

Mach Effect Gravity Assist Warp Drive in Operational Quantum Gravity

Todd J. Desiato

Independent researcher, Statesville, North Carolina, USA

Correspondence: mainengineering@gmail.com

ORCID: 0000-0002-7514-0169

Manuscript version 0.8 · 21 October 2026

Abstract

We derive the work, retained impulse, and ideal thrust-to-power ratio of a prescribed Mach Effect Gravity Assist cycle within Operational Quantum Gravity. The supporting-power and axial matter-wave relations connect the material clock factor to state energy, inertia, and a spatial environmental force. During fast extension, the disk reaches minimum support at maximum speed, then continues outward while decelerating against a restoring gradient. Its lost kinetic energy and positive actuator work supply the restoration energy. A slower return at ambient support completes the cycle. For a quadratic support profile with explicit field evolution during release and stationary spatial restoration, the leading-order outward impulse is twice the restoration energy divided by peak disk speed. Under the stated external-reaction condition, this impulse remains after return and gives the average thrust. Reversible actuation with no separate net field-control cost gives an ideal thrust per electrical watt equal to the inverse force-weighted disk speed. At one metre per second, this benchmark is one newton per watt, approximately 300 million times the ideal photon-rocket value. The main text derives the gradient, force, work, and impulse in physical terms and relates them to terminal electrical power. The result specifies the consequences of the assumed environmental response; a device realization must establish that response and the complete apparatus energy and momentum balances.

Keywords: Operational Quantum Gravity; Mach Effect Gravity Assist; gravitational potential energy; electromagnetic scattering; matter-wave dispersion; thrust-to-power ratio

Mathematics Subject Classification (2020): 83C10; 70H05; 83C40

1 Introduction

This paper derives how the stated Mach Effect Gravity Assist (MEGA) cycle converts work against a supporting-field gradient into a retained impulse within Operational Quantum Gravity (OQG). The engineering question is whether the stack can continue pushing while the disk slows, and how that push becomes average thrust after the disk returns. We answer it by following the material state, the two forces on the disk, the work delivered by the actuator, and the momentum exchanged with the environment through one complete cycle.

Mach's principle connects inertia to surrounding matter. Sciama developed an explicit account of that connection [5], and James F. (Jim) Woodward developed its application to propulsion [6]. The Mach Effect Gravity Assist (MEGA) program was subsequently investigated through the National Aeronautics and Space Administration (NASA) Innovative Advanced Concepts (NIAC) program [1]. Operational Quantum Gravity (OQG) expresses the material connection through balanced supporting exchange with the ambient electromagnetic field. Scattering changes the exchange received by matter, and the resulting matter-wave dispersion determines its storage energy, inertial response, and gravitational force. These relationships provide the governing physics for the application developed here.

Here, vacuum means the low-gas environment of space or an evacuated laboratory chamber. In OQG, matter in either setting remains coupled to the ambient electromagnetic vacuum field that supplies its supporting exchange. We use *traction against the ambient vacuum field* to describe force transmitted through this coupling. Woodward and collaborators used a vacuum chamber for their bench apparatus [2]. Evacuation removes gas while the ambient vacuum field remains part of the OQG description. The field response in each setting depends on its surrounding matter and boundary conditions.

The underlying material relations are developed in the supporting-power manuscript [7]. Sections 3–5 explain how supporting exchange connects scattering to material energy, electromagnetic response, and axial inertia. Appendix A gives the corresponding-state microscopic response framework, while Appendix B specifies the axial material branch. Here we assume that the driven response produces the phase-dependent profile stated in Section 10, and derive the force and work that follow. This separates the prescribed operating load from the calculation needed to realize it with particular sources and boundary conditions.

The essential operation is a powered lift through gravitational potential energy. The outward-moving disk becomes inertially heavier as support decreases. It then slows while the stack continues pushing it back toward ambient support. The lost kinetic energy and the actuator's work both enter the restoration balance. The stack's opposite force acts on the ship throughout this lifting phase. The return takes place at ambient inertia. Whether the outward impulse remains after that return is determined by the total momentum transferred to the external supporting environment.

The calculation gives a conditional ideal advantage over a photon rocket: one newton per net electrical watt at a force-weighted disk speed of one metre per second. The physical sequence comes first in Sections 2–6. Sections 7–9 establish its force, work, and complete-cycle momentum balances. Section 10 then turns the selected support profile into a gradient and an explicit average-thrust formula. Sections 11–12 connect that result to net electrical input and quantify departures from the ideal benchmark. All motion considered here is subluminal.

2 The device and its operating cycle

The device consists of a piezoelectric stack between a massive ship and a much lighter end disk. Applying voltage makes the stack extend and retract. During extension, the stack pushes the disk away from the

ship and applies an opposite force to the ship. The propulsion calculation must determine how much of that ship impulse remains after the disk returns to its starting position and state.

OQG supplies the material and environmental response used in this calculation. Matter maintains its natural state through a balanced exchange with the surrounding electromagnetic field. Scattering and interference change the supporting exchange received by matter. The resulting change in material state alters its clock rate, dimensions, gravitational potential energy, and resistance to acceleration. A spatial variation of that state produces the gravitational dispersion force against which the actuator works.

Let $b > 0$ denote the material clock factor relative to its ambient reference state, where $b = 1$. The axial material branch used here also has longitudinal ruler factor b . When b decreases, the relevant electromagnetic permittivity and permeability increase, the disk's axial inertial mass increases, and its gravitational potential energy decreases. The connection between these changes will be derived after the physical cycle is established.

The outward stroke has two parts. During the first, the disk accelerates outward while its supporting state moves below ambient and its inertia increases. At maximum speed, support is at its minimum. During the second, the disk continues outward while the stack pushes it uphill toward ambient support. It slows to rest as its gravitational potential energy is restored. The stack then retracts at ambient inertia, using a slower, nearly constant-speed return between smooth starting and stopping intervals. This is the operating meaning of “push when it is heavy and pull when it is light.” Figure 1 in Section 10.5 places the motion, field response, contact force, and accumulated impulse on a common time axis.

The second part of extension is the gravity-assist portion of the stroke. Pushing the disk uphill in supporting state requires work, just as lifting a book requires work against gravity. The stack transmits the opposite contact force to the ship. The field and the matter coupled to it receive the environmental reaction momentum. Establishing that exchange across the complete cycle is essential: a change of internal inertia alone does not establish propulsion.

MEGA retains its established expansion, **Mach Effect Gravity Assist**. In the OQG application, *warp* describes the controlled change of electromagnetic response and material clock and ruler standards represented by the effective spacetime geometry. The calculation concerns subluminal mechanical motion.

A pendulum suggests the exchange between kinetic and potential energy, but the driven cycle also exchanges energy and momentum with its environment. The calculation below follows those transfers explicitly. The resulting average thrust depends on the impulse that remains after every part of the apparatus has completed its cycle.

3 Ambient support and gravitational potential energy

The first task is to connect the supporting exchange to a material state. Let P_{ambient} be the balanced supporting power supplied to the reference receiver by the ambient vacuum field at equilibrium, and P_{support} its corresponding supporting power in the changed state. Both powers use the same receiver, exchange channels, and reference units. The OQG supporting-power relation gives

$$b = \sqrt{\frac{P_{\text{support}}}{P_{\text{ambient}}}}, \quad P_{\text{support}} = b^2 P_{\text{ambient}}. \quad (1)$$

Supporting power is the rate of the balanced exchange maintaining the material state. At equilibrium, incoming and outgoing powers are equal, so the exchange does not continually increase the body's energy

or temperature. A reduction in the available exchange establishes a lower value of b . Equation (1) translates that reduction into the material factor used throughout the calculation.

The bare zero-point vacuum and the local ambient field must be distinguished. The former is Lorentz invariant. Matter, boundaries, and driven sources can give the latter a physical rest frame and a direction-dependent spectrum. A covariant field description permits such a material environment. The familiar distinction between invariant zero-point radiation and radiation tied to a source frame illustrates this point [8].

Motion relative to the local environment changes the spectrum sampled by the receiver, including its directional Doppler shifts. Here P_{ambient} remains the equilibrium reference-receiver exchange in fixed reference units. The motion-dependent received exchange enters the changed-state response. A common b means a common fractional supporting change for matter in the same local state, rather than an equal number of watts for bodies of different size. The work calibration below determines the state-energy difference; a spectral calculation must determine the response that produces it.

Let m_0 be the body's ambient reference mass and c_0 the propagation speed in ambient reference standards. Define $E_0 = m_0 c_0^2$ as its reference material energy, U_{rest} as its rest-storage energy, and U_g as its gravitational potential energy relative to ambient support. Their OQG relations are

$$U_{\text{rest}} = E_0 b, \quad U_g = E_0(b - 1). \quad (2)$$

At ambient support, the body retains material energy E_0 and has reference gravitational potential energy zero. Lowering b reduces its rest-storage energy and takes its gravitational potential energy below that reference. Raising b restores the energy difference. The choice of zero for gravitational potential energy does not remove the stored material energy.

For a specified material response, P_{ambient} establishes the natural supported state and its associated energy storage. It is a power, however, so it is not itself an energy reservoir measured in joules. The recoverable gravitational potential energy is determined by Eq. (2), rather than by multiplying the equilibrium exchange rate by an arbitrary time.

This is the OQG connection to Mach's principle. Surrounding matter enters the local supporting configuration that establishes the material state and its inertial response [5, 7]. The common factor b describes matter in that state. Materials may have different susceptibilities and scattering responses, while the same local supporting state has the same fractional scaling relative to each receiver's ambient reference. The next section describes how the received exchange changes.

4 How scattering changes the received support

A material receiver responds to the electromagnetic field present at its location. That field includes radiation arriving from the environment and radiation scattered by other matter. The scattered field has both an amplitude and a phase. It can therefore reinforce or reduce the incident field at the receiver.

Let $\mathbf{E}_{\text{in}}$ and $\mathbf{E}_{\text{sc}}$ denote the incident and scattered electric fields. Angle brackets indicate a statistical average, an asterisk complex conjugation, and Re the real part. The mean-square total field is

$$\langle |\mathbf{E}_{\text{in}} + \mathbf{E}_{\text{sc}}|^2 \rangle = \langle |\mathbf{E}_{\text{in}}|^2 \rangle + \langle |\mathbf{E}_{\text{sc}}|^2 \rangle + 2\text{Re}\langle \mathbf{E}_{\text{in}} \cdot \mathbf{E}_{\text{sc}}^* \rangle. \quad (3)$$

The first two terms are the incident and scattered mean-square fields considered separately. The last term describes their interference. It can be negative: adding a scattered field can reduce the total field received at a particular location. This explains why the loss of supporting exchange cannot be found simply by

adding up positive scattered power. The coupled-field calculation must retain the correlations between incident and scattered fields.

Equation (3) states the criterion for reducing the received mean-square field: the interference term must be negative and exceed the scattered-field intensity in magnitude. The total then falls below the incident value. This reduction comes from the coupled fields and their relative phase. Its microscopic starting point is the charge response of bound matter. A material's *susceptibility* describes how strongly, and with what phase, its charge distribution responds to a driving field. That response determines its dipole moment, scattering, and the field received by neighboring matter. Appendix A develops the susceptibility, scattering, and supporting-power relations used here.

The supporting-power scaling follows from the same material response. In corresponding states, transition energies scale with b , while charge fluctuations and their frequency distribution transform together. The power integral then scales as b^2 , producing Eq. (1). The result depends on transforming the response and the field consistently; changing a frequency while holding the rest of the material response fixed would describe a different calculation.

The proposed physical sequence is therefore specific: matter scatters the ambient field; interference changes the received exchange; the changed exchange establishes the material state; and a spatial variation of that state produces a force. The response framework identifies the quantities needed to calculate that sequence. The cycle analysis below uses a prescribed profile to determine its mechanical consequences.

5 Why the disk becomes inertially heavier

Inertial mass measures how strongly a body resists a change in its motion. Gravitational potential energy measures the energy associated with its position in a supporting configuration. These quantities need not vary in the same direction. OQG relates both to the electromagnetic response of matter through its matter-wave dispersion.

Take the axial coordinate x as positive from the ship toward the disk, and let t be reference time. Define disk velocity $v = dx/dt$. Ship motion within one stroke is neglected to leading order in the small disk-to-ship mass ratio. Comparisons below use the instantaneous ship rest frame at the beginning of the cycle.

The axial branch changes the longitudinal material ruler by b while retaining both transverse ruler factors at one. Thus the disk's center moves during extension, while its own longitudinal material dimension also changes; those are separate displacements. Its diameter remains unchanged. Electromagnetic propagation along the axis samples transverse electric and magnetic response. Denote those coefficients by $\epsilon_{\perp}$ and $\mu_{\perp}$, their ambient values by ϵ_0 and μ_0 , and the axial propagation speed by c_x . Then

$$\epsilon_{\perp} = \frac{\epsilon_0}{b^2}, \quad \mu_{\perp} = \frac{\mu_0}{b^2}, \quad c_x = \frac{1}{\sqrt{\epsilon_{\perp}\mu_{\perp}}} = b^2 c_0. \quad (4)$$

The directional tensor relation is given in Appendix B. It explains why the coefficients relevant to axial propagation can increase while the disk diameter remains fixed. The isotropic family used in Appendix A changes all ruler directions; the device branch changes only the axial ruler. Applying the supporting-power relation to that branch retains the corresponding-state assumptions on the response channels, occupations, and transformed field data.

Let p be the axial canonical momentum and H the disk's total matter-wave energy in reference standards. Canonical momentum is the momentum variable entering the dispersion relation and its force law. The Hamiltonian, which expresses energy in terms of position and momentum, is

$$H(x, p, t) = \sqrt{E_0^2 b(x, t)^2 + c_0^2 b(x, t)^4 p^2}. \quad (5)$$

At rest, this gives $H = E_0 b$, in agreement with Eq. (2). For disk speeds much smaller than c_x , expansion in momentum separates the rest-storage and kinetic terms. Define m_x as the axial inertial coefficient appearing in that kinetic term. Then

$$H \simeq E_0 b + \frac{p^2}{2m_x}, \quad m_x = \frac{m_0}{b^3}, \quad p \simeq m_x v. \quad (6)$$

Equation (6) resolves the apparent contradiction. Lower b lowers rest-storage energy but increases the coefficient relating momentum to velocity. The same disk consequently requires more impulse to acquire a given low velocity. No material is added to it; its response to motion changes.

Table 1 summarizes the linked changes. Each denominator is the corresponding ambient value.

Table 1 Material response ratios on the axial branch relative to ambient support

Quantity	Ratio	Effect of decreasing b
Supporting power	b^2	Less balanced exchange
Transverse permittivity and permeability	b^{-2}	Greater electromagnetic response
Axial propagation speed	b^2	Lower propagation speed
Rest-storage energy	b	Lower stored rest energy
Axial inertial coefficient	b^{-3}	Greater resistance to changing motion

These relations supply the heavy and light states used by the cycle. The gravitational potential energy difference between them remains Eq. (2).

6 Acceleration and the two parts of extension

Gravity and acceleration act through the same local supporting-power response in OQG. The following comparison expresses that equivalence through the material clock factor. To state the comparison, let z measure height from a reference event, g be a locally uniform gravitational acceleration magnitude, and a_{lab} the upward acceleration of a comparison laboratory. Denote their local clock factors by b_g and b_a . With matched local field and thermal states, negligible tides, and small potential differences,

$$b_g(z) = 1 + \frac{gz}{c_0^2}, \quad b_a(z) = 1 + \frac{a_{\text{lab}} z}{c_0^2}. \quad (7)$$

For $a_{\text{lab}} = g$, both laboratories have the same local material scaling, charge response, and supporting-power ratio. Their common supporting response is the microscopic basis of the local equivalence used here. Equation (7) specifies a spatial comparison between matched laboratories; the driven disk also requires the time evolution of its supporting state.

The comparison concerns the spatial variation of the material state. It does not identify the driven state factor with an ordinary Lorentz factor or determine the disk's state from its instantaneous speed. For the operating cycle, the supporting configuration and its work history specify the state reached at each point of the motion.

The distinction becomes clear from work. Define $\Psi = U_g/m_0 = c_0^2(b - 1)$ as gravitational potential energy per unit reference mass. For slow motion generated solely by a stationary supporting gradient, let $\mathbf{a}_{\text{eff}} = -\nabla\Psi$ be the effective acceleration and $\mathbf{x}$ position. Let b_1, b_2 and v_1, v_2 be endpoint material factors and speeds. Near ambient support,

$$\int_1^2 \mathbf{a}_{\text{eff}} \cdot d\mathbf{x} = \frac{v_2^2 - v_1^2}{2} = -c_0^2(b_2 - b_1). \quad (8)$$

The integral accumulates acceleration over displacement. It explains why the material state is connected to the work history and supporting configuration, rather than being inferred from acceleration at one instant. In the driven device, actuator work and changes of the field must also be included. Equation (8) supplies the unpowered reference case; Section 8 supplies the driven energy balance.

Define X as the outward stroke, τ its duration, t_m the transition between outward acceleration and deceleration, v_{max} the maximum outward speed, and b_{min} the minimum material factor. The operating trajectory places $v = v_{\text{max}}$ and $b = b_{\text{min}}$ at $t = t_m$.

First part of extension. The stack accelerates the disk outward. Supporting exchange decreases and b falls from one toward b_{min} . The coefficients in Eq. (4) increase, the axial inertia increases, and gravitational potential energy decreases. The driven material and field response determines the history of this change.

Second part of extension. The disk is still moving outward, but now slows. The stack continues pushing against the environmental force while b rises toward one. The disk begins this part at its greatest inertia and progressively lightens as gravitational potential energy is restored. At maximum extension it reaches $x = X$, $v = 0$, and $b = 1$.

At the transition, maximum velocity can coincide with zero instantaneous acceleration. That does not, by itself, restore the material to ambient support: the supporting configuration and its accumulated energy change still determine b . The source response must produce the required timing of b relative to motion. The smooth example in Appendix C makes the kinematics explicit without treating a displacement command as a solution for the field.

The return then proceeds at ambient inertia. It includes a smooth inward acceleration, a nearly constant-speed interval, and a smooth stop. The starting and stopping impulses are both included. Ambient support on this segment is a specified operating condition: slow motion alone does not establish an exact reset to $b = 1$. Any residual gradient or source reaction belongs in the complete-cycle momentum balance.

7 How a slowing disk continues to push the ship

Force arises when the disk's energy changes with position. The spatial derivative of the Hamiltonian holds momentum and time fixed; it asks how much the energy changes under a small displacement through the supporting configuration. Let F_g be the signed environmental gravitational dispersion force on the disk. With velocity determined by $v = \partial H / \partial p$, Eq. (5) gives

$$F_g = -\frac{\partial H}{\partial x} = -\frac{H + pv}{b} \frac{\partial b}{\partial x}. \quad (9)$$

At instantaneous rest, the force is $-E_0 \partial_x b$. A positive gradient means that outward displacement raises b and gravitational potential energy. The environmental force therefore points inward, opposing the outward push. This is the mathematical form of the book-lifting analogy. The full expression also includes the moving disk's contribution to the energy slope.

Let S be the stack force on the disk, positive outward, and F_{ship} the contact reaction on the ship in the negligible-stack-momentum approximation. A dot denotes differentiation along the disk trajectory. The force balance is

$$\dot{p} = S + F_g, \quad F_{\text{ship}} = -S. \quad (10)$$

During the second part of extension, the disk moves outward while its momentum decreases. This is compatible with a positive outward stack force: the inward environmental force is larger. Consequently, the stack continues applying a forward reaction to the ship even while the disk slows.

A distributed stack also carries momentum. Its instantaneous root reaction can therefore differ from its tip reaction by the time rate of change of that internal momentum. In a repeating cycle the stack returns to its initial momentum, so this contribution integrates to zero. The complete-assembly balance in Section 9 includes the stack and every onboard source component.

To include changing inertia, define acceleration $a = dv/dt$. Differentiating the low-speed momentum in Eq. (6) gives

$$\dot{p} \simeq m_0 \left(\frac{a}{b^3} - \frac{3v\dot{b}}{b^4} \right). \quad (11)$$

The first term describes changing velocity. The second describes changing inertia. During restoration, $a < 0$, $v > 0$, and $\dot{b} > 0$, so both terms reduce outward momentum. Continued pushing requires $S > 0$, or equivalently $F_g < \dot{p} < 0$. This inequality states the required division of force between the environment and the actuator.

For a selected motion and pushing load, Eqs. (9) and (10) specify the necessary spatial response. The material factor along the trajectory also includes explicit field evolution at a fixed position:

$$\frac{\partial b}{\partial x} = \frac{b(S - \dot{p})}{H + pv}, \quad \dot{b} = \frac{\partial b}{\partial t} + v \frac{\partial b}{\partial x}. \quad (12)$$

The second relation separates two ways the disk encounters changing support: the configuration changes with time, or the disk moves through a spatial gradient. During a stationary restoration segment, the first contribution vanishes. The spatial change per unit distance is then the change per unit time divided by the distance travelled per unit time. Section 10 uses precisely this chain-rule calculation to obtain the uphill gradient. The full relation remains necessary when the supporting configuration itself evolves.

8 Lifting work and restoration of the material state

The loss of disk velocity during restoration must appear explicitly in the work balance. Define kinetic energy $K = H - E_0 b$, stack mechanical power $P_{\text{PZT}} = Sv$, and explicit-time field power $P_b = (\partial H / \partial t)_{x,p}$. The subscript on the derivative means position and momentum are held fixed. The exact disk energy balance is

$$\frac{d}{dt}(U_g + K) = P_{\text{PZT}} + P_b, \quad P_b = \frac{H + pv}{b} \frac{\partial b}{\partial t}. \quad (13)$$

Mechanical power is force multiplied by disk velocity. The field term accounts for energy transferred when the supporting configuration changes at the disk's fixed position. It is separate from the work of moving through a spatial gradient. Both contributions are needed during a driven cycle.

Let $K_{\max} = K(t_m)$ be the disk kinetic energy at the transition. Define W_2 as stack work and $E_{b,2}$ as explicit-time field energy delivered to the disk from t_m to τ ; these are the respective time integrals of Sv and P_b . Restoration ends with zero disk velocity and ambient support. Integrating Eq. (13) therefore gives

$$E_0(1 - b_{\min}) = K_{\max} + W_2 + E_{b,2}. \quad (14)$$

The left side is the rise in gravitational potential energy. On the right, the first contribution is kinetic energy surrendered as the disk slows from $v_{\max}$ to zero. The second is positive work supplied by the pushing stack. The third is energy supplied by explicit evolution of the supporting configuration. This is the complete energy ledger for the lifting phase.

The mechanically supplied uphill load case has $E_{b,2} = 0$. Its required actuator work is then

$$W_2 = E_0(1 - b_{\min}) - K_{\max} > 0. \quad (15)$$

The inequality states that the gravitational potential energy rise exceeds the kinetic energy available from slowing the disk. The actuator supplies the difference while its reaction pushes the ship. Thus the second phase can perform positive lifting work despite the disk's decreasing speed. Requiring Eq. (15) is part of defining this operating load case.

The first and second parts must also close as one extension. Let W_{out} be the stack work over the complete outward stroke and $E_{b,\text{out}}$ the corresponding explicit-time field energy delivered to the disk. The disk begins and ends extension at rest with $b = 1$, so its total energy has no net change:

$$W_{\text{out}} + E_{b,\text{out}} = 0. \quad (16)$$

Positive outward actuator work is therefore accompanied by net energy transfer from the disk into the changing environment. In the mechanically supplied restoration case, this transfer occurs through the driven release portion. A purely stationary conservative descent and ascent would return the same energy and yield zero net outward work. Equation (16) identifies the driven exchange required by the positive-work cycle; the source calculation must account for its destination.

8.1 What the stack terminals measure

The moving disk can stretch the piezoelectric stack while the stack's elastic response tends to pull it back. Mechanical stress also generates charge. If that charge cannot flow away, its voltage changes the opposing electromechanical force; the electrical termination therefore affects the stack's effective stiffness [9]. The applied voltage must maintain the required net outward contact force during restoration. A decreasing disk speed does not require the actuator to pull inward.

The charge response makes the electrical energy ledger essential. Let $V(t)$ be the stack terminal voltage and $I(t)$ the terminal current, with positive VI denoting power entering the stack. Let T be the complete cycle period, E_{term} the net terminal energy per cycle, and P_{term} its mean power. The true power factor, denoted PF, is that mean power divided by the product of the root-mean-square voltage and current, V_{rms} and I_{rms} . Then

$$E_{\text{term}} = \int_0^T VI \, dt, \quad P_{\text{term}} = \frac{E_{\text{term}}}{T}, \quad \text{PF} = \frac{P_{\text{term}}}{V_{\text{rms}} I_{\text{rms}}}. \quad (17)$$

This integral counts both energy delivered to the stack and energy returned from it. It applies directly to the asymmetric, nonsinusoidal command; a single phase angle is sufficient only for sinusoidal voltage and current. Induced voltage can accompany electrical storage, recovery, or dissipation. Its magnitude alone does not identify real power or mechanical work.

The disk work is the integral of contact force times disk velocity. Terminal energy also includes changes in electrical and elastic storage, motion within the stack, and losses. These internal stores must return to their starting values, including residual ringing, before a complete-cycle balance is taken. Total input further includes drive losses and any separate field-control work. Section 11 states the ideal conditions under which net electrical input reduces to the outward disk work.

9 The impulse retained after the disk returns

Work measures transferred energy; impulse measures transferred momentum. Let $J_{\text{out}} > 0$ be the magnitude of the ship's outward-stroke reaction impulse, defined as the time integral of the positive pushing force. Because disk momentum starts and ends extension at zero, integrating Eq. (10) gives

$$J_{\text{out}} = \int_0^\tau S \, dt = - \int_0^\tau F_g \, dt. \quad (18)$$

The disk's own momentum excursion cancels between these endpoints. What remains is the environmental impulse transmitted through the stack. The ship receives that impulse toward negative x , while the environment receives the opposite momentum.

Let S_{ret} be the signed stack force on the disk during return and Δp_{ret} its momentum change. For the ideal ambient return, $b = 1$, the environmental force on the disk is zero, and both return endpoints have zero disk velocity. Over the full period T , the return contact impulse is

$$\int_\tau^T S_{\text{ret}} \, dt = \Delta p_{\text{ret}} = 0. \quad (19)$$

The initial pull accelerates the disk inward; the final stopping force supplies the opposite impulse. Their sum is zero. The near-constant-speed interval requires no ideal inertial force. Accordingly, the ideal return does not cancel the outward environmental impulse.

To state the whole-assembly condition, let $F_{\text{env,all}}$ be the signed external force on ship, disk, stack, onboard drive hardware, and assigned local field together. Let J_{net} denote their net impulse toward negative x , $\nu = 1/T$ the cycle frequency, and $\bar{F}$ the corresponding average thrust. Then

$$J_{\text{net}} = - \int_0^T F_{\text{env,all}} \, dt, \quad \bar{F} = \nu J_{\text{net}}. \quad (20)$$

The ideal load case assigns the net external impulse to the disk's extension; other onboard components add no net environmental impulse. All onboard hardware remains inside the assembly boundary. The external supporting field and the matter coupled to it receive the opposing momentum. This is the boundary condition under which the contact impulse in Eq. (18) becomes assembly propulsion.

Let $\Delta P_{\text{assembly}}$ and $\Delta P_{\text{environment}}$ be the signed total momentum changes, with transferred field momentum included consistently. Their balance is

$$\Delta P_{\text{assembly}} + \Delta P_{\text{environment}} = 0. \quad (21)$$

Traction is force per unit area transmitted across a surface. Its integral over the assembly boundary gives the environmental force, with momentum stored in the enclosed field included consistently in Eqs. (20) and (21). In this sense the stack transmits environmental traction to the ship.

A chamber test must account separately for forces through suspension, electrical leads, and chamber hardware. Reactions returned through onboard source hardware remain inside Eq. (20). A device calculation must establish the external transfer by evaluating the complete boundary stress and field momentum.

10 From the supporting profile to average thrust

The preceding balances can now be used as a calculation procedure. First determine the energy needed to restore the disk from its minimum supporting state. Next specify where that restoration occurs along the stroke. The change of energy per unit distance gives the uphill force. Integrating the transmitted force over time gives the impulse, and multiplying by the number of complete cycles per second gives average thrust. Each step below describes a physical quantity that can be compared with a driven model or a measurement.

This section specializes the general equations to a weak supporting change and disk speeds much smaller than the local axial propagation speed. Thus $1 - b_{\min} \ll 1$ and $|v| \ll b^2 c_0$. To leading order, the force coefficient in Eq. (9) is $E_0 = m_0 c_0^2$, and disk kinetic energy is $m_0 v^2/2$. The exact Hamiltonian remains available when these approximations are insufficient. The ship's small motion during one cycle is neglected as in Section 5.

10.1 Calibrate the depth and specify the profile

Define the positive supporting deficit $\delta = 1 - b_{\min}$ and restoration energy ΔU . The latter is the rise in gravitational potential energy between minimum support and the ambient state. Equation (14) calibrates the depth from the disk kinetic energy, the mechanical work supplied during restoration, and any explicit-time field energy delivered on that segment:

$$\Delta U = E_0 \delta = K_{\max} + W_2 + E_{b,2}, \quad \delta = \frac{K_{\max} + W_2 + E_{b,2}}{m_0 c_0^2}. \quad (22)$$

This relation converts joules of restoration energy into a dimensionless supporting change. In the mechanically supplied uphill case, explicit field evolution contributes no restoration energy, so the kinetic energy surrendered by the disk and the stack's positive work supply the full rise. The rise must then exceed the disk's kinetic energy at the transition. The work calibration fixes the depth once those energy contributions are known; it does not choose their values independently.

For a concrete analytic profile, let the supporting deficit during extension have the same normalized shape as the disk's low-speed kinetic energy. It vanishes at the two stationary endpoints and reaches its maximum at the maximum outward speed:

$$b(t) = 1 - \delta \frac{v(t)^2}{v_{\max}^2}, \quad 0 \leq t \leq \tau. \quad (23)$$

The squared-speed factor specifies the shape of this cycle. At the transition, the disk has its greatest speed and its lowest support. During the rest of extension, its speed decreases while support rises. This is a prescribed response along the trajectory, not a universal rule for inferring every body's supporting state from its speed.

To turn that history into a force, the response must also specify how much of the change is spatial and how much is explicit evolution in time. Let $G = -F_g$ denote the inward environmental force as a positive quantity during restoration. Table 2 states the phase conditions used for the calculation.

Table 2 Environmental response for the explicit cycle

Phase	Disk motion and supporting state	Specified field response
Release	Outward acceleration; support falls to its minimum at maximum speed.	Zero spatial force on the disk along its path. Explicit field evolution lowers its state energy.
Restoration	Outward motion continues while speed decreases; support rises to ambient.	A stationary spatial gradient supplies the inward force. Explicit-time field power is zero on this segment.
Ambient return	Smooth inward start, nearly constant speed, smooth stop; support stays at ambient.	Zero environmental force on the disk. Starting and stopping contact impulses cancel.

The table specifies the local response sampled by the disk. Together with the external-reaction condition in Section 9, it defines the load case whose thrust is calculated here. The release segment and restoration segment have different divisions between temporal and spatial change, even though they share one continuous supporting-state history.

10.2 Calculate the uphill gradient and contact force

During restoration, the supporting profile is stationary along the sampled path: the explicit partial time derivative in Eq. (12) is zero. Divide the rate at which support changes by the disk's outward speed to obtain the change in support per unit distance. Differentiating Eq. (23) and applying that chain rule gives

$$\frac{\partial b}{\partial x} = \frac{\dot{b}}{v} = -\frac{2\delta}{v_{\max}^2}a, \quad t_m < t < \tau. \quad (24)$$

The acceleration is negative because the outward-moving disk is slowing. The resulting gradient is positive: each additional outward displacement takes the disk to greater support and higher gravitational potential energy. At a zero-speed endpoint the ratio is evaluated by its finite limiting expression on the right. It does not require dividing two measured zeros.

Multiplying the gradient by reference material energy gives the leading inward force. The stack supplies the difference between that inward load and the rate at which the disk's outward momentum decreases. Substitution into the force balance gives

$$G = -\frac{2\Delta U}{v_{\max}^2}a, \quad S \simeq (m_0 - \frac{2\Delta U}{v_{\max}^2})a. \quad (25)$$

These two forces explain the powered uphill phase. The field pulls inward more strongly than the stack pushes outward, so the disk decelerates. Nevertheless, the stack force and disk velocity are both outward, and the stack does positive work. With $K_{\max} \simeq m_0 v_{\max}^2/2$, positive pushing throughout the decelerating segment requires $\Delta U > K_{\max}$. This is the same condition obtained from the work balance in Eq. (15).

The gradient is calibrated through energy and displacement, rather than by assigning a separate material-dependent factor to the disk. A very small supporting deficit can still represent a finite restoring force because its energy scale is $m_0 c_0^2$ and its spatial variation can occur over a short stroke. The inertia change and the gradient force are related consequences of the state, but they are distinct terms in the force balance.

10.3 Obtain the impulse and the average thrust

The disk begins and ends extension with zero momentum. Its temporary momentum gain therefore cancels when the contact force is integrated over the whole outward stroke. What remains is the inward environmental impulse transmitted through the stack. In Table 2 that environmental force acts during restoration. Integrating Eq. (25) as velocity falls from its maximum to zero gives

$$J_{\text{out}} = \int_{t_m}^{\tau} G dt = \frac{2\Delta U}{v_{\text{max}}}, \quad F = vJ_{\text{net}} = \frac{2v\Delta U}{v_{\text{max}}}. \quad (26)$$

The last equality uses the ambient return and the whole-assembly external-reaction condition of Section 9. It gives a thrust magnitude toward the ship, opposite positive disk displacement. The first expression is impulse per outward stroke; the second multiplies that impulse by the complete-cycle repetition rate. The rate must include the return interval. Using the inverse extension time instead would overcount the strokes.

The units also expose the result's physical meaning: energy divided by speed is momentum, and momentum delivered per second is force. For the selected profile, a larger restoration energy at fixed peak speed gives a proportionally larger impulse. A smaller peak speed at fixed restoration energy gives more impulse per joule, but maintaining the same field depth and cycle frequency is an additional operating requirement.

The work follows the same sequence. Let W_1 be stack work during release. With zero spatial force there, the stack supplies the disk's kinetic energy while explicit field evolution transfers the state-energy decrease to the environment. During restoration, the disk gives up its kinetic energy and the stack supplies the remaining lift. To leading order,

$$W_1 \simeq K_{\text{max}}, \quad W_2 \simeq \Delta U - K_{\text{max}}, \quad W_{\text{out}} \simeq \Delta U. \quad (27)$$

Thus the positive work over extension equals the restoration energy for this specified response. The field receives that energy during release, while restoration returns the disk to its starting state. The cycle does not accumulate an uncounted energy store in the disk. Electrical input equals this mechanical work only under the reversible recovery and control-work conditions stated in Section 11.

Define the force-weighted disk speed v_{eff} as outward work divided by outward impulse. It is the speed that converts the integrated pushing impulse into the work delivered over that same stroke. Equations (26) and (27) give

$$v_{\text{eff}} = \frac{W_{\text{out}}}{J_{\text{out}}} \simeq \frac{v_{\text{max}}}{2}. \quad (28)$$

For this profile, the force-weighted speed is half the peak speed. Under ideal electrical recovery with no additional net control cost, mean input power is $v\Delta U$, so the thrust per watt is twice the inverse peak speed. Section 11 expresses the same result in the more general form of inverse force-weighted speed. This is why the peak speed and the speed used in the power comparison must be named separately.

10.4 What the phase timing accomplishes

The retained impulse depends on the division between temporal release and spatial restoration in Table 2. The same supporting-state history alone would not select that division. For comparison, applying the stationary-gradient interpretation of Eq. (24) throughout both parts of extension would give an outward assisting force during release and an inward force during restoration. Their leading impulses would sum to

$$\int_0^\tau G dt = -\frac{2\Delta U}{v_{\max}^2} [v(\tau) - v(0)] = 0. \quad (29)$$

The cancellation uses the equal endpoint velocities and does not depend on acceleration symmetry. Table 2 instead assigns the release energy change to explicit field evolution without that assisting spatial impulse. This is the precise field-response condition that permits the restoration impulse in Eq. (26) to remain. The ambient return then closes the motion while its own starting and stopping impulses cancel.

The resulting derivation is complete for the prescribed load. A driven source model must establish this phase response and the external momentum exchange of Section 9. Electrical recovery, source reactions, and any return-field residual then enter the measured complete-cycle ratios of Section 12.

10.5 Reading the complete cycle

Figure 1 evaluates the cycle in normalized form using the smooth extension and return command in Appendix C. For illustration, extension occupies one quarter of the period and the restoration energy is twice the disk kinetic energy at the transition. This choice satisfies the positive-push condition. It is a stated plotting input, not an independently derived field depth.

Let S_{pk} be peak outward stack force, $|P_b|_{pk}$ peak magnitude of explicit-time field power, and $J_+(t)$ accumulated ship contact impulse toward negative x . The final impulse used to normalize panel (e) is J_{net} under the external-reaction condition. All signs follow Sections 7–9.

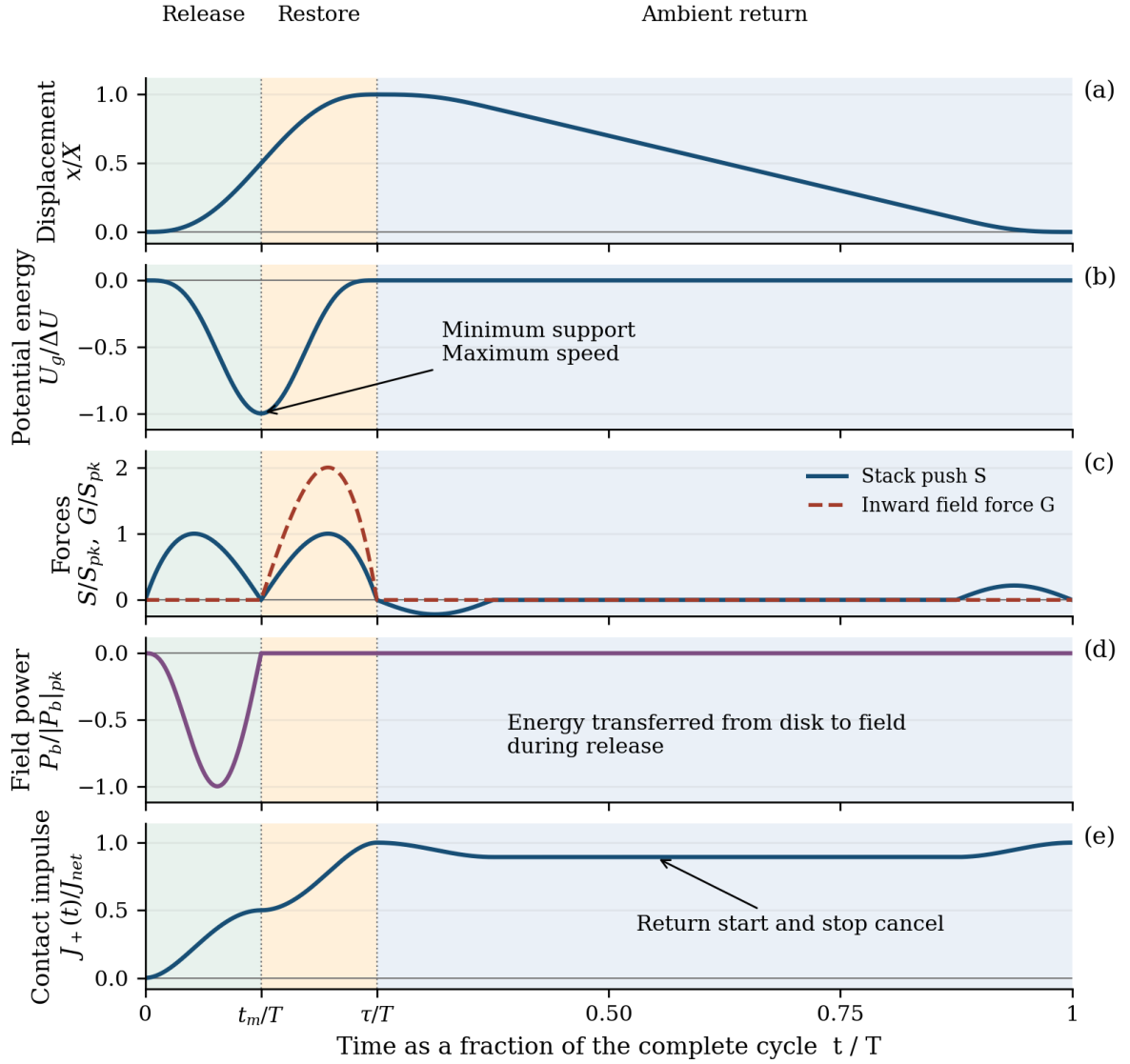


Figure 1 The prescribed cycle and its energy and momentum transfers. Panel (a) shows continued outward displacement during restoration. Panel (b) reaches minimum support and potential energy at maximum speed. Panel (c) separates the outward stack push from the larger inward environmental force. Panel (d) locates the explicit field-energy transfer in release. Panel (e) shows the outward impulse retained after the return's starting and stopping impulses cancel. These are analytical load traces, not measured hardware response.

The force panel makes the deceleration mechanism visible: both forces are present during restoration, and the inward one is larger. The disk slows while the actuator continues pushing. The field-power panel shows where the release energy goes, while the final impulse panel includes the entire return. Read together, the panels connect the supporting change to the force, work, and average-thrust results.

11 Ideal cycle advantage over a photon rocket

The comparison uses equal net electrical input, the same cycle interval, and the same reference frame. Let $E_{\text{elec}} > 0$ be net electrical energy supplied per cycle after recovered electrical energy is subtracted. Define E_{env} as net energy gained by the external supporting environment, Q_{heat} as apparatus heat, and $\Delta K_{\text{assembly}}$ as the assembly's bulk kinetic-energy increase. With internal storage restored each cycle,

$$E_{\text{elec}} = \Delta K_{\text{assembly}} + E_{\text{env}} + Q_{\text{heat}}. \quad (30)$$

Heat is excluded from E_{env} to avoid double counting. The environment term is signed and is negative when it supplies energy. The recoverable gravitational potential energy excursion is already included in the cycle accounting.

The ideal benchmark uses the following specified load case: outward pushing delivers the impulse in Eq. (18); the ambient return satisfies Eq. (19); other onboard components add no net external impulse; and the actuator has reversible energy recovery with zero losses. The benchmark also assigns no separate net electrical cost to controlling the supporting configuration beyond the stack work. This is an explicit ideal load condition; a device calculation must include any additional control work.

The leading-order ship-rest approximation neglects the ship's small velocity change within the cycle. The return begins and ends with the same disk kinetic energy, so its net mechanical work is zero. Stored electrical and elastic energy also returns to its starting value. Under these conditions,

$$J_{\text{net}} = J_{\text{out}}, \quad E_{\text{elec}} = W_{\text{out}}. \quad (31)$$

These equalities establish the two cycle factors for the specified benchmark. The environment still receives the outward work through Eq. (16), and the opposite momentum through Eq. (21). Energy and momentum transfer remain part of the ideal load case.

Let $P_{\text{elec}} = vE_{\text{elec}}$ be mean net electrical input. The force-weighted disk speed v_{eff} , introduced for the explicit profile in Section 10, has a general definition: divide the outward work by the outward impulse. With positive outward pushing, this weights each disk speed by the force applied at that speed:

$$v_{\text{eff}} = \frac{W_{\text{out}}}{J_{\text{out}}} = \frac{\int_0^\tau S v dt}{\int_0^\tau S dt}, \quad \frac{\bar{F}}{\bar{P}_{\text{elec}}} = \frac{1}{v_{\text{eff}}}. \quad (32)$$

The velocity in this equation is the moving disk's velocity at the actuator, not the propagation speed of radiation or the ship's cruising velocity. Force applied over a small mechanical displacement transfers substantial impulse for a given amount of work. The return and environmental conditions in Eq. (31) are what retain that advantage as net assembly thrust.

Let J_γ and F_γ be the impulse and mean thrust of an ideal photon rocket converting the same electrical input completely into a collimated outgoing beam. Photon momentum is energy divided by c_0 [3], so

$$J_\gamma = \frac{E_{\text{elec}}}{c_0}, \quad \frac{F_\gamma}{\bar{P}_{\text{elec}}} = \frac{1}{c_0}. \quad (33)$$

Define $\mathcal{R}_{\text{ideal}}$ as the ideal MEGA cycle's thrust-to-power ratio divided by this photon-rocket ratio. Combining Eqs. (32) and (33) gives the central ideal-cycle result:

$$\mathcal{R}_{\text{ideal}} = \frac{c_0}{v_{\text{eff}}}. \quad (34)$$

The specified ideal cycle exceeds the photon-rocket thrust per input watt by the ratio of light speed to force-weighted disk speed. For the illustrative kinematic benchmark $v_{\text{eff}} = 1 \text{ m s}^{-1}$ and $c_0 = 299\,792\,458 \text{ m s}^{-1}$, its thrust-to-power ratio is 1 N/W . The ideal photon rocket gives $3.336 \times 10^{-9} \text{ N/W}$. The enhancement is approximately 3.00×10^8 , **or 8.48 orders of magnitude.**

For the quadratic profile of Section 10, the force-weighted speed is half the peak speed. Thus this one-metre-per-second benchmark corresponds to a peak speed of two metres per second for that profile. The example specifies a kinematic ratio; it does not assign an actuator stroke, drive voltage, achieved field depth, or measured thrust. Those quantities determine the absolute force and electrical demand of a particular implementation.

12 Quantifying departures from the ideal cycle

The ideal result provides a reference against which the complete device can be evaluated. Define r_J as net assembly impulse divided by outward impulse, and r_E as outward mechanical work divided by net electrical input. These are accounting ratios, not independently assumed efficiencies:

$$r_J = \frac{J_{\text{net}}}{J_{\text{out}}}, \quad r_E = \frac{W_{\text{out}}}{E_{\text{elec}}}. \quad (35)$$

The first ratio includes every return and source reaction. The second includes electrical recovery, drive losses, additional control work, and energy transferred to the environment, with internal storage restored each cycle. Let $\mathcal{R}_{\text{elec}}$ denote the actual calculated thrust-to-electrical-power enhancement over the photon reference. Then

$$\mathcal{R}_{\text{elec}} = r_J r_E \frac{c_0}{v_{\text{eff}}}. \quad (36)$$

At the same illustrative speed of one metre per second, Table 3 shows the sensitivity to the specified products of impulse retention and work-to-input ratio. These are comparison cases, not evaluated properties of the assembly.

Table 3 Photon-rocket comparison at a force-weighted disk speed of one metre per second

Specified $r_J r_E$	Thrust per net electrical watt	Photon-rocket enhancement
1	1 N/W	3.00×10^8
10^{-2}	10^{-2} N/W	3.00×10^6
10^{-6}	10^{-6} N/W	3.00×10^2

Let $N > 0$ be the required number of orders of magnitude of enhancement. The precise cycle requirement is

$$\mathcal{R}_{\text{elec}} \geq 10^N \Leftrightarrow r_J r_E \geq 10^N \frac{v_{\text{eff}}}{c_0}. \quad (37)$$

For the one-metre-per-second benchmark, exceeding the photon reference requires $r_J r_E > 3.336 \times 10^{-9}$; exceeding it by six orders requires $r_J r_E \geq 3.336 \times 10^{-3}$. These inequalities identify the bound a device calculation must establish. They quantify the available mechanical factor without assigning unevaluated values to the retained impulse or electrical work.

For a particular assembly, one driven solution must determine the scattering and supporting response, the spatial gradient in Eq. (12), the lifting work in Eq. (15), the total boundary impulse in Eq. (20), and net input energy in Eq. (30). The terminal measurement in Eq. (17) supplies the electrical energy at the stack; it must be combined with the drive and source-control accounting to obtain total input. These quantities then determine the two ratios in Eq. (35).

Changing stroke duration or repetition frequency also changes mechanical loading and material response. A smaller disk velocity increases the ratio in Eq. (34), but does not alone establish a larger absolute thrust. The achieved force, stroke, and cycle frequency must be evaluated together. Equation (37) supplies the quantitative acceptance criterion for that operating solution.

13 Conclusions

The preferred cycle places minimum support at maximum disk speed and restores ambient support during the remainder of extension. The inward environmental force can exceed the positive outward stack force, allowing the disk to slow while the actuator continues doing uphill work. The disk's lost kinetic energy and the actuator's work together supply the restoration energy.

The main-body derivation connects that work calibration to a spatial gradient, a contact force, and a complete-cycle impulse. For the quadratic supporting profile and phase response of Section 10, the leading-order average thrust is $2\nu\Delta U/v_{\max}$ under the stated external-reaction condition. The result uses the full cycle period, includes the return start and stop, and keeps all onboard source components inside the assembly boundary.

Integrating terminal voltage times current measures net stack electrical energy, including recovery. Drive losses and field-control input complete the electrical ledger. Reversible recovery with no separate net control cost gives the ideal reduction to outward mechanical work.

Under those conditions, the thrust-to-power ratio is the inverse force-weighted disk speed. The illustrative one-metre-per-second benchmark gives one newton per watt, approximately 3.00×10^8 times the ideal photon-rocket ratio. For the explicit profile, the peak speed is twice that force-weighted speed. Equations (35)–(37) quantify how complete-cycle impulse retention and electrical input change the benchmark.

The accomplishment is a conditional performance derivation for a specified environmental response, with its force, energy, and momentum requirements stated together. A device prediction must establish that response and its reactions in a consistent driven solution. Within OQG, the same material-state factor connects supporting exchange, inertia, and gravitational potential energy; the geometry records the resulting changes of clocks, rulers, and electromagnetic response.

Matter tells matter how to scale. Geometry is just bookkeeping.

Appendix A Microscopic response and supporting power

The main text uses the supporting-power relation in Eq. (1). This appendix develops the microscopic chain connecting bound charge response, scattering, and supporting-power scaling used in Sections 3 and 4. Let Q be a nuclear charge coordinate, $|0\rangle$ its ground state, and $|n\rangle$ an excited state. Let Δ_n be the excitation energy and $q_n = \langle 0|Q|n\rangle$ the charge matrix element. Define angular frequency ω , reduced Planck constant $\hbar$, and imaginary unit i . The causal charge susceptibility χ^R , relating induced charge to its conjugate applied voltage, is

$$\chi^R(\omega) = 2 \sum_{n \neq 0} \frac{\Delta_n |q_n|^2}{\Delta_n^2 - \hbar^2(\omega + i0^+)^2}. \quad (38)$$

The infinitesimal 0^+ selects response after the applied disturbance. The excitation energies and matrix elements contain the bound-state physics. Under the common OQG scaling, the energies scale by b and the charge matrix elements remain fixed. With subscript 0 denoting reference values and subscript b the changed state,

$$\Delta_{n,b} = b\Delta_{n,0}, \quad q_{n,b} = q_{n,0}, \quad \chi_b^R(b\omega) = b^{-1}\chi_0^R(\omega). \quad (39)$$

Let ℓ_d be the effective dipole length, so that dipole moment is $\ell_d Q$. In the isotropic common state, let ϵ and μ be the electromagnetic coefficients and $c = (\epsilon\mu)^{-1/2}$ the propagation speed. Define volume polarizability $\alpha = \ell_d^2 \chi^R / \epsilon$, with radiation reaction included consistently, and wave number $k = \omega/c$. The electric-dipole scattering cross section σ_{sc} , an effective scattering area, is

$$\sigma_{sc} = \frac{k^4 |\alpha|^2}{6\pi}. \quad (40)$$

In this common family, $\ell_{d,b} = b\ell_{d,0}$ and $\epsilon_b = \epsilon_0/b^2$, while $\mu_b = \mu_0/b^2$. Thus $\alpha_b(b\omega) = b^3\alpha_0(\omega)$ and $k_b = k_0/b$. Substitution in Eq. (40) gives $\sigma_{sc,b}(b\omega) = b^2\sigma_{sc,0}(\omega)$. The scattered field then enters the interference balance in Eq. (3).

Let $R(\omega)$ be the dissipative response matrix for the retained exchange channels and $S_Q(\omega)$ their symmetrized, two-sided charge spectral-density matrix. The trace Tr adds the channel contributions. Balanced supporting power is

$$P_{\text{support}} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega^2 \text{Tr}[R(\omega)S_Q(\omega)]. \quad (41)$$

The integral sums the power exchanged across the response spectrum. For corresponding ground states, or thermal states with corresponding dimensionless occupations, the transformations are $R_b(b\omega) = R_0(\omega)$ and $S_{Q,b}(b\omega) = b^{-1}S_{Q,0}(\omega)$. The integration measure contributes b , the squared frequency contributes b^2 , and the spectrum contributes b^{-1} . Their product is b^2 , giving Eq. (1). Channel definitions and spectral cutoffs are transformed consistently with the corresponding states.

Appendix B Directional electromagnetic coefficients

The axial device branch follows the directional transformation of the supporting-power manuscript. Define the dimensionless ruler matrix $\mathbf{R}$ and its determinant $\mathcal{V}$, the material volume ratio. Let $\boldsymbol{\epsilon}$ and $\boldsymbol{\mu}$ be the full electromagnetic tensors. Superscript T means transpose and diag lists the axial and two transverse components. The transformation is

$$\mathbf{R} = \text{diag}(b, 1, 1), \quad \mathcal{V} = b, \quad \frac{\boldsymbol{\epsilon}}{\epsilon_0} = \frac{\boldsymbol{\mu}}{\mu_0} = \frac{\mathbf{R}\mathbf{R}^T}{b\mathcal{V}} = \text{diag}(1, b^{-2}, b^{-2}). \quad (42)$$

For reference disk diameter D , thickness h , and density ρ , the reference mass is $m_0 = \rho\pi D^2 h/4$. The axial branch retains diameter D and changes material thickness to bh . Its transverse coefficients give Eq. (4). This directional statement specifies which electromagnetic response changes while preserving the stipulated disk geometry.

Applying the supporting-power relation to this axial family retains the corresponding-state conditions of Appendix A: the response channels, state occupations, spectral data, and boundary fields transform consistently with the material map. The tensor relation specifies the directional coefficients. The phase-dependent supporting profile used in Section 10 is an additional driven-response condition.

Appendix C A smooth example of the extension command

A displacement command illustrates the acceleration scale required of the actuator. It does not prescribe the material field response. Define normalized extension time $s = t/\tau$. A smooth command with zero velocity and acceleration at both endpoints is

$$x = X(10s^3 - 15s^4 + 6s^5), \quad 0 \leq s \leq 1. \quad (43)$$

Maximum velocity occurs at $t_m = \tau/2$, where the displacement is $X/2$. Let $a_{\max}$ be the peak magnitude of acceleration. Differentiation gives

$$v_{\max} = \frac{15X}{8\tau}, \quad a_{\max} = \frac{10X}{\sqrt{3}\tau^2}. \quad (44)$$

At fixed stroke, reducing extension time by ten increases maximum velocity by ten and peak acceleration by one hundred. This is why the short extension can demand much greater acceleration than the return. Loaded actuator response, drive current, preload, and strength determine the achievable motion [4]. The source response must also establish the required supporting configuration and positive contact force. The motion command does not determine when b reaches its minimum or how far it falls.

For Figure 1, set $\tau = T/4$ and $\Delta U = 2K_{\max}$. The return uses two equal ramp intervals of duration $t_r = (T - \tau)/6$ with a constant-speed interval between them. Define the positive plateau speed $v_r = X/(T - \tau - t_r)$, so the integrated inward displacement is exactly X . Let u be elapsed time within either ramp, with $0 \leq u \leq t_r$. The starting and stopping velocity commands are

$$v_{\text{start}}(u) = -\frac{v_r}{2}[1 - \cos(\pi u/t_r)], \quad v_{\text{stop}}(u) = -\frac{v_r}{2}[1 + \cos(\pi u/t_r)]. \quad (45)$$

Between the ramps, velocity is $-v_r$. Integration returns the disk from $x = X$ to $x = 0$, with zero velocity and acceleration at both endpoints. Equation (23) and Table 2 specify the supporting response; Eqs. (13), (25), and (26) give the plotted power, force, and impulse.

Statements and Declarations

Funding. This work was self-funded by the author. No funds, grants, or other external support were received during the preparation of this manuscript.

Conflict of Interest. On behalf of all authors, the corresponding author states that there is no conflict of interest.

Author contributions. Todd J. Desiato conceived the OQG program and the material-scaling interpretation, directed the theoretical development, reviewed the calculations and literature, and prepared the manuscript.

Data availability. This theoretical study generated no new experimental or observational datasets. The analytical derivations, operating conditions, and numerical comparison inputs are contained in the article. Table 3 follows directly from Eqs. (33)–(37). Figure 1 evaluates the normalized prescribed cycle described in Section 10.5 and Appendix C; it is not a measured trace or a prediction of a particular actuator.

Code availability. The reported comparisons and normalized figure are evaluations of the equations and inputs stated in the article. They require no external experimental dataset or proprietary simulation

package. The figure’s extension, return command, support profile, and phase conditions are specified so that the traces can be independently reconstructed.

Ethics approval and consent to participate. Not applicable. This study involved no human participants, human data, animals, or biological material.

Consent for publication. Not applicable.

Use of generative artificial intelligence. The author used OpenAI ChatGPT and Codex for literature discovery, mathematical cross-checking, numerical evaluation, organization, drafting, and editing. Grok was used for a critical review of the draft. The author is responsible for reviewing the scientific arguments, equations, references, and final text and for the content of the manuscript.

Supplementary information. The average-thrust derivation is included in the main text, Section 10. Supporting microscopic, directional, and motion-command mathematics is included in Appendices A–C. No separate supplementary files accompany this article.

References

- [1] Fearn, H.: Mach effects for in space propulsion: interstellar mission. NASA Innovative Advanced Concepts project description (2017). <https://www.nasa.gov/general/mach-effects-for-in-space-propulsion-interstellar-mission/>. Accessed 20 October 2026
- [2] Fearn, H., van Rossum, N., Wanser, K., Woodward, J.F.: Theory of a Mach Effect Thruster II. *J. Mod. Phys.* **6**, 1868–1880 (2015). <https://doi.org/10.4236/jmp.2015.613192>
- [3] NASA Goddard Space Flight Center: Ask an Astrophysicist: Do photons have mass? Response by Andy Ptak, 31 July 1996. https://imagine.gsfc.nasa.gov/ask_astro/relativity.html. Accessed 20 October 2026
- [4] Physik Instrumente: Dynamic operation. Piezo actuator technical documentation. <https://www.physikinstrumente.com/en/expertise/technology/piezo-technology/properties-piezo-actuators/dynamic-operation>. Accessed 20 October 2026
- [5] Sciama, D.W.: On the origin of inertia. *Mon. Not. R. Astron. Soc.* **113**, 34–42 (1953). <https://doi.org/10.1093/mnras/113.1.34>
- [6] Woodward, J.F.: *Making Starships and Stargates: The Science of Interstellar Transport and Absurdly Benign Wormholes*. Springer, New York (2013). <https://doi.org/10.1007/978-1-4614-5623-0>
- [7] Desiato, T.J.: Operational Quantum Gravity: From Electromagnetic Power Balance to Gravitational Response. Preprint, version 2.2 (2026).
- [8] Ford, G.W., O’Connell, R.F.: Lorentz Transformation of Blackbody Radiation. *Phys. Rev. E* **88**, 044101 (2013). <https://doi.org/10.1103/PhysRevE.88.044101>
- [9] Physik Instrumente: Piezo Motion Control Tutorial: Mechanical Considerations. <https://www.pi-usa.us/en/products/piezo-flexure-nanopositioners/piezo-motion-control-tutorial/tutorial-4-22>. Accessed 21 October 2026