary to explore this possibility.

We further suggest that some of the relevant physical phenomena may have appeared in experimental contexts long before their underlying mechanisms were properly understood. The Biefeld–Brown effect, particularly in asymmetric capacitors, represents an important example within this line of investigation. If explanations based on ionic wind and parasitic electromagnetic interactions with surrounding structures are set aside, the possibility remains, within our hypothesis, of investigating additional mechanisms involving electromagnetic correlations and the effective geometry of the dielectric medium.

The central principle underlying this approach is that one cannot trivially transport an internally defined geometry through an externally prescribed background geometry. The relationship between the two geometrical structures must itself be treated as a physical problem. By investigating this relationship through electromagnetic correlators, constitutive material properties, and affine transport, we seek to establish a new theoretical perspective in which the engineering of effective geometry may be explored through electromagnetic and material systems, rather than exclusively through the conventional generation of gravitational curvature.

# 3 Electromagnetic Correlation Geometry

Kempf's work, (Achim Kempf (2021), "Replacing the Notion of Spacetime Distance by the Notion of Correlation," Frontiers in Physics, 9, 655857), provides an important conceptual foundation for the present research. He develops the proposition that correlations need not be regarded merely as quantities measured between events whose spacetime geometry is already known. Instead, correlations may be treated as a more fundamental structure

from which spacetime distance, geometry, and ultimately the conventional descriptions of fields and matter emerge as approximate representations in appropriate physical regimes.

This perspective is particularly relevant to the present investigation, in which the internal geometric structure of a finite electromagnetic system is not introduced as an independently prescribed geometry. Rather, it is proposed to arise from the spatial and temporal organization of its electromagnetic field correlations.

# 4 Kempf's Central Ideas and their use in the present work

Three aspects of Kempf's framework are particularly relevant to the present research.

## 4.1 Correlation Strength Encodes Spacetime Distance

Kempf starts from the observation that quantum-field fluctuations are generally more strongly correlated at shorter spacetime separations. The two-point Feynman propagator can therefore encode information about the distance between its endpoints.

In this interpretation, the propagator is not simply a field-theoretic quantity defined on a pre-existing spacetime. Its short-distance behaviour also carries information about the geometric relations that characterize that spacetime.

## 4.2 Infinitesimal Correlations Contain Metric Information

The local metric can be recovered from the behaviour of a two-point function in the coincidence limit, where its two arguments approach one another.

This is important for the elementary two-point segment considered in our model. It indicates that the reconstruction of local geometric information does not necessarily require beginning with an extended object or an independently specified spatial geometry. An infinitesimal neighbourhood of two correlated points can, under the appropriate assumptions, contain sufficient information for local metric reconstruction.

For the present electromagnetic system, this motivates treating the correlation between two infinitesimally separated field-measurement points as the elementary geometric element. A distributed family of such two-point correlations can then provide the basis for reconstructing the effective geometry associated with the entire dielectric structure.

## 4.3 Correlators May Be More Fundamental Than Spacetime

Kempf advances the broader proposition that abstract two-point and higher-point correlators may constitute the primary structure, with quantum fields and curved spacetime emerging as approximate representations in suitable physical regimes.

This reverses the usual explanatory order: rather than assuming spacetime first and defining correlations within it, one investigates whether the correlation structure itself can provide the information from which spacetime geometry is reconstructed.

This aspect is especially significant for our hypothesis. The electromagnetic correlations are not being treated merely as observables inside an otherwise independently defined internal geometry. They are the quantities from which that geometry is proposed to emerge.

## 4.4 The Principal Mathematical Relationships

There are two important levels of metric reconstruction in Kempf's discussion: reconstruction from Synge's world function and reconstruction from the short-distance behaviour of the Feynman propagator.

## 4.5 Reconstruction from Synge's World Function

Let $\sigma(x, x')$ denote Synge's world function, conventionally defined as one-half the signed squared geodesic distance between two points.

With the usual coincidence-limit convention, the metric is obtained from

$$\boxed{g_{\mu\nu}(x) = -\lim_{x' \to x} \nabla_\mu \nabla_{\nu'} \sigma(x, x').} \tag{1}$$

This relationship expresses the local metric through a two-point geometric quantity.

The unprimed covariant derivative, $\nabla_\mu$, acts at the point $x$, while the primed covariant derivative, $\nabla_{\nu'}$, acts at the point $x'$. Their coincidence limit extracts the local geometric information associated with the two-point separation.

The significance for the present model is that a local metric can be associated with the limiting behaviour of a two-point relationship. The geometric description is therefore connected directly to the structure of relations between neighbouring points.

## 4.6 Reconstruction from the Feynman Propagator

For a massless scalar field in $D$ spacetime dimensions, the short-distance singularity of the propagator has the characteristic dependence

$$G_F(x, x') \sim [\sigma(x, x')]^{-\frac{D-2}{2}}, \tag{2}$$

up to normalization, signature, and propagator conventions.

Consequently, the corresponding inverse power of the propagator recovers a quantity proportional to the world function:

$$G_F(x, x')^{-\frac{2}{D-2}} \propto \sigma(x, x'). \tag{3}$$

The resulting metric reconstruction therefore has the structure

$$\boxed{g_{\mu\nu}(x) \propto \lim_{x' \to x} \nabla_\mu \nabla_{\nu'} \left[ G_F(x, x')^{-\frac{2}{D-2}} \right].} \tag{4}$$

The proportionality constant is fixed by the propagator normalization and the conventions used for the world function and the metric.

In four spacetime dimensions, $D = 4$, so that

$$-\frac{2}{D-2} = -1. \tag{5}$$

The inverse power is therefore simply $-1$, giving the inverse-propagator construction that motivates the metric-reconstruction expression employed in our framework.

Kempf emphasizes that the propagator contains information about both the light-cone structure and the remaining conformal scale of the metric. He also discusses why the reconstruction can be independent of the choice of vacuum representation, despite differences between propagators by homogeneous solutions.

These observations support the broader idea that local geometric information can be encoded in field correlations rather than requiring the metric to be treated as the sole fundamental input.

A further point relevant to our construction is Kempf's discussion of tensorial and spinorial propagators. After suitable scalar contractions, these objects may also provide sources of metric information.

This is particularly interesting for our electromagnetic approach, because our starting quantity is not a scalar-field propagator but a tensor-valued electric-field correlation function. The extension from scalar propagators to tensor-valued electromagnetic correlations is therefore a natural direction of investigation, although the appropriate contraction, transport prescription, and reconstruction conditions must be specified for the electromagnetic case.

## 4.7 Relevance to the Present Electromagnetic Correlation Geometry

The connection between Kempf's framework and the present investigation can be expressed through the replacement of an independently prescribed internal metric by a metric reconstructed from electromagnetic correlations.

In the present model, the fundamental two-point quantity is the electromagnetic correlator

$$W_{\mu\nu\rho'\sigma'}(x,x') = \left\langle \hat{F}_{\mu\nu}(x)\hat{F}_{\rho'\sigma'}(x') \right\rangle_c . \tag{6}$$

A suitable tensor contraction and scalar projection define the corresponding scalarized correlator,

$$G(x,x') = \mathcal{N}\, h^{\mu\rho}(x) P_\rho{}^{\nu'}(x,x') W_{\mu\nu'}(x,x'). \tag{7}$$

The internal metric is then proposed to follow from the inverse-correlator reconstruction,

$$\boxed{q_{\mu\nu}(x) = -\frac{1}{8\pi^2} \lim_{x'\to x} \nabla_\mu \nabla_{\nu'} \left[ G(x,x')^{-1} \right].} \tag{8}$$

This construction adopts the central conceptual direction of Kempf's proposal while investigating its application to an electromagnetic system with an asymmetric dielectric structure.

The resulting geometry is not assumed independently of the electromagnetic field. Instead, it is reconstructed from the spatial and temporal organization of the field correlations.

Once the internal metric has been obtained, its affine connection and curvature can be calculated, allowing the correlation-defined segment to be transported and its differential deformation to be investigated within an external spacetime.

The distinction is that Kempf provides the conceptual and mathematical motivation for correlation-based metric reconstruction, whereas the present research investigates a

specific electromagnetic realization involving the full tensor-valued two-point function, its scalar projection, and the subsequent construction of an internal geometrical transport framework.

Let two points defining the segment be $P$ and $Q$, with their positions specified along the electric field inside the asymmetric dielectric.

The starting correlation strength is

$$\boxed{\mathcal{C}(P,Q) = \left(\frac{V_{PQ}}{L_{PQ}}\right)^2 + \frac{\hbar c}{\pi^2 \epsilon_0 R_{PQ}^4}} \tag{9}$$

Here, the quantum term is retained as the contribution associated with the electromagnetic vacuum two-point correlation.

However, the fundamental object to be transported is not this scalar correlation strength alone. The relevant quantity is the electromagnetic two-point function,

$$\boxed{W_{\mu\nu\rho'\sigma'}(P,Q) = \left\langle \hat{F}_{\mu\nu}(P)\hat{F}_{\rho'\sigma'}(Q)\right\rangle_c} \tag{10}$$

The indices at $P$ and $Q$ belong to different tangent spaces. This distinction is essential because the correlator connects two distinct spacetime points.

For the asymmetric capacitor, the electromagnetic field operator includes both the applied field and its quantum fluctuations:

$$\hat{F}_{\mu\nu} = F_{\mu\nu}^{\text{applied}} + \delta\hat{F}_{\mu\nu}. \tag{11}$$

Consequently, the two-point function retains the classical contribution, the quantum contribution, and their cross-correlations wherever they are nonzero.

No tensor components are discarded, and the correlator is not replaced by a single averaged electromagnetic field.

2. Reconstructing the Internal Segment

The correlation between $P$ and $Q$ defines the geometric separation through a correlation-distance reconstruction.

For a scalarized correlator $G(P,Q)$, the four-dimensional reconstruction prescription is

$$\boxed{q_{\mu\nu}(P) = -\frac{1}{8\pi^2}\lim_{Q\to P}\nabla_\mu\nabla_{\nu'}\left[G(P,Q)^{-1}\right]} \tag{12}$$

If the scalarized electromagnetic correlator is identified with the proposed correlation strength, the corresponding matching condition is

$$\boxed{\mathcal{N}\, h^{\mu\rho}(x)\, P_{\rho}{}^{\nu'}(x,x')\, W_{\mu\nu'}(x,x') = \left(\frac{V(x,x')}{L(x,x')}\right)^{2} + \frac{\hbar c}{\pi^{2}\epsilon_{0} R(x,x')^{4}}.}$$

(13)

Under this identification, the proposed correlation-distance metric reconstruction takes the explicit form

$$\boxed{q_{\mu\nu}(x) = -\frac{1}{8\pi^{2}} \lim_{x'\to x} \nabla_{\mu}\nabla_{\nu'} \left[ \left( \left(\frac{V(x,x')}{L(x,x')}\right)^{2} + \frac{\hbar c}{\pi^{2}\epsilon_{0} R(x,x')^{4}} \right)^{-1} \right].} \tag{14}$$

The normalization and tensor projection must be fixed consistently with the chosen electromagnetic correlator. Equation (4) is the scalar-correlator reconstruction prescription; its application to electromagnetism requires specifying the corresponding projection of the full two-point function.

The resulting $q_{\mu\nu}$ is the internal metric reconstructed from the correlation structure, rather than an independently imposed metric.

For the complete segment, the two-point geometry must be retained before taking the coincidence limit. In particular, the finite separation between $P$ and $Q$ must remain available throughout the transport calculation.

The internal line element is then obtained from the reconstructed geometry:

$$d\ell_{q}^{2} = q_{\mu\nu}\, dx^{\mu} dx^{\nu}. \tag{15}$$

Because the dielectric is asymmetric, the correlation geometry need not be spatially uniform. Its longitudinal and transverse variations must both be retained.

## 4.8 Constructing the Internal Affine Connection

The internal affine connection is calculated directly from the reconstructed geometry and its derivatives:

$$\boxed{\mathcal{A}^{\rho}{}_{\mu\nu} = \frac{1}{2} q^{\rho\lambda} \left( \partial_\mu q_{\lambda\nu} + \partial_\nu q_{\lambda\mu} - \partial_\lambda q_{\mu\nu} \right)} \tag{16}$$

This is the torsion-free, metric-compatible connection associated with $q_{\mu\nu}$.

Since the internal metric is reconstructed from the electromagnetic correlators, its derivatives necessarily carry the spatial and temporal variations of the underlying field correlations.

In particular, differentiating the correlator-derived geometry requires retaining the derivatives of the applied electric field, the derivatives of the quantum two-point function, mixed derivatives involving both spacetime arguments, derivatives of the asymmetric dielectric profile, and derivatives of the inverse correlation entering the reconstruction.

No uniform-field approximation is introduced.

## 4.9 Embedding the Segment in the External Spacetime

Let the external spacetime have metric $g_{\mu\nu}$ and Levi-Civita connection $\Gamma^{\rho}{}_{\mu\nu}$.

The internal and external connections are generally different. Their difference is defined by

$$\boxed{C^{\rho}{}_{\mu\nu} = \mathcal{A}^{\rho}{}_{\mu\nu} - \Gamma^{\rho}{}_{\mu\nu}} \tag{17}$$

This difference tensor is the central quantity governing transport. It describes how parallel transport in the internal correlation geometry differs from parallel transport in the external spacetime.

For any vector carried by the segment,

$$\boxed{\frac{D_{\mathcal{A}} X^{\rho}}{d\tau} = \frac{D_g X^{\rho}}{d\tau} + C^{\rho}{}_{\mu\nu} u^{\mu} X^{\nu}} \tag{18}$$

where $u^{\mu}$ denotes the transport tangent.

This relation applies to every vector in the transported structure, rather than exclusively to its reference point.

## 4.10 Transporting the Complete Segment

The segment is described by its two endpoints and their connecting separation vector. Let $x_P^\mu(\tau)$ and $x_Q^\mu(\tau)$ denote the transported endpoint trajectories, with their local separation represented by $\xi^\mu$.

The segment follows the internal affine connection according to the autoparallel equation

$$\boxed{\frac{d^2x^\rho}{d\tau^2} + \mathcal{A}^\rho{}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 0} \tag{19}$$

Expressed in terms of the external spacetime connection, this becomes

$$\boxed{\frac{D_g u^\rho}{d\tau} = -C^\rho{}_{\mu\nu}u^\mu u^\nu} \tag{20}$$

Thus, the internal-external connection difference determines the external covariant acceleration of the transported trajectory under the prescribed affine transport law.

The segment need not be identified with a conventional material point.

## 4.11 Transporting the Segment's Internal Deformation

The separation vector itself evolves according to the curvature of the internal connection:

$$\boxed{\frac{D_{\mathcal{A}}^2\xi^\rho}{d\tau^2} = -R[\mathcal{A}]^\rho{}_{\sigma\mu\nu}u^\sigma\xi^\mu u^\nu} \tag{21}$$

This equation governs the relative evolution of neighbouring elements of the segment.

The internal curvature tensor is defined by

$$\boxed{\begin{aligned} R[\mathcal{A}]^\rho{}_{\sigma\mu\nu} = \partial_\mu\mathcal{A}^\rho{}_{\nu\sigma} - \partial_\nu\mathcal{A}^\rho{}_{\mu\sigma} \\ + \mathcal{A}^\rho{}_{\mu\lambda}\mathcal{A}^\lambda{}_{\nu\sigma} - \mathcal{A}^\rho{}_{\nu\lambda}\mathcal{A}^\lambda{}_{\mu\sigma}. \end{aligned}} \tag{22}$$

It contains first derivatives of the connection and products of connection coefficients. Since the connection is constructed from the correlation-derived metric, the resulting curvature incorporates second derivatives of the correlation geometry.

This dependence establishes why the complete electromagnetic two-point correlator and its derivatives must be retained throughout the calculation.

## 4.12 Expressing Internal Curvature Through External Curvature

Using the connection difference defined in equation (7), the internal curvature can be expressed in terms of the external curvature and the derivatives and products of $C^{\rho}{}_{\mu\nu}$:

$$\boxed{\begin{aligned} R[\mathcal{A}]^{\rho}{}_{\sigma\mu\nu} = {} & R[g]^{\rho}{}_{\sigma\mu\nu} \\ & + 2\nabla_{[\mu} C^{\rho}{}_{\nu]\sigma} \\ & + 2C^{\rho}{}_{[\mu|\lambda|} C^{\lambda}{}_{\nu]\sigma}. \end{aligned}} \tag{23}$$

This identity provides the central relationship for transporting the correlation-defined segment through an external gravitational spacetime.

It contains three distinct contributions: the curvature of the external spacetime, the spacetime variation of the internal-external connection difference, and the nonlinear products of the connection difference.

The quadratic contribution is essential and cannot be discarded merely because the first-order connection difference is small.

## 4.13 Incorporating the Electromagnetic Two-Point Correlator into the Transport Equations

The connection difference introduced in equation (7) is not an independent input. It must be calculated from the electromagnetic two-point correlator through the reconstruction of the internal geometry.

The complete dependency is therefore

$$\boxed{W_{\mu\nu\rho'\sigma'}(x,x') \longrightarrow G(x,x') \longrightarrow q_{\mu\nu} \longrightarrow \mathcal{A}^{\rho}{}_{\mu\nu} \longrightarrow C^{\rho}{}_{\mu\nu} \longrightarrow R[\mathcal{A}].} \tag{24}$$

Here, $G(x,x')$ denotes the scalarized correlator obtained through the specified tensor projection of the electromagnetic two-point function $W_{\mu\nu\rho'\sigma'}(x,x')$.

This dependency makes explicit that the internal metric, its affine connection, the connection difference, and the resulting curvature are all derived quantities determined by the electromagnetic correlations.

Accordingly, the segment transport equations are not equations with an independently prescribed connection. They are equations whose geometric coefficients are functionals of the electromagnetic two-point correlator.

The reference trajectory consequently takes the form

$$\boxed{\frac{D_g u^\rho}{d\tau} = -\left[\mathcal{A}^\rho{}_{\mu\nu}[W] - \Gamma^\rho{}_{\mu\nu}[g]\right] u^\mu u^\nu.} \tag{25}$$

The notation $\mathcal{A}^\rho{}_{\mu\nu}[W]$ emphasizes that the internal affine connection is calculated from the electromagnetic two-point function through the correlation-distance reconstruction.

Likewise, the complete segment deformation equation becomes

$$\boxed{\frac{D^2_{\mathcal{A}[W]}\xi^\rho}{d\tau^2} = -R[\mathcal{A}[W]]^\rho{}_{\sigma\mu\nu} u^\sigma \xi^\mu u^\nu.} \tag{26}$$

Thus, both the reference trajectory and the relative deformation of the segment are determined by the same underlying electromagnetic two-point correlator.

The correlator is therefore the common geometrical input to the entire transport system, rather than an auxiliary quantity introduced only at the initial stage.

## 4.14 The Complete Correlator-Dependent Transport System

The full calculation is organized as a sequence of geometrical constructions and transport relations, beginning with the electromagnetic two-point function and culminating in the evolution of the finite spacetime segment.

The initial stage consists of determining the electromagnetic two-point function,

$$W_{\mu\nu\rho'\sigma'}(P, Q), \tag{27}$$

including the contributions associated with the applied electric field and its quantum fluctuations. This correlator provides the fundamental information from which the internal correlation geometry is reconstructed.

The second stage consists of the correlation-distance reconstruction,

$$W_{\mu\nu\rho'\sigma'}(P, Q) \longrightarrow G(P, Q) \longrightarrow q_{\mu\nu}, \tag{28}$$

which defines the internal geometry of the segment.

The third stage involves differentiating the reconstructed metric to obtain the internal affine connection,

$$q_{\mu\nu} \longrightarrow \mathcal{A}^{\rho}{}_{\mu\nu}. \tag{29}$$

The fourth stage establishes the relationship between the internal and external geometries through the connection difference,

$$C^{\rho}{}_{\mu\nu} = \mathcal{A}^{\rho}{}_{\mu\nu} - \Gamma^{\rho}{}_{\mu\nu}. \tag{30}$$

The fifth stage determines the reference trajectory through the external covariant transport equation,

$$\frac{D_g u^{\rho}}{d\tau} = -C^{\rho}{}_{\mu\nu} u^{\mu} u^{\nu}. \tag{31}$$

The sixth stage calculates the internal curvature,

$$R[\mathcal{A}]^{\rho}{}_{\sigma\mu\nu}, \tag{32}$$

which incorporates the derivatives and nonlinear products of the internal connection.

The seventh stage expresses this curvature relative to the external gravitational background,

$$\begin{aligned} R[\mathcal{A}]^{\rho}{}_{\sigma\mu\nu} = R[g]^{\rho}{}_{\sigma\mu\nu} + 2\nabla_{[\mu} C^{\rho}{}_{\nu]\sigma} \\ + 2C^{\rho}{}_{[\mu|\lambda|} C^{\lambda}{}_{\nu]\sigma}. \end{aligned} \tag{33}$$

Finally, the eighth stage determines the differential evolution of the entire segment through its separation vector,

$$\frac{D^2_{\mathcal{A}} \xi^{\rho}}{d\tau^2} = -R[\mathcal{A}]^{\rho}{}_{\sigma\mu\nu} u^{\sigma} \xi^{\mu} u^{\nu}. \tag{34}$$

These stages constitute a single correlator-dependent geometrical transport construction. Each quantity is derived from the preceding one, and the final deformation equation retains the dependence on the electromagnetic two-point function through the reconstructed metric, connection, and curvature.

## 4.15 Transporting the Correlation-Defined Segment

Let $x, x'$ denote the two endpoints in the original correlation geometry, and let $X, X'$ denote their corresponding transported locations in the external spacetime.

The transport procedure acts on the complete electromagnetic two-point function, rather than independently transporting a reconstructed scalar metric. The transported correlator is defined by

$$\boxed{\widetilde{W}_{\alpha\beta'}(X,X') = J_\alpha{}^\mu(X,x)\, J_{\beta'}{}^{\nu'}(X',x')\, W_{\mu\nu'}(x,x').} \tag{35}$$

Here, $J_\alpha{}^\mu(X,x)$ and $J_{\beta'}{}^{\nu'}(X',x')$ denote the transport maps associated with the two endpoints, respectively. These maps act independently at their corresponding spacetime points and are not assumed to be identical.

The scalar correlator in the transported geometry is subsequently reconstructed from the transported tensor:

$$\boxed{\widetilde{G}(X,X') = \mathcal{N}\, \widetilde{h}^{\alpha\gamma}(X)\, \widetilde{P}_\gamma{}^{\beta'}(X,X')\, \widetilde{W}_{\alpha\beta'}(X,X').} \tag{36}$$

Here, $\mathcal{N}$ denotes the normalization associated with the scalar projection, $\widetilde{h}^{\alpha\gamma}$ specifies the relevant tensor contraction, and $\widetilde{P}_\gamma{}^{\beta'}$ provides the appropriate bitensorial identification between the tangent spaces at $X$ and $X'$.

The transported internal metric is then reconstructed from the transported scalar correlator:

$$\boxed{\widetilde{q}_{\alpha\beta}(X) = -\frac{1}{8\pi^2} \lim_{X'\to X} \widetilde{\nabla}_\alpha \widetilde{\nabla}_{\beta'} \left[\widetilde{G}(X,X')^{-1}\right].} \tag{37}$$

The essential feature of this construction is the ordering of the operations: the electromagnetic two-point correlator is transported first, and the internal geometry is reconstructed afterward. Consequently, the resulting metric incorporates the transformation of the complete two-point correlation structure between the original and transported endpoints.

## 4.16 Transporting the Complete Derivative Structure

The reconstruction of the transported geometry requires retaining not only the transported two-point function but also its complete derivative structure. In particular, the first covariant derivative is

$$\widetilde{\nabla}_\lambda \widetilde{W}_{\mu\nu'}(X,X'), \tag{38}$$

while the second covariant derivative is

$$\widetilde{\nabla}_\kappa \widetilde{\nabla}_\lambda \widetilde{W}_{\mu\nu'}(X,X'). \tag{39}$$

These derivatives contain contributions from the spacetime dependence of both endpoint transport maps, the derivatives of the original electromagnetic correlator, and the mixed products generated by differentiating the transported tensor.

Schematically, the derivative structure therefore retains

$$\partial J\, J\, W, \qquad J\, \partial J\, W, \qquad J\, J\, \nabla W, \tag{40}$$

together with the corresponding second-derivative contributions,

$$\partial^2 J\, J\, W, \qquad J\, \partial^2 J\, W, \qquad \partial J\, \partial J\, W, \tag{41}$$

and the terms involving first and second derivatives of the original correlator.

All contributions arising from the differentiation of the transported two-point function must be retained. In particular, no mixed derivative terms or products of derivatives of the endpoint transport maps are discarded.

This ensures that the reconstructed metric and its subsequent derivatives consistently incorporate the full spacetime dependence of the transported electromagnetic correlation structure.

## 4.17 Reconstructing the Transported Connection and Curvature

Once the transported internal metric has been obtained, the corresponding internal affine connection follows from its covariant derivatives:

$$\boxed{\widetilde{\mathcal{A}}^{\rho}{}_{\mu\nu} = \frac{1}{2}\widetilde{q}^{\rho\lambda}\left(\widetilde{\nabla}_{\mu}\widetilde{q}_{\lambda\nu} + \widetilde{\nabla}_{\nu}\widetilde{q}_{\lambda\mu} - \widetilde{\nabla}_{\lambda}\widetilde{q}_{\mu\nu}\right).} \tag{42}$$

Here, $\widetilde{q}^{\rho\lambda}$ denotes the inverse transported internal metric, and $\widetilde{\nabla}_{\mu}$ is the covariant derivative associated with the specified background connection used in the transported representation.

The transported connection is therefore determined by the transported correlator through the sequence

$$\boxed{\widetilde{W} \longrightarrow \widetilde{G} \longrightarrow \widetilde{q}_{\mu\nu} \longrightarrow \widetilde{\mathcal{A}}^{\rho}{}_{\mu\nu}.} \tag{43}$$

The corresponding transported curvature is constructed from this connection:

$$\boxed{\begin{aligned}\widetilde{R}[\widetilde{\mathcal{A}}]^{\rho}{}_{\sigma\mu\nu} = \widetilde{\nabla}_{\mu}\widetilde{\mathcal{A}}^{\rho}{}_{\nu\sigma} - \widetilde{\nabla}_{\nu}\widetilde{\mathcal{A}}^{\rho}{}_{\mu\sigma} \\ + \widetilde{\mathcal{A}}^{\rho}{}_{\mu\lambda}\widetilde{\mathcal{A}}^{\lambda}{}_{\nu\sigma} - \widetilde{\mathcal{A}}^{\rho}{}_{\nu\lambda}\widetilde{\mathcal{A}}^{\lambda}{}_{\mu\sigma}.\end{aligned}} \tag{44}$$

Thus, the transported curvature is not an independently prescribed geometrical quantity. It is derived from the transported electromagnetic two-point function through the reconstructed metric and affine connection.

The complete procedure consequently preserves the dependence of the internal geometry on the original electromagnetic correlations while allowing the entire finite segment, including its endpoint correlations and differential structure, to be described in the external spacetime.

Can a finite device whose internal spacetime geometry is reconstructed from electromagnetic correlations follow an orbit in a planet's external gravitational field, while its internal geometry modifies the orbital radius and orbital frequency?

Furthermore, when the internal geometry is switched off, does the device retain its instantaneous orbital velocity and position, rather than undergoing an artificial reset?

These questions can be investigated without assuming the outcome in advance or dismissing the proposed mechanism. However, it is necessary to distinguish a mathematically derived prediction from a physical hypothesis and to establish precisely which equations determine the resulting dynamics.

## 4.18 Explicit Expansion of the Two-Endpoint Transport

We begin with the complete transported electromagnetic two-point tensor:

$$\widetilde{W}_{\alpha\beta'}(X,X') = J_{\alpha}{}^{\mu}(X,x)J_{\beta'}{}^{\nu'}(X',x') \quad \times \left\langle \hat{F}_{\mu\rho}(x)U^{\rho}(x)\hat{F}_{\nu'\sigma'}(x')U^{\sigma'}(x')\right\rangle_{c}.$$

For clarity, let the endpoint maps be smooth and locally invertible, with

$$\mathrm{x}=\mathrm{x}(\mathrm{X}), \qquad \mathrm{x'}=\mathrm{x'}(\mathrm{X'}).$$

The corresponding Jacobian maps act independently at the two endpoints.

The derivative with respect to the first endpoint is

$$\partial_{\kappa}\widetilde{W}_{\alpha\beta'} = \left(\partial_{\kappa}J_{\alpha}{}^{\mu}\right)J_{\beta'}{}^{\nu'}W_{\mu\nu'} \quad + J_{\alpha}{}^{\mu}J_{\beta'}{}^{\nu'}\frac{\partial x^{\lambda}}{\partial X^{\kappa}}\partial_{\lambda}W_{\mu\nu'}.$$

Similarly, differentiation with respect to the second endpoint gives

$$\partial_{\lambda'}\widetilde{W}_{\alpha\beta'} = J_{\alpha}{}^{\mu}\left(\partial_{\lambda'}J_{\beta'}{}^{\nu'}\right)W_{\mu\nu'} \quad + J_{\alpha}{}^{\mu}J_{\beta'}{}^{\nu'}\frac{\partial x'^{\sigma'}}{\partial X'^{\lambda'}}\partial_{\sigma'}W_{\mu\nu'}.$$

The mixed second derivative is particularly important because it retains the contributions arising from the independent differentiation of both endpoint transport maps and the original two-point correlator.

Explicitly,

$$\boxed{\begin{aligned}&\partial_\kappa \partial_{\lambda'} \widetilde{W}_{\alpha\beta'} \\ &= (\partial_\kappa J_\alpha{}^\mu) \left(\partial_{\lambda'} J_{\beta'}{}^{\nu'}\right) W_{\mu\nu'} \\ &+ (\partial_\kappa J_\alpha{}^\mu)\, J_{\beta'}{}^{\nu'} \frac{\partial x'^{\sigma'}}{\partial X'^{\lambda'}} \partial_{\sigma'} W_{\mu\nu'} \\ &+ J_\alpha{}^\mu \left(\partial_{\lambda'} J_{\beta'}{}^{\nu'}\right) \frac{\partial x^\lambda}{\partial X^\kappa} \partial_\lambda W_{\mu\nu'} \\ &+ J_\alpha{}^\mu J_{\beta'}{}^{\nu'} \frac{\partial x^\lambda}{\partial X^\kappa} \frac{\partial x'^{\sigma'}}{\partial X'^{\lambda'}} \partial_\lambda \partial_{\sigma'} W_{\mu\nu'}.\end{aligned}} \tag{49}$$

Equation (49) contains four distinct contributions to the mixed derivative structure:

Simultaneous differentiation of both endpoint transport maps.

Differentiation of the first endpoint transport map combined with differentiation of the original correlator at the second endpoint.

Differentiation of the second endpoint transport map combined with differentiation of the original correlator at the first endpoint.

Simultaneous differentiation of the original correlator with respect to both of its spacetime arguments.

All four contributions must be retained when reconstructing the transported correlation geometry. In particular, the mixed derivative terms cannot be omitted without changing the derivative structure of the transported two-point function.

The resulting expression provides the explicit two-endpoint derivative structure required for the subsequent reconstruction of the transported scalar correlator, internal metric, affine connection, and curvature.

## 4.19 Physical and Geometrical Interpretation

The central result is that the electromagnetic correlation structure of the asymmetric dielectric defines an internal geometry through the two-point correlator. This geometry determines its own affine connection and curvature, which are then compared with those of the external spacetime.

The connection difference governs the transport of the reference trajectory, while the internal curvature determines the differential evolution of the separation vector and therefore the deformation of the complete finite segment.

The object under consideration is consequently not the transport of a capacitor as a conventional material body. Rather, the calculation concerns the evolution of a finite spacetime segment whose geometry is defined by electromagnetic two-point correlations.

The electromagnetic correlator is the underlying quantity from which the entire geometrical transport structure is constructed. Its spatial dependence, tensorial structure, finite-separation information, and derivatives enter successively into the reconstructed metric, the internal affine connection, the connection difference, and the curvature governing segment deformation.

# 5 Correlation-Defined Geometry and Its Effects on Orbital Dynamics

## 5.1 External spacetime and correlation-defined geometry

We consider an object moving in the external Schwarzschild spacetime of a planet of mass $M$, while its internal configuration is described by a correlation-defined geometry. The external and internal geometries are treated as distinct.

We retain the working relationship

$$q_{\mu\nu}(r) = \frac{1}{s(r)} g_{\mu\nu}(r), \qquad s(r) > 0, \tag{50}$$

where $s(r)$ is the correlation-strength factor, assumed to increase with voltage.

The assumptions are as follows:

- The external spacetime is described by the Schwarzschild geometry around a planet of mass $M$. - The internal correlation geometry is $q_{\mu\nu} = s(r)^{-1} g_{\mu\nu}$. - The correlation-strength factor $s(r)$ increases with voltage. - The center follows an affine geodesic of the correlation-defined geometry while the internal configuration is active. - When the internal configuration is switched off, the center continues with its acquired position and velocity in the external spacetime.

The external Schwarzschild line element is

$$ds_E^2 = -f(r)c^2dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2, \tag{51}$$

where

$$f(r) = 1 - \frac{2GM}{c^2 r}. \tag{52}$$

In the equatorial plane, $\theta = \pi/2$, the line element becomes

$$ds_E^2 = -fc^2 dt^2 + \frac{dr^2}{f} + r^2 d\varphi^2. \tag{53}$$

The correlation-defined line element is

$$ds_K^2 = \frac{1}{s(r)} ds_E^2. \tag{54}$$

Explicitly,

$$ds_K^2 = \frac{1}{s(r)} \left[ -fc^2 dt^2 + \frac{dr^2}{f} + r^2 d\varphi^2 \right]. \tag{55}$$

Define

$$\psi(r) = -\frac{1}{2} \ln s(r). \tag{56}$$

Then

$$ds_K^2 = e^{2\psi} ds_E^2. \tag{57}$$

The difference between the external and correlation-defined connections is

$$C^{\rho}{}_{\mu\nu} = \delta^{\rho}_{\mu} \partial_{\nu} \psi + \delta^{\rho}_{\nu} \partial_{\mu} \psi - g_{\mu\nu} \partial^{\rho} \psi. \tag{58}$$

Because the correlation strength varies radially, the connection difference is nonzero. Consequently, the correlation-defined geometry changes the geodesic equations followed by the center of the internal configuration.

## 5.2 Radial and circumferential radius transformations

The correlation-defined geometry changes not only the orbital velocity but also the geometric meaning of radius. In particular, the radial proper distance and the circumferential radius must be calculated independently.

Starting from the equatorial correlation-defined line element,

$$ds_K^2 = \frac{1}{s(r)}\left[-fc^2dt^2 + \frac{dr^2}{f} + r^2d\varphi^2\right], \tag{59}$$

the radial proper-distance element, obtained by setting $dt = d\varphi = 0$, is

$$ds_K^2 = \frac{dr^2}{f(r)s(r)}. \tag{60}$$

Therefore,

$$\boxed{d\ell_K = \frac{dr}{\sqrt{f(r)s(r)}}} \tag{61}$$

The circumferential radius is obtained from the angular component of the metric. At fixed $r$ and $t$, the circumference is

$$C_K(r) = 2\pi\sqrt{q_{\varphi\varphi}}. \tag{62}$$

Since

$$q_{\varphi\varphi} = \frac{r^2}{s(r)}, \tag{63}$$

we obtain

$$C_K(r) = 2\pi\sqrt{\frac{r^2}{s(r)}}. \tag{64}$$

Thus, the circumferential radius in the correlation-defined geometry is

$$\boxed{R_K(r) = \frac{r}{\sqrt{s(r)}}} \tag{65}$$

and the corresponding circumference is

$$\boxed{C_K(r) = 2\pi R_K(r) = \frac{2\pi r}{\sqrt{s(r)}}.} \tag{66}$$

This establishes that the same external coordinate position $r$ corresponds to a different circumferential radius in the correlation-defined geometry.

The radial distance is also changed:

$$d\ell_E = \frac{dr}{\sqrt{f(r)}}, \qquad d\ell_K = \frac{dr}{\sqrt{f(r)s(r)}}. \tag{67}$$

The radial conversion is therefore not generally identical to the circumferential rescaling. The two are determined by different metric components.

## 5.3 Derivation of the radius transformation

Starting from

$$R_K(r) = \frac{r}{\sqrt{s(r)}}, \tag{68}$$

differentiate with respect to the external radial coordinate $r$:

$$\frac{dR_K}{dr} = \frac{d}{dr}\left(rs^{-1/2}\right). \tag{69}$$

Applying the product rule gives

$$\frac{dR_K}{dr} = s^{-1/2} - \frac{r}{2}s^{-3/2}s'. \tag{70}$$

Factoring out $s^{-1/2}$,

$$\boxed{\frac{dR_K}{dr} = \frac{1}{\sqrt{s}}\left(1 - \frac{r}{2}\frac{s'}{s}\right)} \tag{71}$$

Define

$$\alpha(r) = \frac{r}{2}\frac{s'}{s}. \tag{72}$$

Then

$$\boxed{dR_K = \frac{1-\alpha}{\sqrt{s}}\,dr.} \tag{73}$$

The inverse transformation is

$$\boxed{\frac{dr}{dR_K} = \frac{\sqrt{s}}{1-\alpha}} \tag{74}$$

provided

$$1 - \alpha \neq 0. \tag{75}$$

The radial derivative therefore explicitly demonstrates how the correlation-strength gradient changes the relationship between the external coordinate radius and the circumferential radius of the internal geometry.

The sign of the correlation gradient determines the behavior of this transformation:

- If $s' < 0$, then $R_K$ increases more rapidly with external $r$. - If $s' > 0$, then $R_K$ increases more slowly. - If $s' = 0$, the radius is rescaled by a constant factor.

Thus, a spatially varying correlation strength changes the radius itself, rather than merely changing the numerical value of the orbital velocity.

## 5.4 Rewriting the contained geometry in terms of the transformed radius

Using $R_K$ as the radial coordinate, the correlation-defined metric can be written as

$$\boxed{ds_K^2 = -A(R_K)c^2dt^2 + B(R_K)dR_K^2 + R_K^2 d\varphi^2} \tag{76}$$

where

$$A = \frac{f}{s}, \qquad B = \frac{1}{f(1-\alpha)^2}. \tag{77}$$

To obtain the radial coefficient, use

$$dr = \frac{\sqrt{s}}{1-\alpha}\, dR_K. \tag{78}$$

The original radial metric component is

$$\frac{dr^2}{fs}. \tag{79}$$

Substituting the radial transformation gives

$$\frac{dr^2}{fs} = \frac{1}{fs}\left(\frac{\sqrt{s}}{1-\alpha}\right)^2 dR_K^2. \tag{80}$$

Hence,

$$B = \frac{1}{f(1-\alpha)^2}. \tag{81}$$

The angular component is exactly $R_K^2$, by the definition of the circumferential radius.

The transformed geometry therefore contains both a modified time component and a modified radial component. Its angular component is expressed directly in terms of the transformed circumferential radius.

## 5.5 Orbital equations in the correlation-defined geometry

We take the orbit in the equatorial plane:

$$\theta = \frac{\pi}{2}, \qquad \dot{\theta} = 0. \tag{82}$$

A dot denotes differentiation with respect to an affine parameter of the internal geometry.

The time equation is

$$\ddot{t} + \left(\frac{f'}{f} + 2\psi'\right)\dot{r}\dot{t} = 0. \tag{83}$$

It gives the conserved quantity

$$\boxed{E_K = s^{-1} f c^2 \dot{t}.} \tag{84}$$

The angular equation is

$$\ddot{\varphi} + 2\left(\frac{1}{r} + \psi'\right)\dot{r}\dot{\varphi} = 0, \tag{85}$$

giving

$$\boxed{L_K = s^{-1} r^2 \dot{\varphi}.} \tag{86}$$

The radial equation is

$$\boxed{\begin{aligned}&\ddot{r}+\left(\frac{ff'c^2}{2}+f^2c^2\psi'\right)\dot{t}^2\\&+\left(-\frac{f'}{2f}+\psi'\right)\dot{r}^2\\&-fr(1+r\psi')\dot{\varphi}^2=0.\end{aligned}} \tag{87}$$

The correlation gradient enters the time, radial, and angular equations. It therefore modifies the trajectory through the connection, rather than merely changing the numerical value of the interval.

## 5.6 Circular orbit with the correlation geometry active

For a circular orbit,

$$\dot{r}=0, \qquad \ddot{r}=0. \tag{88}$$

The radial equation becomes

$$\left(\frac{ff'c^2}{2}+f^2c^2\psi'\right)\dot{t}^2=fr(1+r\psi')\dot{\varphi}^2. \tag{89}$$

Therefore, the angular velocity measured in external coordinate time is

$$\boxed{\Omega_K^2=\left(\frac{d\varphi}{dt}\right)^2=\frac{c^2}{r}\frac{f'/2+f\psi'}{1+r\psi'}.}$$

Since

$$\psi'=-\frac{s'}{2s}, \tag{90}$$

we obtain

$$\boxed{\Omega_K^2=\frac{c^2}{r}\frac{f'-f\,s'/s}{2-r\,s'/s}.} \tag{91}$$

This is the angular frequency obtained directly from the affine geodesic equations in the correlation-defined geometry.

## 5.7 Circular-orbit condition using the transformed radius

The same circular-orbit condition can be expressed using the transformed metric

$$ds_K^2 = -A(R_K)c^2dt^2 + B(R_K)dR_K^2 + R_K^2 d\varphi^2. \tag{92}$$

For a circular orbit, the radial geodesic condition gives

$$\Omega_K^2 = \frac{c^2}{2R_K}\frac{dA}{dR_K}. \tag{93}$$

Using

$$A = \frac{f}{s} \tag{94}$$

and the radius transformation

$$\frac{dR_K}{dr} = \frac{1}{\sqrt{s}}\left(1 - \frac{r}{2}\frac{s'}{s}\right), \tag{95}$$

we have

$$\frac{dA}{dR_K} = \frac{dA}{dr}\frac{dr}{dR_K}. \tag{96}$$

The derivative of $A$ with respect to $r$ is

$$\frac{dA}{dr} = \frac{d}{dr}\left(\frac{f}{s}\right) = \frac{f'}{s} - \frac{fs'}{s^2}. \tag{97}$$

Using

$$\frac{dr}{dR_K} = \frac{\sqrt{s}}{1 - \frac{r}{2}(s'/s)}, \tag{98}$$

and

$$R_K = \frac{r}{\sqrt{s}}, \tag{99}$$

the circular-orbit condition becomes

$$\boxed{\Omega_K^2 = \frac{c^2}{2r}\frac{f' - f(s'/s)}{1 - \frac{r}{2}(s'/s)}} \tag{100}$$

This is the coordinate angular frequency with the changing circumferential radius explicitly included.

The locally measured tangential speed is

$$v_K^2 = \frac{R_K^2 \Omega_K^2}{A}. \tag{101}$$

Therefore,

$$\boxed{v_K^2 = \frac{c^2 r}{2f}\frac{f' - f(s'/s)}{1 - \frac{r}{2}(s'/s)}} \tag{102}$$

Both the circumference radius and the radial geometry enter this result.

## 5.8 Comparison with the ordinary Schwarzschild orbit

When the internal geometry is switched off, $s$ is constant, so

$$s' = 0. \tag{103}$$

The ordinary Schwarzschild circular-orbit frequency is

$$\boxed{\Omega_E^2 = \frac{c^2 f'}{2r} = \frac{GM}{r^3}.} \tag{104}$$

The ratio of the two frequencies is

$$\boxed{\frac{\Omega_K^2}{\Omega_E^2} = \frac{1 - f(s'/s)/f'}{1 - r(s'/s)/2}.} \tag{105}$$

This expression gives the frequency difference between the correlation-defined orbit and the ordinary Schwarzschild orbit at the same external coordinate radius.

Now consider the voltage assumption

$$\frac{ds}{dV} > 0. \tag{106}$$

If voltage and correlation strength are higher toward the planet, then

$$\frac{dV}{dr} < 0 \quad \Longrightarrow \quad \frac{s'}{s} < 0. \tag{107}$$

For an ordinary weak-field planetary orbit,

$$r \gg \frac{GM}{c^2}. \tag{108}$$

In that regime, a negative correlation gradient increases the orbital frequency in this model:

$$\boxed{\Omega_K > \Omega_E.} \tag{109}$$

The object therefore moves faster than the expected Schwarzschild circular orbit at the same external coordinate radius, within the stated model and its allowed circular-orbit range.

The locally measured tangential velocity is

$$v_K = \frac{r\Omega_K}{\sqrt{f}}. \tag{110}$$

Thus,

$$\boxed{v_K^2 = \frac{c^2 r}{f}\frac{f' - f\,s'/s}{2 - r\,s'/s}.} \tag{111}$$

For the ordinary Schwarzschild orbit,

$$v_E^2 = \frac{c^2 r f'}{2f}. \tag{112}$$

The comparison is therefore between the correlation-modified frequency and velocity and their ordinary Schwarzschild counterparts.

These are results for the radial conformal correlation model, not universal consequences of every possible correlation profile.

| **Configuration** | **Correlation gradient** | **Orbital result** |
|---|---|---|
| Weaker outward correlation | $s' < 0$ | Higher orbital frequency |
| Uniform correlation | $s' = 0$ | Ordinary Schwarzschild orbit |
| Stronger outward correlation | $s' > 0$ | Lower orbital frequency, within the allowed |

Table 1: Effect of correlation gradient on orbital frequency.

## 5.9 Radial motion and orbital stability

The circular-orbit equation alone does not determine whether an orbit is stable. The radial equation must also be considered.

Let a dot denote differentiation with respect to proper time in the contained geometry. The conserved quantities are

$$E_K = \frac{f}{s}c^2\dot{t}, \qquad L_K = \frac{r^2}{s}\dot{\varphi}. \tag{113}$$

The full radial equation is

$$\boxed{\dot{r}^2 = s^2\left(\frac{E_K^2}{c^2} - \frac{fL_K^2}{r^2}\right) - fsc^2.} \tag{114}$$

Define the radial function

$$\mathcal{F}(r) = s^2\left(\frac{E_K^2}{c^2} - \frac{fL_K^2}{r^2}\right) - fsc^2. \tag{115}$$

Then

$$\dot{r}^2 = \mathcal{F}(r). \tag{116}$$

A circular orbit at $r = r_0$ requires

$$\mathcal{F}(r_0) = 0, \qquad \mathcal{F}'(r_0) = 0. \tag{117}$$

For stability, the radial perturbation must be restoring:

$$\boxed{\mathcal{F}''(r_0) < 0.} \tag{118}$$

This condition depends on the profile of $s(r)$, including its derivatives. Thus, two correlation profiles can produce the same local circular speed but different orbital stability.

## 5.10 Switch-off of the correlation-defined geometry

Suppose the internal geometry is active at radius $r_0$, and the center has acquired a tangential velocity

$$v_1 = v_K(r_0). \tag{119}$$

The correlation-defined geometry is now switched off. The external gravitational field remains, but the internal correlation contribution disappears. The subsequent trajectory follows the external Schwarzschild geometry, with the acquired position and velocity retained as initial conditions.

The initial conditions are

$$r(0) = r_0, \qquad \dot{r}(0) = 0, \qquad v_\varphi(0) = v_1. \tag{120}$$

In the Newtonian approximation, the specific orbital energy after switch-off is

$$\varepsilon_1 = \frac{v_1^2}{2} - \frac{GM}{r_0}. \tag{121}$$

The new semimajor axis is

$$\boxed{a_1 = \frac{GM}{2GM/r_0 - v_1^2}.} \tag{122}$$

The eccentricity is

$$\boxed{e_1 = \frac{r_0 v_1^2}{GM} - 1} \tag{123}$$

for the case in which $v_1$ is above the ordinary circular velocity and below escape velocity.

Consequently,

$$r_{\text{peri}} = r_0, \qquad r_{\text{apo}} = a_1(1 + e_1). \tag{124}$$

The post-switch-off trajectory is now an ellipse with a larger apoapsis than the original circular orbit.

If

$$v_1 \geq \sqrt{\frac{2GM}{r_0}}, \tag{125}$$

the post-switch-off trajectory is unbound.

Equivalently, define the circular and escape velocities by

$$v_c = \sqrt{\frac{GM}{r_0}}, \qquad v_{\text{esc}} = \sqrt{\frac{2GM}{r_0}}. \tag{126}$$

For a tangential release with zero radial velocity and

$$v_c < v_1 < v_{\text{esc}}, \tag{45127}$$

the resulting orbit is elliptical:

$$\boxed{a = \frac{GM}{2GM/r_0 - v_1^2}} \tag{128}$$

$$\boxed{e = \frac{r_0 v_1^2}{GM} - 1} \tag{129}$$

and

$$r_{\text{peri}} = r_0, \qquad r_{\text{apo}} = a(1+e). \tag{130}$$

If

$$v_1 \geq v_{\text{esc}}, \tag{131}$$

the resulting orbit is unbound.

The important distinction is that the correlation-defined geometry can change the internal circumference radius and radial distance while active, while the transport dynamics can change the center's velocity. After switch-off, the acquired velocity determines the new external orbit.

## 5.11 Illustrative radial correlation profile

For a simple illustrative profile, take

$$s(r) = s_0 \exp\left(\frac{\beta}{r}\right). \tag{132}$$

Then

$$\frac{s'}{s} = -\frac{\beta}{r^2}, \qquad \alpha = -\frac{\beta}{2r}. \tag{133}$$

The contained circumferential radius becomes

$$\boxed{R_K = r \exp\left(-\frac{\beta}{2r}\right).} \tag{134}$$

The radial distance is

$$\boxed{d\ell_K = \frac{\exp[-\beta/(2r)]}{\sqrt{f}}\, dr.} \tag{134}$$

The orbital frequency becomes

$$\boxed{\Omega_K^2 = \frac{c^2}{2r}\frac{f' + f\beta/r^2}{1 + \beta/(2r)}.} \tag{135}$$

For $\beta > 0$, the correlation strength increases inward. The orbit's circumferential radius is smaller than the external Schwarzschild circumferential radius at the same $r$, while the angular-frequency expression also changes.

This example demonstrates that the correlation profile changes the radial and angular geometry simultaneously.

## 5.12 The three stages of the proposed orbital effect

The proposed orbital mechanism consists of three stages.

Stage 1: Internal geometry switched off

The object follows an ordinary circular orbit in the external Schwarzschild geometry:

$$v = v_E, \qquad r = r_0. \tag{136}$$

Stage 2: Internal geometry switched on

The correlation gradient modifies the orbital frequency:

$$v_K > v_E \tag{137}$$

for the inward-increasing correlation profile considered above.

The internal circumference radius is also changed:

$$R_K(r) = \frac{r}{\sqrt{s(r)}}. \tag{138}$$

The radial proper-distance element becomes

$$d\ell_K = \frac{dr}{\sqrt{f(r)s(r)}}. \tag{139}$$

Thus, both the geometry of the orbit and the orbital dynamics are modified.

Stage 3: Internal geometry switched off again

The acquired velocity remains. The external gravitational field now produces a larger elliptical orbit, or an unbound trajectory if the acquired velocity reaches or exceeds escape velocity.

## 5.13 Comparison of the external and contained geometries

The geometric and dynamical quantities can be compared as follows:

| **Quantity** | **External Geometry** | **Contained Correlation Geometr** |
|---|---|---|
| Circumferential radius | $r$ | $\frac{r}{\sqrt{s}}$ |
| Radial proper distance | $\frac{dr}{\sqrt{f}}$ | $\frac{dr}{\sqrt{fs}}$ |
| Temporal metric component | $-fc^2dt^2$ | $-\frac{fc^2dt^2}{s}$ |
| Circular orbital frequency | $\Omega^2 = \frac{c^2 f'}{2r}$ | Modified by $\frac{s'}{s}$ |
| Orbital stability | Schwarzschild effective potential | Depends on $s$, $s'$, and $s''$ |

Table 2: Comparison of geometric and orbital properties between the external and contained correlation geometries.

The comparison demonstrates that the correlation profile changes the geometry in which the complete orbit is defined. It changes the circumference assigned to a radius, the radial distance between neighboring positions, the circular-orbit condition, and the stability condition.

## 5.14 Central result

The correlation-defined geometry produces a specific relationship between the spatial correlation gradient and the orbital frequency:

$$\boxed{\frac{\Omega_K^2}{\Omega_E^2} = \frac{1 - f(s'/s)/f'}{1 - r(s'/s)/2}.} \tag{140}$$

It predicts a change in orbital dynamics when the correlation strength varies spatially.

At the same time, the circumferential radius is transformed according to

$$\boxed{R_K(r) = \frac{r}{\sqrt{s(r)}},} \tag{141}$$

with radial derivative

$$\boxed{\frac{dR_K}{dr} = \frac{1}{\sqrt{s}}\left(1 - \frac{r}{2}\frac{s'}{s}\right).} \tag{142}$$

The radial proper-distance element is independently transformed according to

$$\boxed{d\ell_K = \frac{dr}{\sqrt{f(r)s(r)}}.} \tag{143}$$

Consequently, the effect is not restricted to a change in orbital speed. The correlation profile changes the circumference radius, the radial distance, the orbital frequency, the locally measured tangential velocity, and the stability condition. When the internal geometry is switched off, the acquired velocity and radial state determine the subsequent orbit in the external Schwarzschild geometry.

A spatially uniform increase in correlation strength does not produce the same connection-induced orbital effect, because the gradient is what changes the connection and therefore the trajectory. A spatially varying correlation profile, by contrast, changes both the geometry of the orbit and the dynamical equations governing its motion.

# 6 Spatial Correlations of the Electric Field in a Voltage-Biased Dielectric Capacitor

## 6.1 Physical system and field decomposition

Consider a parallel-plate capacitor containing a dielectric medium, with conducting plates located at $z = 0$ and $z = d$. A voltage difference $V$ is applied between the plates, establishing an electric field within the intervening region. Under the ideal parallel-plate approximation, the classical electric field is approximately uniform and directed perpendicular to the plates:

$$\mathbf{E}_{\mathrm{dc}}(\mathbf{r}; V) \simeq \frac{V}{d}\,\hat{\mathbf{z}}. \tag{144}$$

To describe both the applied field and its quantum fluctuations, the electric-field operator is decomposed as

$$\boxed{\hat{\mathbf{E}}(\mathbf{r}, t) = \mathbf{E}_{\mathrm{dc}}(\mathbf{r}; V)\hat{I} + \hat{\mathbf{E}}_{\mathrm{quant}}(\mathbf{r}, t)} \tag{145}$$

where $\hat{I}$ denotes the identity operator and $\hat{\mathbf{E}}_{\mathrm{quant}}$ describes the fluctuating electromagnetic field around the voltage-dependent field configuration.

This decomposition distinguishes the macroscopic electromagnetic field established by the applied voltage from the quantum fluctuations associated with the electromagnetic modes of the system. The voltage-dependent contribution represents a coherent displacement of the field, whereas the quantum contribution contains the fluctuations and correlations determined by the electromagnetic state, material response, and boundary conditions.

The central physical quantity of interest is therefore not exclusively the electric-field magnitude at an individual position, but also the statistical relationship between field amplitudes evaluated at distinct spatial positions and, in general, at different times. Such relationships are characterized by two-point correlation functions.

## 6.2 Meaning and physical interpretation of two-point correlators

An electric-field measurement at a single point provides local information concerning the field amplitude, intensity, or fluctuation level at that position. A two-point correlator provides additional information by characterizing the relationship between the electromagnetic field evaluated at two distinct spatial positions.

For two observation points $\mathbf{r}_1$ and $\mathbf{r}_2$, their spatial separation is defined as

$$R = |\mathbf{r}_1 - \mathbf{r}_2|. \tag{146}$$

A two-point correlation function quantifies the statistical relationship between the field at $(\mathbf{r}_1, t_1)$ and the field at $(\mathbf{r}_2, t_2)$. Its dependence on $R$ characterizes the spatial structure and range of the correlations, while its dependence on the temporal separation $t_1 - t_2$ describes their evolution.

Within quantum electrodynamics, the electric field is represented by an operator, and its correlation functions are defined through expectation values evaluated in a specified quantum state. The choice of operator ordering determines which physical properties of the electromagnetic field are described.

The positive- and negative-frequency components of the electric-field operator are defined by

$$\hat{E}(\mathbf{r}, t) = \hat{E}^{(+)}(\mathbf{r}, t) + \hat{E}^{(-)}(\mathbf{r}, t), \qquad \hat{E}^{(-)} = \left(\hat{E}^{(+)}\right)^{\dagger}. \tag{147}$$

A commonly employed first-order correlation function is

$$\boxed{G^{(1)}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) = \left\langle \hat{E}^{(-)}(\mathbf{r}_1, t_1)\hat{E}^{(+)}(\mathbf{r}_2, t_2)\right\rangle.} \tag{148}$$

The expectation value is taken with respect to the quantum state describing the electromagnetic field. This correlator characterizes first-order spatial and temporal coherence by relating the field amplitude at one position to the conjugate field amplitude at another.

In optical and microwave measurements, first-order correlations are associated with interference and coherence phenomena. They determine whether electromagnetic amplitudes sampled at different positions can produce a stable interference contribution.

The diagonal quantity,

$$G^{(1)}(\mathbf{r}, t; \mathbf{r}, t), \tag{149}$$

is associated with the local normally ordered field intensity, whereas the off-diagonal quantity,

$$G^{(1)}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2), \qquad \mathbf{r}_1 \neq \mathbf{r}_2, \tag{150}$$

contains information about coherence and correlations between spatially separated points.

It is important to distinguish the first-order correlator from a connected fluctuation correlator. The latter removes the contribution associated with the product of the mean fields, thereby isolating correlations arising from fluctuations around the coherent electromagnetic background. This distinction becomes essential when separating the voltage-induced classical displacement from the nonclassical contribution.

## 6.3 Classical displacement and quantum contributions to the correlator

Using the field decomposition introduced in Eq. (A.2), the first-order correlation function can be separated into contributions associated with the voltage-dependent classical field and the quantum fluctuations.

For a displaced quantum state with vanishing mean fluctuation field, the resulting expression is

$$\boxed{\begin{aligned} G^{(1)}(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2) &= E^{*}_{\mathrm{dc}}(\mathbf{r}_1;V)E_{\mathrm{dc}}(\mathbf{r}_2;V) \\ &\quad + \left\langle \hat{E}^{(-)}_{\mathrm{quant}}(\mathbf{r}_1,t_1)\hat{E}^{(+)}_{\mathrm{quant}}(\mathbf{r}_2,t_2)\right\rangle . \end{aligned}} \tag{151}$$

The first term represents the coherent contribution associated with the applied voltage. For an ideal parallel-plate capacitor, its magnitude scales as

$$G^{(1)}_{\mathrm{classical}} \propto \left(\frac{V}{d}\right)^2 . \tag{152}$$

The second term describes the quantum correlation contribution, which contains the spatial and temporal structure of the electromagnetic fluctuations. Its properties depend on the quantum state, the electromagnetic mode structure, and the boundary conditions imposed by the conducting plates.

Equation (A.6) establishes an important distinction between the coherent field amplitude and the quantum fluctuation structure. The total two-point correlation function contains both contributions and therefore carries information beyond the local electric-field magnitude.

The connected first-order correlation function can be defined by subtracting the product of the mean field amplitudes:

$$\begin{aligned} G^{(1)}_{\mathrm{conn}} &= G^{(1)} \\ &\quad - \left\langle \hat{E}^{(-)}(\mathbf{r}_1,t_1)\right\rangle \left\langle \hat{E}^{(+)}(\mathbf{r}_2,t_2)\right\rangle . \end{aligned} \tag{153}$$

This quantity isolates correlations beyond the coherent mean field and is consequently useful for investigating the spatial organization of the quantum fluctuations themselves.

An important consequence of this decomposition is that the applied voltage constitutes a direct control parameter of the total two-point correlation function. Since the electric-field operator contains an explicitly voltage-dependent coherent contribution, the corresponding correlator necessarily acquires a voltage-dependent component.

For the ideal capacitor geometry, this direct contribution is proportional to $V^2/d^2$. Consequently, even if the quantum fluctuation covariance remains unchanged under a purely coherent displacement, the total first-order correlation function changes with the applied voltage.

This direct dependence must be distinguished from a second, potentially more significant mechanism: voltage-induced modifications of the quantum fluctuation correlations themselves.

## 6.4 Spatial dependence of vacuum correlations and the role of capacitor boundaries

Even in the absence of an applied voltage, the electromagnetic vacuum is characterized by nonvanishing field fluctuations. The ground state of the electromagnetic field possesses zero-point fluctuations that exhibit spatial correlations between distinct observation points.

For the free electromagnetic field, the equal-time vacuum correlation of electric-field components exhibits a characteristic inverse-fourth-power dependence on the separation between the points. In one commonly used convention, the nonlocal correlation is expressed as

$$\begin{aligned} &\left\langle 0 \left| \hat{E}_i(\mathbf{r},t)\hat{E}_j(\mathbf{r}',t) \right| 0 \right\rangle \\ &\qquad = -\frac{4\hbar c}{\pi}\left(\delta_{ij} - 2\hat{R}_i\hat{R}_j\right)\frac{1}{R^4}, \qquad R \neq 0, \end{aligned} \tag{154}$$

where

$$\mathbf{R} = \mathbf{r} - \mathbf{r}', \qquad R = |\mathbf{R}|, \qquad \widehat{\mathbf{R}} = \frac{\mathbf{R}}{R}. \tag{155}$$

The tensor factor describes the dependence on the orientation of the electric-field components relative to the separation direction. The characteristic spatial dependence is proportional to $R^{-4}$. The coincident-point singularity is excluded from this expression. See Passante and Rizzuto (2023).

In the capacitor geometry, the conducting plates impose electromagnetic boundary conditions that modify the allowed electromagnetic modes and consequently alter the spatial structure of the field correlations. The two-point correlation function inside the capacitor is therefore not generally identical to its free-space counterpart. Instead, it must incorporate the plate separation, the electromagnetic mode structure, and the positions of the observation points relative to the boundaries.

The quantum correlation function is consequently more generally expressed as

$$G^{(1)}_{\text{quant}} = G^{(1)}_{\text{quant}}(\mathbf{r}_1, \mathbf{r}_2; t_1, t_2; d). \tag{156}$$

In free space, translational invariance permits the spatial dependence to be expressed in terms of the separation vector alone. Between conducting plates, however, translational symmetry is restricted by the boundaries. Consequently, two pairs of observation points having identical separations need not exhibit identical correlations if their positions relative to the plates differ.

The conducting boundaries therefore play a dual physical role. They establish the macroscopic capacitor geometry while simultaneously determining the electromagnetic mode structure from which the quantum correlations emerge. This modification of vacuum fluctuations by conducting boundaries is closely related to the physical mechanisms underlying Casimir phenomena.

## 6.5 Voltage-induced displacement and voltage-dependent quantum correlations

When a voltage is applied across the capacitor, the electromagnetic field acquires a coherent component associated with the imposed electric-field configuration. Within the decomposition of Eq. (A.2), this contribution is represented by the voltage-dependent displacement $E_{\text{dc}}(\mathbf{r}; V)$.

For the ideal parallel-plate configuration, the coherent contribution to the first-order correlation scales as $V^2/d^2$, as established in Eq. (A.7). Increasing the applied voltage therefore modifies the coherent component of the total two-point correlation function.

However, the voltage dependence need not be restricted to the classical contribution. If the applied voltage modifies the electromagnetic mode dynamics, the effective Hamiltonian, the boundary conditions, or the coupling between electromagnetic and material degrees of freedom, it can also modify the quantum state and its associated fluctuation correlations.

This distinction is particularly relevant in systems exhibiting nonclassical electromagnetic

states. In a displaced quantum state, the coherent field amplitude and the fluctuations around that amplitude can be treated separately. If the field undergoes additional squeezing, the fluctuations become redistributed between conjugate field quadratures. One quadrature exhibits reduced fluctuations, while the conjugate quadrature exhibits increased fluctuations, subject to the uncertainty principle.

The resulting electromagnetic correlation structure can therefore contain two physically distinct but potentially coupled contributions:

1. Coherent contribution: Associated with the voltage-induced displacement of the electric field. For an ideal parallel-plate capacitor, this contribution provides an approximately uniform background between the plates.
2. Nonclassical contribution: Associated with quantum fluctuations and their spatial correlations. This contribution can exhibit spatially structured behavior, including squeezing-dependent modifications of the field-quadrature covariance.

The total correlation function can consequently be represented in the general form

$$\boxed{G^{(1)}_{\text{total}}(R;V) = G^{(1)}_{\text{classical}}(V) + G^{(1)}_{\text{quantum}}(R;V).} \tag{157}$$

The explicit voltage dependence of the quantum contribution is retained in this expression. The notation $R$ represents a simplified description of the spatial separation, whereas the complete capacitor geometry may require separate dependence on both observation positions, as indicated in Eq. (A.10).

The physical distinction between direct voltage dependence and voltage-controlled quantum correlations is important.

A purely mathematical displacement of a harmonic field, with the underlying Hamiltonian and quantum state of its fluctuations unchanged, modifies the mean field but leaves the connected covariance unchanged. Therefore, the existence of a voltage-dependent coherent contribution does not, by itself, establish voltage-dependent squeezing.

However, when the applied voltage modifies the physical dynamics of the electromagnetic system, the quantum covariance can also become voltage-dependent. This can occur through voltage-controlled parametric interactions, nonlinear coupling, changes in effective mode frequencies, or interactions between electromagnetic modes and quantum material degrees of freedom.

A particularly relevant example is provided by Mendes and Mora (2015), who demonstrated that parametric driving of a quantum conductor can produce a squeezed steady state of a microwave cavity, with the squeezing controlled by voltage excitation. Their results

establish a physical mechanism through which electrical control can influence nonclassical electromagnetic correlations.

Furthermore, the spatial structure of squeezed electromagnetic fields has been investigated by Lugiato and Gatti (1993), who demonstrated that squeezing in a degenerate parametric oscillator can exhibit spatial dependence rather than being characterized exclusively by a single global squeezing parameter.

These results provide a theoretical and experimental motivation for investigating voltage-dependent spatial correlations in dielectric electromagnetic systems. They suggest that the applied voltage may act not only as a control parameter of the coherent field displacement but also, under appropriate dynamical conditions, as a control parameter of the nonclassical correlation structure.

Accordingly, the voltage-dependent quantum contribution should not be assumed constant a priori. Its dependence on voltage must instead be determined from the electromagnetic dynamics, material coupling, and quantum state of the system.

## 6.6 Interpretation of the spatial-correlation profile

The qualitative behavior of the correlation function can be investigated by considering its dependence on the separation $R$ between two observation points.

For an ideal uniform applied field, the coherent contribution is approximately independent of the separation:

$$G^{(1)}_{\text{classical}}(R;V) \sim \left(\frac{V}{d}\right)^2 . \tag{158}$$

It therefore appears as an approximately constant background in a simplified spatial representation.

The quantum contribution, in contrast, depends on the spatial correlations of the electromagnetic field and is modified by the conducting boundaries and the quantum state. In a squeezed state, the relevant field-quadrature covariance may contain a reduced-fluctuation contribution over a characteristic spatial range. Its magnitude and spatial profile depend on the participating electromagnetic modes and on the mechanism responsible for producing the squeezing.

The total spatial correlation can therefore be represented schematically as

$$\boxed{G_{\text{total}}(R;V) = C_{\text{cl}}(V) + C_{\text{q}}(R;V).} \tag{159}$$

This representation distinguishes an approximately constant coherent background from a spatially varying quantum contribution.

The latter may itself depend on the applied voltage when the voltage modifies the electromagnetic dynamics or the quantum state. Consequently, changes in voltage may influence both the background level and the spatial structure of the total correlation function.

## 6.7 Choice of correlation function for the theoretical development

A further distinction is necessary when selecting the appropriate correlation function for a quantitative description of the electromagnetic vacuum.

The first-order correlator introduced in Eq. (A.5) is normally ordered. Consequently, for the vacuum state, its expectation value vanishes:

$$\left\langle 0 \left| \hat{E}^{(-)}(\mathbf{r}_1, t_1) \hat{E}^{(+)}(\mathbf{r}_2, t_2) \right| 0 \right\rangle = 0. \tag{160}$$

Nevertheless, the electromagnetic vacuum possesses nonzero field fluctuations and nonzero spatial correlations. These are described by unsymmetrized vacuum expectation values, such as the one introduced in Eq. (A.9), or by the symmetrized field correlation function,

$$\boxed{C_{ij}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) = \frac{1}{2} \left\langle \left\{ \delta\hat{E}_i(\mathbf{r}_1, t_1), \delta\hat{E}_j(\mathbf{r}_2, t_2) \right\} \right\rangle .} \tag{161}$$

Here, the fluctuation operator is defined as

$$\delta\hat{\mathbf{E}} = \hat{\mathbf{E}} - \langle \hat{\mathbf{E}} \rangle, \tag{162}$$

and the braces denote the anticommutator.

The distinction between these correlation functions identifies their respective physical roles:

- $G^{(1)}$ describes first-order coherence and is appropriate for normally ordered field correlations and the coherent contribution.
- The symmetrized or unsymmetrized field covariance describes vacuum and quantum fluctuations, including their spatial correlations.
- Quadrature covariance functions provide a particularly useful description for identifying and quantifying squeezing.

For a voltage-displaced state, the mean field and the fluctuation correlations can be treated separately. The coherent displacement determines the classical contribution, whereas the covariance describes the quantum fluctuations around that displaced field.

If the voltage additionally controls the dynamics responsible for squeezing, the covariance itself becomes voltage-dependent. In this case, the total correlation structure contains both the direct voltage-induced coherent contribution and an additional voltage-dependent modification of the quantum fluctuations.

This distinction provides a consistent mathematical foundation for investigating the dependence of electromagnetic correlations on the applied voltage, spatial separation, and nonclassical state of the field.

## 6.8 Experimental and theoretical context

Several studies provide relevant theoretical and experimental foundations for the physical mechanisms discussed in this appendix.

**Vacuum electric-field correlations.** Benea-Chelmus et al. experimentally investigated temporal and spatial correlations of electromagnetic vacuum fluctuations using electro-optic detection. Their work provides an important experimental reference for the interpretation and measurement of two-point field correlations.

Benea-Chelmus et al., Electric field correlation measurements on the electromagnetic vacuum state, Nature 568, 202–206 (2019).

DOI: 10.1038/s41586-019-1083-9.

**Nonlocal vacuum correlations.** Passante and Rizzuto examined spatial and temporal correlations of electromagnetic vacuum fields, including their equal-time spatial dependence and their relationship to Casimir–Polder interactions.

Passante and Rizzuto, Nonlocal Static and Dynamical Vacuum Field Correlations and Casimir–Polder Interactions, Entropy 25, 1424 (2023).

DOI: 10.3390/e25101424.

**Voltage-controlled squeezing.** Mendes and Mora demonstrated voltage-controlled generation of squeezed microwave cavity states through coupling to a driven quantum conductor. Their analysis provides a relevant example of electrical control influencing nonclassical electromagnetic fluctuations.

Mendes and Mora, Cavity squeezing by a quantum conductor, New Journal of Physics 17, 113014 (2015).

DOI: 10.1088/1367-2630/17/11/113014.

**Spatial structure of squeezed fields.** Lugiato and Gatti investigated the spatial distribution of squeezing in the field generated by a degenerate parametric oscillator, demonstrating the relevance of spatially dependent quantum correlations.

Lugiato and Gatti, Spatial structure of a squeezed vacuum, Physical Review Letters 70, 3868 (1993).

DOI: 10.1103/PhysRevLett.70.3868.

**Quantum fluctuations in dielectric media.** Gruner and Welsch investigated ground-state electric-field fluctuations in dispersive and lossy dielectric media, providing a theoretical reference for describing electromagnetic correlations in material environments.

Gruner and Welsch, Correlation of radiation-field ground-state fluctuations in a dispersive and lossy dielectric, Physical Review A 51, 3246–3256 (1995).

DOI: 10.1103/PhysRevA.51.3246.

Taken together, these studies establish the relevance of spatially correlated vacuum fluctuations, boundary-dependent electromagnetic correlations, voltage-controlled nonclassical states, and quantum fluctuations in dielectric environments. They provide a foundation for developing a more complete correlation-based description of voltage-biased electromagnetic systems.

## 6.9 Implications for the subsequent theoretical development

The preceding discussion establishes the physical quantities required to investigate spatial electromagnetic correlations inside a voltage-biased dielectric capacitor. The applied voltage defines a coherent electric-field displacement, while the electromagnetic field retains quantum fluctuations whose spatial structure depends on the observation positions, electromagnetic mode configuration, material properties, and boundary conditions imposed by the conducting plates.

The central object for the subsequent theoretical development is therefore a voltage-dependent two-point electric-field correlation function:

$$\boxed{G^{(1)}(R;V) = G^{(1)}_{\text{classical}}(V) + G^{(1)}_{\text{quantum}}(R;V).} \tag{163}$$

This expression emphasizes two potentially distinct mechanisms through which the applied voltage can influence the correlation structure.

First, the voltage directly modifies the coherent field displacement and therefore changes the classical contribution to the total correlator. This dependence follows from the field decomposition and is present independently of any modification of the quantum fluctuation state.

Second, when the voltage modifies the underlying electromagnetic dynamics or the coupling between electromagnetic and material degrees of freedom, it may also influence the quantum covariance. Such a mechanism can introduce voltage-dependent spatial correlations and, under suitable conditions, squeezing-dependent modifications of the electromagnetic field.