

# Operational Quantum Gravity

## From Electromagnetic Power Balance to Gravitational Response

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### Abstract

The power balance between matter and its electromagnetic environment provides the starting point for a material account of gravity. Incoming and outgoing powers balance at equilibrium, but their common value can change when surrounding matter scatters the shared field. We develop the hypothesis that this loss of available supporting power produces the material response measured as gravity. A calculation for two resonant dipoles shows how the field scattered by one dipole interferes with the incident field at the other, reducing absorbed power by 0.07546% at the stated parameters. We derive nuclear capacitance and inductance from spectral response. Within the stated corresponding-state family, we prove that bound states remain stable as their dimensions and clock frequencies change, and that the supporting-power ratio equals the square of the clock-frequency ratio. The resulting change of complete material energy supplies recoverable gravitational work: lifting increases that storage, while its spatial gradient produces a dispersion force. The associated matter-wave Hamiltonian generates geodesic motion. The same long-wavelength spin-2 response yields propagating permittivity and permeability disturbances with two gravitational-wave polarizations and positive energy. Matching the interaction strength to measured gravity gives an Earth supporting-power deficit of 1.39 parts per billion, with consistent clock and work comparisons. Allowing clock and ruler factors to differ also reconstructs the Schwarzschild exterior under explicit response conditions. Under the stated conditions, the supporting-power law links stable material scaling, recoverable gravitational work, gravitational potential energy stored in matter, and geodesic motion. The resulting geometric description represents the dynamics generated by the material Hamiltonian.

Keywords: electromagnetic scattering; nuclear response; supporting power; recoverable energy storage; dispersion forces; matter-wave scaling

# 1 Introduction

Matter in equilibrium can exchange energy continuously while its average stored energy remains constant. Power entering its internal motion balances power leaving through radiation and other coupled channels. The common value of those two powers can nevertheless differ between environments. Operational Quantum Gravity (OQG) takes this balanced exchange as the starting point for understanding how the presence of other matter changes a body's size, clock rate, and recoverable energy.

The physical hypothesis is that scattering reduces the power available to support that equilibrium. The shared electromagnetic field drives charge and current responses in surrounding matter, which reradiates a scattered field. At a receiving body, the incident and scattered fields interfere and change the power entering its internal motion. The body then adjusts to a new balance. In OQG, the loss of available supporting power is the source of gravity: it controls the material response, and the spatial variation of the resulting stored energy supplies the force.

This power interpretation develops the author's engineering model [6] and the electromagnetic energy-flow approach of Electrogravimagnetics [25, 26]. Puthoff's particle–vacuum equilibrium calculation [21] provides an explicit example of balanced absorption and radiation reaction. Dicke's electromagnetic account of gravitational standards [8] and microscopic descriptions of electromagnetic coefficients [15, 27] provide related context. Fluctuation–dissipation theory [4] and scattering methods [14, 22] allow the exchange, stored energy, and force to be calculated from the same response. The contribution of this paper is the supporting-power theorem and its connection to stable matter-wave scaling, nuclear storage, recoverable work, and geodesic motion. The resonant-dipole absorption inequality supplies an explicit electromagnetic example of reduced reception within the scattering framework of [16].

The first task is to establish that scattering can reduce received power. We use the resonant-dipole model of Luis-Hita and colleagues [16] to separate the direct field, the field scattered by a second dipole, and their interference. The calculated negative interference exceeds the added scattered intensity. This example makes the sign of the change explicit. Its role is to establish an electromagnetic mechanism for reduced reception; the nuclear application requires the balanced exchange of the complete material system.

The next task is to determine what such a change makes matter do. Nuclear excitation energies and charge-transition strengths define capacitance and inductance, while the spatial charge and current patterns connect these components to permittivity and permeability. We then prove the scaling of stable bound states and their supporting power. This establishes how a lower exchange level can accompany smaller dimensions, slower clocks, and changed energy without losing stable binding. The complete-body energy gives the work of lifting and the force on freely moving matter.

The derivation proceeds from exchange and interference in Section 2 to nuclear storage in Section 3. Section 3.5 assembles the coupled equilibrium conditions and relates a small isotropic load to supporting power on the common branch. Sections 4–6 establish stable scaling, recoverable work, and geodesic motion. Section 4.5 makes the collective wave dynamics explicit and connects its electromagnetic response to the radiative sector of WTDiff. Section 4.6 quantizes this response and identifies its massless spin-2 quanta. The gravitational strength is calibrated to measurement before the Earth comparisons in Section 7. Section 8 extends the clock and ruler response to the exact spherical exterior under stated

conditions. The purpose is to make each link between power balance and gravitational motion explicit, so that the collective nuclear calculation has a definite set of material consequences to reproduce.

## 2 Power balance and electromagnetic scattering

We first define the supporting-power comparison, then calculate how scattering can change the received power. The field, its sources, and the receiving currents must be treated together so that interference and energy conservation enter the same account.

### 2.1 The reference balance and the physical hypothesis

Consider a receiving body in a shared electromagnetic environment. Its internal charge and current fluctuations absorb energy and return energy to the field. At equilibrium these exchanges balance. Other matter responds to the same field and reradiates it; the resulting interference changes the field seen by the receiver. The OQG hypothesis identifies the associated reduction of balanced supporting power with the change that controls the receiver's material state. The storage response and its force are developed after the exchange calculation.

Throughout this paper, the local ambient field means the electromagnetic environment sampled by matter, including fluctuations, fields scattered by surrounding matter, and imposed source and boundary conditions. The bare zero-point vacuum is Lorentz invariant. Matter and boundaries can give the local ambient spectrum a physical rest frame; its measured spectrum can then depend on motion relative to those sources [28]. The field and matter equations remain Lorentz covariant: they retain their form when all participating quantities are transformed consistently. Supporting power is the balanced exchange of the specified receiver with this environment.

Compare the same receiving system with and without the additional source contribution, using fixed reference standards. Let  $P_0 = P_{\text{bal},0}$  be the reference balanced supporting power,  $P_b = P_{\text{bal}}$  the changed value, and  $\Delta P_{\text{scat}} = P_0 - P_b$  the reduction. The relative power  $p = P_b/P_0$  and fractional deficit  $\delta_p = 1 - p$  describe the same comparison. On the contraction branch,  $0 < p \leq 1$ , so

|                                                                                                                                       |
|---------------------------------------------------------------------------------------------------------------------------------------|
| $P_b = P_0 - \Delta P_{\text{scat}}, \quad \frac{P_b}{P_0} = 1 - \delta_p, \quad \frac{P_0}{P_b} = \frac{1}{1 - \delta_p}. \quad (1)$ |
|---------------------------------------------------------------------------------------------------------------------------------------|

Here  $P_{\text{in}}$  is power entering the material modes retained in the calculation and  $P_{\text{out}}$  is power leaving them. Each stationary state satisfies  $P_{\text{in}} = P_{\text{out}}$ ; its supporting power is this common value. The outward channels include radiation and the associated reaction on the emitting currents. Thus a power loss between equilibria means a lower exchange level. Energy released or supplied while the body adjusts is included in the changes of storage, radiation, and mechanical work.

Equation (1) is the power budget in watts. It compares two settled states of the same system. The material constitutive laws, which relate fields to stored energy, will express the response through  $P_0/P_b$  and its reciprocal. The calculation therefore has two connected parts: determine the change of exchange, then determine the material equilibrium sustained by the changed exchange.

A subscript 0 denotes the reference state throughout. We distinguish two positive material factors:  $f$  is the ratio of transition frequencies or energies, and  $b$  is the ratio of corresponding lengths. Their equality,

$f = b$ , defines the common branch on which clocks and sizes change by the same fraction. Charge and  $\hbar = h/(2\pi)$  remain fixed, with  $h$  denoting Planck's constant. We use  $\omega$  for a selected angular frequency and  $\nu$  for the angular-frequency integration variable. Both are in radians per second; ordinary frequency in hertz is  $\nu_{\text{ph}} = \omega/(2\pi)$ . The Fourier convention is  $e^{-i\nu t}$ , with reference time  $t$  and  $i^2 = -1$ .

The reference exchange  $P_0$  is held fixed throughout each comparison and its spatial derivatives. It is the receiver's reference balanced power, rather than an integral of bare vacuum energy or the net difference between incoming and outgoing powers. On the common matched branch, bodies in the same local material state share the fractional ratio  $p$  and scale factor  $b$ , even when their total reference exchanges in watts differ.

The electrical description uses capacitance  $C$ , in farads, for charge storage and inductance  $L$ , in henries, for current storage. Radiation resistance  $R_{\text{rad}}$ , in ohms, represents the power carried away by the field for a given current; the total dissipative resistance  $R$  includes all assigned outward channels. Coupled modes require matrices of these quantities. Permittivity  $\epsilon$  and permeability  $\mu$  describe the corresponding field storage, with reference values  $\epsilon_0$  and  $\mu_0$ . The reference propagation speed is  $c_0 = (\epsilon_0\mu_0)^{-1/2}$ , and  $c$  denotes the changed speed in the same comparison standards.

Figure 1 shows why this is a coupled equilibrium problem. The nuclear state determines the responding currents and their storage. Those currents exchange energy through the electric field  $\mathbf{E}$  and magnetic field strength  $\mathbf{H}$ , whose energy flux is the Poynting vector  $\mathbf{S}$ , given by  $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ . Scattering changes the exchange, and the adjusted material state changes the currents that participate in it. Retarded coupling includes the finite propagation delay.

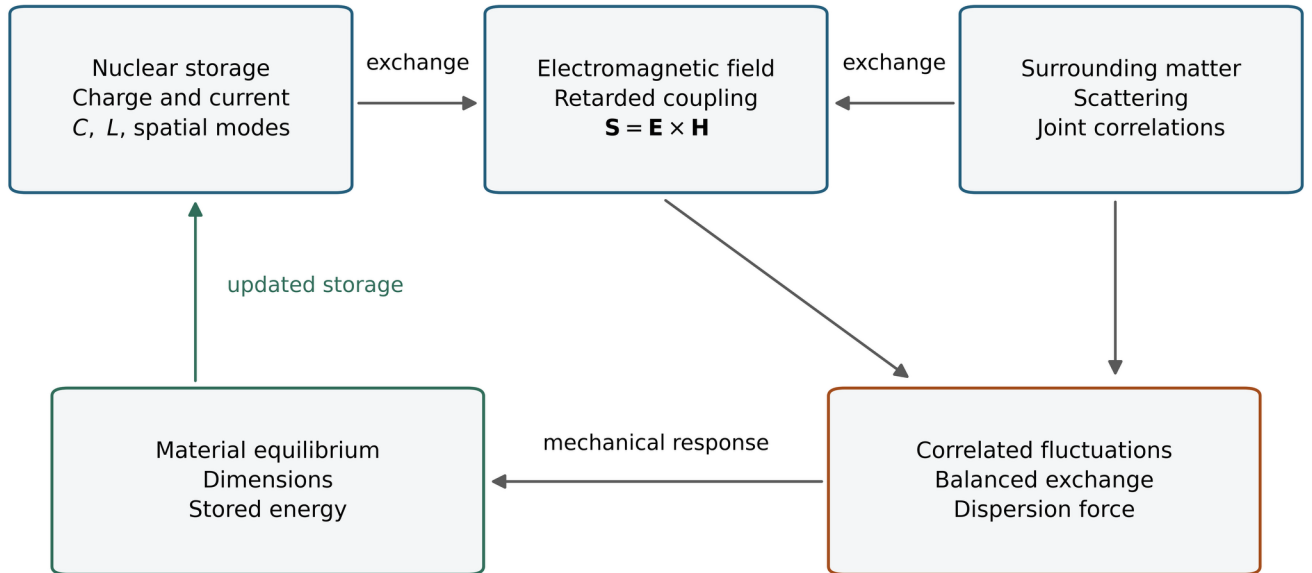


*Fig. 1 Nuclear storage and electromagnetic exchange. Nuclear charge and current fluctuations communicate through one shared field. Scattering changes their correlations and supporting power; the material storage and dimensions adjust. The return arrow indicates that the adjusted material response must be used in the field calculation.*

## 2.2 Energy flow between responding currents

To calculate the exchange, we need both the energy carried by the field and the work it does on matter. Let  $\mathbf{E}$  be electric field,  $\mathbf{H}$  magnetic field strength,  $\mathbf{B}$  magnetic flux density, and  $\mathbf{J}$  current density. The Poynting vector  $\mathbf{S}$  measures local energy flow in watts per square meter. With position  $\mathbf{x}$ , electromagnetic energy  $U_{\text{EM}}$  in volume  $V$ , boundary  $\partial V$ , and outward area element  $d\mathbf{A}$ , energy conservation takes the following form. Bold symbols denote spatial vectors; in the microscopic vacuum description,  $\mathbf{H} = \mathbf{B}/\mu_0$ .

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}, \quad -\oint_{\partial V} \mathbf{S} \cdot d\mathbf{A} = \frac{dU_{\text{EM}}}{dt} + \int_V \mathbf{J} \cdot \mathbf{E} d^3x. \quad (2)$$

The left side is net power entering the volume. It either changes the field energy inside or does work on the currents. This balance applies during adjustment as well as at a stationary endpoint.

For a traveling wave in a lossless, nondispersive medium, the energy flux is the propagation speed times the wave energy density. EGM IV uses this relation for spectral power density  $S_\omega$  and energy density  $U_\omega$ , with  $S_\omega = cU_\omega$  [25, Section 3.4.2, Eq. (12)]; EGM V proposes corresponding Poynting-vector simulations [26]. Equation (2) also includes the near fields and overlapping currents needed for the present coupled system.

We now describe the actual patterns that transmit and receive that power. Resolve the charge density  $\rho$  and current density  $\mathbf{J}$  into modes indexed by  $a$ . Each mode has charge amplitude  $Q_a$ , current amplitude  $I_a$ , and spatial profiles  $r_a(\mathbf{x})$  and  $\mathbf{j}_a(\mathbf{x})$  at position  $\mathbf{x}$ . A dot denotes a time derivative and  $\nabla \cdot$  denotes divergence. The following choice of profiles enforces charge conservation:

$$\rho = \sum_a Q_a r_a(\mathbf{x}), \quad \mathbf{J} = \sum_a I_a \mathbf{j}_a(\mathbf{x}), \quad \nabla \cdot \mathbf{j}_a = -\dot{r}_a, \quad I_a = \dot{Q}_a. \quad (3)$$

The profiles  $r_a$  and  $\mathbf{j}_a$ , with units  $\text{m}^{-3}$  and  $\text{m}^{-2}$ , retain the geometry of the charges and currents. Their field propagates through the reference environment according to the outgoing electric Green tensor  $\mathbf{G}_0$ . This tensor is the field produced by a localized current source, including near-field structure, propagation delay, and outgoing radiation. Define wave number  $k = \nu/c_0$ , identity tensor  $\mathbb{I}$ , and the point-source distribution  $\delta^{(3)}$ . With  $\nabla \times$  denoting curl, the field equation is

$$[\nabla \times \nabla \times - k^2] \mathbf{G}_0 = \mathbb{I} \delta^{(3)}, \quad \mathbf{E} = i\mu_0 \nu \mathbf{G}_0 \mathbf{J}. \quad (4)$$

The receiving voltage is obtained by projecting this arriving field onto the receiving current pattern. Multiplication by  $\mathbf{G}_0$  includes integration over source position. Let  $v_{a \leftarrow b}$  be the voltage in mode  $a$  due to source current  $I_b$ . Their mutual impedance  $Z_{ab}$ , in ohms, is the voltage-to-current coupling. For receiving position  $\mathbf{x}$ , source position  $\mathbf{y}$ , and convention  $v_{a \leftarrow b} = -Z_{ab} I_b$ ,

$$Z_{ab}(\nu) = -i\mu_0 \nu \iint d^3x d^3y \mathbf{j}_a(\mathbf{x}) \cdot \mathbf{G}_0(\mathbf{x}, \mathbf{y}; \nu) \mathbf{j}_b(\mathbf{y}). \quad (5)$$

Mutual impedance describes both recoverable energy exchange and radiation between the modes. For reciprocal media and compatible mode definitions, exchanging the source and receiver preserves their transfer relation. Each isolated response already includes its assigned self-field; the mutual term adds the field of the other currents.

The same accounting can be expressed directly in terms of field response, without introducing circuit modes. This is useful for ensuring that electromagnetic feedback is included once. The proper current response  $\tilde{\Pi}$  maps the total potential to induced current, whereas  $\Pi$  maps an externally applied probe potential to that current. The free propagator  $D_0$  converts current to potential before material feedback; the dressed propagator  $D$  includes the feedback. Operator products include the required integrations and sums [24]:

$$\Pi = \tilde{\Pi} + \tilde{\Pi} D_0 \Pi, \quad D = D_0 + D_0 \tilde{\Pi} D. \quad (6)$$

The first identity distinguishes the response to the total field from the response measured with an external probe. The second reconstructs propagation after matter has responded. They express the same source–field feedback used in the mutual-impedance description.

The far field shows how the mode geometry enters radiation. Let  $\mathbf{n}$  be a unit observation direction and  $r$  the distance from a localized source. The form factor  $\mathbf{g}_a(\mathbf{n}, \nu) = \int d^3x \mathbf{j}_a(\mathbf{x}) e^{-i\mathbf{k}\mathbf{n}\cdot\mathbf{x}}$  adds contributions from different parts of the current pattern with their propagation phases. The projector  $P_T = \mathbb{I} - \mathbf{n}\mathbf{n}^T$  keeps the components perpendicular to  $\mathbf{n}$ ;  $T$  denotes transpose. Write the distant field as  $\mathbf{E}_{\text{far}}$ , with  $O(r^{-2})$  representing terms that fall at least as fast as  $r^{-2}$ . Then

$$\mathbf{E}_{\text{far}}(r\mathbf{n}, \nu) = \frac{i\mu_0\nu e^{ikr}}{4\pi r} P_T \sum_a I_a \mathbf{g}_a + O(r^{-2}). \quad (7)$$

The field amplitude falls as  $1/r$ , while its traveling-wave energy flux falls as  $1/r^2$ . The expanding sphere carries the same total radiated power. Figure 2 shows this geometry and the propagation phase, with the radius normalized to reference distance  $r_{\min}$ . The current form factor determines how the radiation is distributed over direction.

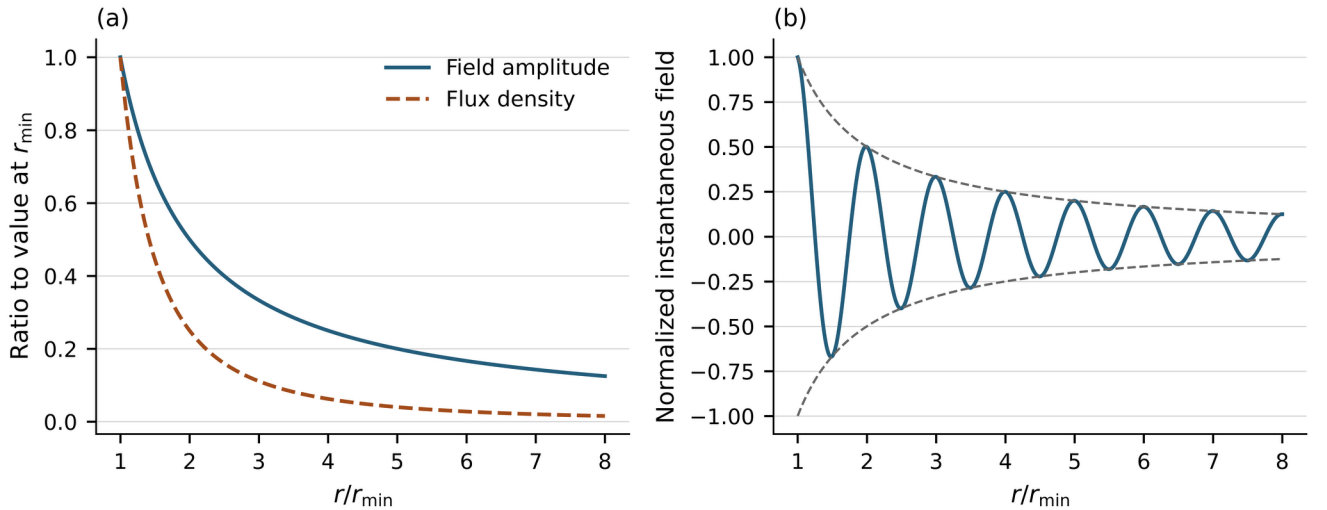


**Fig. 2** Retarded Maxwell propagation. The left panel compares normalized far-field amplitude and traveling-wave flux;  $r_{\min}$  is the reference distance used to normalize the plotted radius. The right panel shows the phase delay accumulated during propagation. The full Green tensor also retains near-field terms and finite source form factors.

To obtain the total outward power, collect the harmonic current amplitudes and phases in the column  $I$ . The radiation-resistance matrix  $R^{\text{rad}}$  then gives  $P_{\text{out}} = I^\dagger R^{\text{rad}} I / 2$ . A star denotes complex conjugation and a dagger conjugate transpose. Integration over solid angle  $d\Omega_{\mathbf{n}}$  gives each matrix entry:

$$R_{ab}^{\text{rad}}(\nu) = \frac{\nu^2}{16\pi^2 \epsilon_0 c_0^3} \int d\Omega_{\mathbf{n}} \mathbf{g}_a^* \cdot P_T \mathbf{g}_b. \quad (8)$$

This is a Gram matrix: its entries are overlaps of the radiating current patterns. It is therefore positive semidefinite, so the total outward power is nonnegative for every current distribution. Off-diagonal entries retain interference between different currents. The formula applies to an individual mode, a correlated cluster, or an extended source.

We now have a common description of transmission, reception, and radiation using the same currents. The mutual impedance gives the field-induced voltage, and the radiation matrix gives the outward power. Keeping their phases allows us to calculate the interference responsible for the next result.

### 2.3 Interference reduces the received power

Consider two identical absorbing electric dipoles in the random-field model of Luis-Hita and colleagues [16]. Each dipole is a localized charge displacement that responds linearly to the electric field. In the common rest frame of the dipoles, the prescribed incident field is homogeneous and isotropic, with the same statistics everywhere and no preferred direction, and is concentrated in a narrow frequency band. Each dipole receives this incident field together with the field scattered by its partner. We calculate their interference before taking the average received intensity.

A receiver moving relative to this field generally samples direction-dependent Doppler shifts and aberration. Its received power must be calculated from the transformed field correlations and receiving response together [28]. The following example evaluates the stated stationary configuration.

Let  $r$  be the separation,  $k$  the wave number, and  $x = kr$  their dimensionless product. The volume-normalized polarizability  $\alpha_{\text{pol}}$  measures induced dipole response and has units of volume. At resonance write  $k^3 \alpha_{\text{pol}} = ia$ , defining the positive dimensionless response strength  $a$ . Resolve the field into transverse ( $T$ ) and longitudinal ( $L$ ) components relative to the line joining the dipoles. The dimensionless propagator  $g_i = u_i + iv_i$  for polarization component  $i$  has real part  $u_i$  and imaginary part  $v_i$ ; this index  $i$  is distinct from the imaginary unit. Its two forms are

$$\begin{aligned} g_T(x) &= \frac{e^{ix}}{4\pi x} \left( 1 + \frac{i}{x} - \frac{1}{x^2} \right), \\ g_L(x) &= \frac{e^{ix}}{4\pi x} \left( -\frac{2i}{x} + \frac{2}{x^2} \right), \quad g_i = u_i + iv_i. \end{aligned} \quad (9)$$

For one polarization component, let  $E_1^{(0)}$  and  $E_2^{(0)}$  be the incident complex field amplitudes at the two positions. Angle brackets denote a statistical average. Set each incident variance to one. Although the

mean field vanishes, the product of fields sampled at the two positions can have a nonzero average. Angular averaging over transverse incident plane waves gives

$$\begin{aligned}\langle E_1^{(0)} \rangle &= \langle E_2^{(0)} \rangle = 0, & \langle |E_1^{(0)}|^2 \rangle &= \langle |E_2^{(0)}|^2 \rangle = 1, \\ \langle E_1^{(0)} E_2^{(0)*} \rangle &= 6\pi v_i.\end{aligned}\tag{10}$$

Both dipoles sample one shared random field, so their incident fields remain correlated. The correlation  $6\pi v_i$  retains the phase information needed to calculate interference. A zero mean field therefore does not imply a zero interference contribution.

Let  $E_1$  and  $E_2$  be the total fields at the two receivers for the selected component. Each equals its incident field plus the field scattered by its partner. Solving these two coupled equations gives the field at receiver 1:

$$\begin{aligned}E_1 &= E_1^{(0)} + iag_i E_2, & E_2 &= E_2^{(0)} + iag_i E_1, \\ E_1 &= \frac{E_1^{(0)} + iag_i E_2^{(0)}}{1 + a^2 g_i^2}.\end{aligned}\tag{11}$$

The numerator adds the incident and scattered amplitudes. The denominator includes their repeated exchange between the dipoles. Define the nonzero denominator factor  $\mathcal{D}_i = |1 + a^2 g_i^2|^2$  and the absorbed-power ratio  $Q_i$  for component  $i$ . With the receiver polarizability unchanged, this equals the local intensity ratio. The polarization-averaged absorbed powers in the coupled and reference states are  $P_{\text{absorbed}}$  and  $P_{\text{absorbed},0}$ .

**Proposition 1 Scattering reduces receiver absorption in the resonant example.** For  $0 < a \leq 4\pi$  and a finite coupled response,  $Q_i \leq 1$ . The component and polarization-averaged ratios are

$$Q_i = \frac{1 + a^2 |g_i|^2 - 12\pi a v_i^2}{\mathcal{D}_i}, \quad \frac{P_{\text{absorbed}}}{P_{\text{absorbed},0}} = \frac{2Q_T + Q_L}{3}.\tag{12}$$

These are powers entering the receiver's absorptive channels, compared with the same receiver in the incident field without its partner. The reference fixes what is meant by a reduction.

**Proof.** Expand the squared magnitude of the local-field numerator. The direct contribution is one and the scattered contribution is  $a^2 |g_i|^2$ . With  $\text{Re}$  denoting the real part, their interference term is

$$2\text{Re}\left[-iag_i^* \langle E_1^{(0)} E_2^{(0)*} \rangle\right] = -12\pi a v_i^2.\tag{13}$$

Division by  $\mathcal{D}_i$  gives the first ratio in the proposition. Equal incident variance and isotropy give two transverse contributions and one longitudinal contribution, proving the second ratio. To determine the sign without discarding repeated scattering, subtract the numerator from its exact denominator. Writing  $g_i = u_i + iv_i$  gives

$$1 - Q_i = \frac{a[(12\pi - 3a)v_i^2 + au_i^2 + a^3(u_i^2 + v_i^2)^2]}{\mathcal{D}_i} \geq 0.\tag{14}$$

The denominator is positive and each term in the numerator is nonnegative in the stated interval. Averaging preserves the inequality. The power reduction is therefore proved from the correlated field solution. At infinite separation the propagators vanish and the reference ratio is recovered.

The partner adds positive scattered intensity, but its overlap with the direct field contributes a larger negative interference term in the stated resonant interval. The resulting local intensity is smaller. This changes the receiver's share of the exchange; conservation concerns the complete source, field, and receiving systems.

For the illustrative response strength  $a = 0.01$  and separation  $kr = 1$ , the three contributions average to the values in Table 1.

**Table 1** Contributions to the normalized absorbed receiver power

| Contribution                                                | Value           |
|-------------------------------------------------------------|-----------------|
| Direct field, including the repeated-scattering denominator | 0.999996577613  |
| Positive scattered intensity                                | +0.000002110842 |
| Interference                                                | −0.000753327938 |
| Sum: $P_{\text{absorbed}}/P_{\text{absorbed},0}$            | 0.999245360517  |
| Deficit relative to the reference                           | 0.000754639483  |

Their sum gives a reduction of 0.07546395%. Both its sign and its size follow from the correlated field solution, including repeated scattering. The example therefore establishes the first required step: another responding body can reduce power received from a shared electromagnetic field.

The ratio becomes a power in watts once the incident energy density and receiving area are specified. Let  $\sigma_{\text{absorbed}}$  be the absorption cross section and  $w$  the total incident electromagnetic energy per unit volume. The reaction-corrected cross section separates absorbed energy from reradiated energy. Multiplying it by the wave-flux scale  $cw$  gives the reference absorbed power:

$$k^2 \sigma_{\text{absorbed}} = a - \frac{a^2}{6\pi}, \quad P_{\text{absorbed},0} = cw \sigma_{\text{absorbed}}. \quad (15)$$

The reradiated contribution  $a^2/(6\pi)$  is removed in calculating  $\sigma_{\text{absorbed}}$ . The reference power multiplied by the fractional deficit is the received-power reduction in watts. This conversion uses the same polarizability and radiation correction as the local-field calculation.

For the nuclear application, the relevant quantity is the change in the complete body's balanced supporting exchange. The dipole example establishes how interference can reduce reception. Calculating the nuclear equilibrium additionally requires its current spectrum and the compensating fluctuations associated with its outward channels. Once that balanced-power change is known, Eq. (1) gives  $P_b$ . The power theorem in Section 4 then gives  $f = \sqrt{p}$ ; on the common branch it also determines the size. The next subsection supplies the equilibrium accounting needed for this connection.

## 2.4 Fluctuations and radiation reaction maintain the balance

An equilibrium calculation must include both the energy accepted by matter and the fluctuations it returns to the field. We first express these two contributions in wave channels, then calculate their balance in material coordinates. A port is an incoming or outgoing wave channel whose amplitude squared is power. If  $S$  maps incoming to outgoing amplitudes, the absorption and emission operators of a passive body are  $\mathcal{A}$  and  $\mathcal{E}$ :

$$\mathcal{A} = \mathbb{I} - S^\dagger S, \quad \mathcal{E} = \mathbb{I} - S S^\dagger. \quad (16)$$

Absorption measures power entering the internal material channels; emission accounts for their contribution back to the field. The associated fluctuations must be included with the propagating waves. Incident, scattered, and reaction fields are contributions to this one correlated field.

To calculate the material balance, eliminate the field while retaining its reaction on the charges. Here  $\mathcal{D}$  denotes the inverse charge-response matrix, rather than the potential propagator used in Eq. (6). Its conservative part  $\mathcal{A}_H$  and dissipative resistance  $R$  obey  $\mathcal{D} = \mathcal{A}_H - i\nu R$ . For a matrix  $X$ , define  $\text{Im}X = (X - X^\dagger)/(2i)$ , giving  $R = -\text{Im}\mathcal{D}/\nu$ . The subscript  $H$  identifies the Hermitian part  $\mathcal{A}_H$ , equal to its conjugate transpose. The charge susceptibility is  $\chi = \mathcal{D}^{-1}$ , in farads. Fluctuating generalized voltages  $\xi$  then produce charges  $Q = \chi\xi$ .

The fluctuation–dissipation relation specifies the voltage noise associated with that same resistance. In the common equilibrium rest frame, at temperature  $T$ , let  $k_B$  be Boltzmann's constant and  $S_\xi(\nu)$  the voltage spectral matrix per angular-frequency interval. We use two-sided spectra and symmetrized quantum products, averaging the two operator orders. With hyperbolic cotangent  $\coth$ , the relation is [4]

$$S_\xi(\nu) = \hbar\nu\coth\left(\frac{\hbar\nu}{2k_B T}\right)R(\nu), \quad \text{Im}\chi = \nu\chi R\chi^\dagger. \quad (17)$$

At zero temperature,  $\nu\coth[\hbar\nu/(2k_B T)]$  tends to  $|\nu|$ . Let  $S_Q$  be the induced charge spectrum and  $P_{\text{diss}}$  the outward power through the retained dissipative channels. The trace  $\text{Tr}$  sums the mode contributions, with their correlations retained in the matrix products. The input and outward powers are then

$$\begin{aligned} S_Q &= \chi S_\xi \chi^\dagger = \hbar\coth\left(\frac{\hbar\nu}{2k_B T}\right)\text{Im}\chi, \\ P_{\text{in}} &= \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \nu \text{Tr}(S_\xi \text{Im}\chi), \\ P_{\text{diss}} &= \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \nu^2 \text{Tr}(R S_Q) = P_{\text{in}}. \end{aligned} \quad (18)$$

**Power-balance identity.** Substituting  $S_Q = \chi S_\xi \chi^\dagger$  into the dissipative integral and using  $\text{Im}\chi = \nu\chi R\chi^\dagger$ , together with the common-temperature fluctuation relation, gives the input integral. Thus the same causal response fixes fluctuations and reaction and proves  $P_{\text{in}} = P_{\text{diss}}$ . We use the physical high-frequency response, or a consistently matched subtraction, to define convergent exchange integrals.

A single oscillator makes the normalization concrete. In Puthoff's nonrelativistic equilibrium example [21], let displacement be  $x$ , charge  $q$ , mass  $m$ , mode length  $\ell$ , and restoring angular frequency  $\omega_0$ . Express

the motion as charge  $Q = qx/\ell$  and current  $I = \dot{Q}$ . The inductance  $L$  represents the moving charge's inertia, the capacitance  $C$  its restoring compliance, and  $R_{\text{rad}}(\nu)$  its radiation resistance:

$$L = \frac{m\ell^2}{q^2}, \quad C = \frac{q^2}{m\omega_0^2\ell^2}, \quad R_{\text{rad}}(\nu) = \frac{\ell^2\nu^2}{6\pi\epsilon_0 c_0^3}. \quad (19)$$

When radiative damping is weak and the response is concentrated near  $\omega_0$ , the mean-square current is  $\langle I^2 \rangle = \hbar\omega_0/(2L)$ . Define the dimensionless radiative coupling  $g_{\text{rad}} = R_{\text{rad}}(\omega_0)/(L\omega_0)$ , which compares the radiative load with the oscillator's inductive scale. The balanced supporting power is

$$P_{\text{bal}} = R_{\text{rad}}(\omega_0)\langle I^2 \rangle = g_{\text{rad}} \frac{\hbar\omega_0^2}{2}, \quad g_{\text{rad}} = \frac{R_{\text{rad}}(\omega_0)}{L\omega_0} = \frac{q^2\omega_0}{6\pi\epsilon_0 mc_0^3}. \quad (20)$$

The same oscillator thus has equal absorbed and radiated powers and an explicit supporting power in watts [21]. The dimensionless radiative coupling converts the squared-frequency scale into that power. For nuclear modes, the current profiles in the radiation matrix replace the point-oscillator approximation while preserving the same balance.

## 2.5 The exchange also carries an interaction force

Zero net heat flow at equilibrium does not require zero force. Correlated fields can store an interaction energy that changes with separation. To relate exchange to this energy, use the isolated-body scattering operators  $T_1, T_2$ , which map incident fields to induced sources. They describe the same current response as above. The propagator  $W = (\mathbb{I} - G_0 T_1 G_0 T_2)^{-1} G_0$  includes repeated scattering between the bodies. Let  $\mathcal{E}_1$  represent emission from body 1,  $\mathcal{A}_2$  absorption by body 2, and  $\mathcal{T}_{12}$  their dimensionless transfer factor. In the conventions of [14],

$$\begin{aligned} \mathcal{E}_1 &= \text{Im}T_1 - T_1(\text{Im}G_0)T_1^\dagger, \\ \mathcal{A}_2 &= \text{Im}T_2 - T_2^\dagger(\text{Im}G_0)T_2, \\ \mathcal{T}_{12} &= 4\text{Tr}(\mathcal{A}_2 W \mathcal{E}_1 W^\dagger). \end{aligned} \quad (21)$$

Let  $n_1(\nu)$  be the mean thermal occupation of a mode of angular frequency  $\nu$  in body 1. Its one-way power is the integral of  $\hbar\nu n_1(\nu)\mathcal{T}_{12}(\nu)$  over positive  $d\nu/(2\pi)$ . Reverse exchange and the surrounding field complete the balance. At a common equilibrium temperature the net heat transfer vanishes, while the field correlations can still exert stress.

The equilibrium interaction energy can be obtained directly from the same scattering response. Let  $\Delta F_{\text{EM}}(x)$  be the interaction free energy relative to isolated bodies; at zero temperature it is the interaction energy. The coordinate  $x$  may describe separation or material deformation. The calculation uses imaginary angular frequency  $\nu = i\zeta$ , with real  $\zeta \geq 0$ , and  $\log\det$ , the logarithm of a determinant. For two disjoint linear bodies at zero temperature,

$$\Delta F_{\text{EM}}(x) = \frac{\hbar}{2\pi} \int_0^\infty d\zeta \log\det[\mathbb{I} - G_0 T_1 G_0 T_2](i\zeta; x). \quad (22)$$

All separation and deformation dependence must be retained before differentiating. The determinant sums repeated scattering, and its many-body form includes the other bodies and environmental channels.

The interaction energy identifies a dispersion force [22]. Let  $M = G_0 T_1 G_0 T_2$  be one round trip between the bodies, and let  $F_x^{\text{disp}}$  be the force conjugate to coordinate  $x$ , positive toward increasing  $x$ . Retaining every geometric and material dependence on that coordinate gives

$$F_x^{\text{disp}} = -\partial_x \Delta F_{\text{EM}} = \frac{\hbar}{2\pi} \int_0^\infty d\zeta \text{Tr}[(\mathbb{I} - M)^{-1} \partial_x M]. \quad (23)$$

This is a dispersion force: a force arising from correlated fluctuations and the bodies' frequency-dependent response [22]. Power measures their exchange per unit time, whereas force measures how their interaction energy changes with position. OQG connects the change of supporting power to material storage through the theorems in Sections 4 and 5.

We now return to the prescribed resonant random field of Section 2.3. Its attractive force is calculated for that field state [16]. Let  $F_r$  be the outward-positive radial force,  $U_{\text{scat}}(r)$  its interaction potential relative to infinite separation, and  $\mathcal{K}_{\text{EM}}$  its positive strength in joule meters. In the absorption-dominated regime  $a/(6\pi) \ll 1$  and weak repeated-scattering limit  $\max_i |ag_i|^2 \ll 1$ , the leading result is

$$F_r = -\frac{\mathcal{K}_{\text{EM}}}{r^2}, \quad U_{\text{scat}}(r) = -\frac{\mathcal{K}_{\text{EM}}}{r}, \quad \mathcal{K}_{\text{EM}} \simeq \frac{w\sigma_{\text{absorbed}}^2}{4\pi} > 0. \quad (24)$$

**Proposition 2 Conservative inverse-distance interaction.** A static central force with the displayed leading coefficient has potential  $-\mathcal{K}_{\text{EM}}/r$  and recoverable quasistatic work  $U_{\text{scat}}(r_2) - U_{\text{scat}}(r_1)$ . Integrating  $dU/dr = -F_r$  and applying the boundary value at infinity proves both statements. The coefficient has units of energy times length. Its sign establishes attraction, while its distance dependence supplies the inverse-distance interaction energy used in the subsequent storage correspondence.

The resonant example supplies two separately calculated quantities: a received-power deficit and a conservative attractive interaction with an inverse-distance potential. Their role in OQG is fixed by the storage hypothesis: the changed nuclear supporting exchange selects the material state whose energy supplies gravitational work. The following sections establish the material consequences of that identification. The overall strength is calibrated with measured  $G_0/c_0^4$ , where  $G_0$  is the reference gravitational constant.

### 3 Nuclear storage and the adjusted equilibrium

A power change has mechanical consequences only through the response of the receiving matter. We therefore need to know how its internal motion stores energy and resists deformation. This section obtains electrical storage quantities from the nuclear spectrum, relates them to the field coefficients, and evaluates a passive model of loading and restoration.

#### 3.1 The nuclear spectrum defines capacitance and inductance

Nuclear charge motion takes place within strongly bound matter. Its response to an electromagnetic field depends on the interacting nuclear states and their excitation energies, which retain the strong-interaction physics. We express that response as capacitance and inductance so that the internal storage can be connected to the exchange calculated in Section 2.

Let  $\delta\rho_{\text{em}}(\mathbf{x})$  be the nuclear charge density relative to its mean. Real, dimensionless weights  $w_a(\mathbf{x})$  select a charge-displacement pattern within the nucleus. Its amplitude  $Q_a$ , in coulombs, responds to a generalized voltage  $v_a$ , defined by its coupling to that pattern. If  $H$  is the nuclear Hamiltonian, the probe changes it by  $\delta H$ :

$$Q_a = \int d^3x w_a(\mathbf{x}) \delta\rho_{\text{em}}(\mathbf{x}), \quad \delta H = -\sum_a v_a Q_a. \quad (25)$$

A uniform weight measures conserved total charge; a nonuniform weight resolves motion of charge inside the nucleus. Retaining this normalization ensures that the same mode is used when calculating storage, radiation, and reception.

The retarded susceptibility  $\chi_{ab}^R$  measures how charge in channel  $a$  changes after a small voltage acts in channel  $b$ . Retarded, denoted by  $R$ , means that the response follows the perturbation. Let  $[A, B] = AB - BA$  denote a commutator,  $\theta(t)$  the step function vanishing for  $t < 0$ , and angle brackets the expectation in the specified state. Repeated channel indices are summed:

$$\begin{aligned} \chi_{ab}^R(t) &= \frac{i}{\hbar} \theta(t) \langle [Q_a(t), Q_b(0)] \rangle, \\ \delta\langle Q_a \rangle(\nu) &= \chi_{ab}^R(\nu) v_b(\nu). \end{aligned} \quad (26)$$

In frequency space this charge-per-voltage response has units of farads. For one reciprocal channel with a unique ground state  $|0\rangle$ , denote the excited states by  $|n\rangle$  and their energies by  $E_n$ ;  $\langle n|$  is the conjugate state. The excitation gaps  $\Delta_n = E_n - E_0$  specify the energy required for each transition, and the matrix elements  $q_n = \langle 0|Q|n\rangle$  specify how strongly charge responds through it. The infinitesimal prescription  $i0^+$  enforces causality. The resulting spectral response is

$$\chi^R(\nu) = 2 \sum_{n>0} \frac{\Delta_n |q_n|^2}{\Delta_n^2 - \hbar^2(\nu + i0^+)^2}. \quad (27)$$

Continuum states enter by integration, with physical renormalization understood. The interacting energies and matrix elements retain nuclear binding and confinement. Background-field and current-matrix-element methods provide ways to evaluate them [7, 12].

Below unresolved resonances, a frequency expansion separates static charge storage from the inertia of charge motion. Assume the required spectral sums are finite. The zero-frequency susceptibility defines capacitance  $C$ ; its next coefficient  $D_2$ , multiplying  $\nu^2$ , has units of farad seconds squared. Expanding the inverse response then identifies the inductance  $L$ :

$$\begin{aligned} \chi^R(\nu) &= C + \nu^2 D_2 + \dots, & [\chi^R(\nu)]^{-1} &= C^{-1} - \nu^2 L + \dots, \\ C &= 2 \sum_n \frac{|q_n|^2}{\Delta_n}, & D_2 &= 2\hbar^2 \sum_n \frac{|q_n|^2}{\Delta_n^3}, & L &= C^{-1} D_2 C^{-1}. \end{aligned} \quad (28)$$

The scalar sums show which nuclear transitions contribute to storage. The matrix definition of  $L$  applies on the nonsingular responsive subspace, and positive spectral weights give positive conservative storage. The units of  $C$ ,  $D_2$ , and  $L$  are farads, farad seconds squared, and henries. Near nuclear resonances the full susceptibility, rather than this low-frequency expansion, supplies the response.

Collect the charges in  $Q$  and their conjugate fluxes  $\Phi_a$ , in webers, in  $\Phi$ . Charge and flux form the canonical pair expressed by the commutator below. The recoverable quadratic storage energy is  $H_{\text{st}}$ . With  $\delta_{ab}$  equal to one when  $a = b$  and zero otherwise,

$$H_{\text{st}} = \frac{1}{2} Q^T C^{-1} Q + \frac{1}{2} \Phi^T L^{-1} \Phi, \quad [Q_a, \Phi_b] = i\hbar \delta_{ab}, \quad I = L^{-1} \Phi = \dot{Q}. \quad (29)$$

For an isolated harmonic mode,  $\omega = (LC)^{-1/2}$  and the ground-state energy is  $\hbar\omega/2$ . The LC form now has a nuclear meaning: the spectrum determines its storage coefficients, and the charge and current patterns determine how it couples to the field.

The nucleon-to- $\Delta(1232)$  excitation places this response on a physical frequency scale [17]. The  $\Delta(1232)$  is a short-lived excited nucleon resonance. Let  $M_\Delta$  be its nominal centroid mass and  $M_p$  the proton mass; one megaelectronvolt (MeV) is  $10^6$  electronvolts. The internal gap  $\Delta E = (M_\Delta - M_p)c_0^2$  corresponds to ordinary frequency  $\nu_{\text{gap}} = \Delta E/h$ . With  $M_\Delta c_0^2 = 1232$  MeV and  $M_p c_0^2 = 938.272088$  MeV,

$$\Delta E = 293.728 \text{ MeV}, \quad \nu_{\text{gap}} = \frac{\Delta E}{h} = 7.102 \times 10^{22} \text{ Hz}. \quad (30)$$

The internal gap must be distinguished from a real photon's absorption or emission energy because recoil also carries energy. For a stationary proton, the absorption resonance is  $(M_\Delta^2 - M_p^2)c_0^2/(2M_p) = 339.704$  MeV, or  $8.214 \times 10^{22}$  Hz; emission in the resonance rest frame gives  $(M_\Delta^2 - M_p^2)c_0^2/(2M_\Delta) = 258.713$  MeV. Figure 3 compares these quantities. The internal-gap wavelength is about 4.22 fm, with one femtometer equal to  $10^{-15}$  meter. The calculation locates the nuclear response near  $10^{23}$  Hz; the spectral weights determine how that band contributes to equilibrium.

The nuclear state therefore supplies both a storage response and the currents that communicate through the field. To obtain permittivity and permeability from those components, we must also retain their spatial geometry.

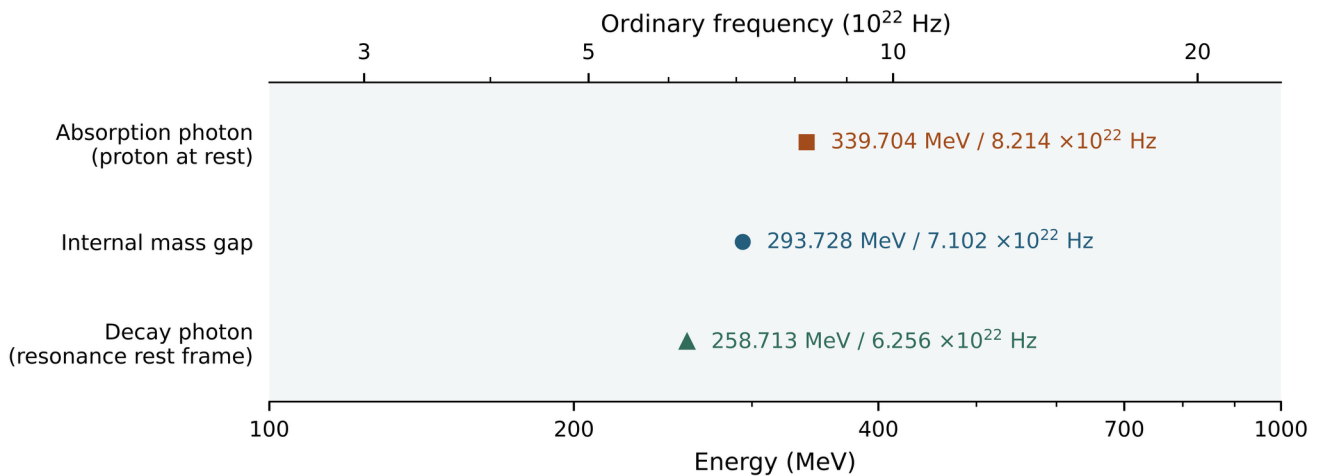


**Fig. 3** Nuclear response scales obtained from the same nominal proton and resonance masses. The markers distinguish the internal gap, stationary-proton absorption energy, and resonance-rest-frame decay energy. The upper axis converts energy to ordinary frequency. The shaded interval indicates the nuclear energy range used for orientation.

### 3.2 From component storage to field coefficients

Capacitance and inductance describe energy stored in specified charge and current patterns. Permittivity and permeability describe the same storage in field variables. Let  $\mathbf{D}$  be electric displacement, in coulombs per square meter,  $\mathbf{B}$  magnetic flux density, in webers per square meter, and  $u$  recoverable energy density. Bold  $\mathbf{D}$  denotes the field and is distinct from the propagator  $D$ . In a direction-dependent response,  $\epsilon$  and  $\mu$  are tensors with Cartesian indices  $i, j$ . In the rest frame of the settled material response, take a local, reciprocal, conservative constitutive law with no mixed electric–magnetic storage term:

$$u(\mathbf{D}, \mathbf{B}) = \frac{1}{2} \mathbf{D}^T \epsilon^{-1} \mathbf{D} + \frac{1}{2} \mathbf{B}^T \mu^{-1} \mathbf{B},$$

$$(\epsilon^{-1})_{ij} = \frac{\partial^2 u}{\partial D_i \partial D_j}, \quad (\mu^{-1})_{ij} = \frac{\partial^2 u}{\partial B_i \partial B_j}. \quad (31)$$

These second derivatives measure the energy cost of changing the fields. Lower electric or magnetic curvature corresponds to higher permittivity or permeability. The full coefficients retain the reference Maxwell contribution [24]. For dispersive response, the expression refers to its conservative limits; for nonlocal response, the derivatives become spatial kernels.

To express this field energy in component variables, use the profiles  $\mathbf{D} = \sum_a Q_a \mathbf{d}_a$  and  $\mathbf{B} = \sum_a \Phi_a \boldsymbol{\beta}_a$ . The patterns  $\mathbf{d}_a, \boldsymbol{\beta}_a$  have units  $\text{m}^{-2}$ . Integrating their energy defines electric capacitance  $C_E$  and magnetic inductance  $L_B$ :

$$(C_E^{-1})_{ab} = \int d^3x \mathbf{d}_a^T \epsilon^{-1} \mathbf{d}_b,$$

$$(L_B^{-1})_{ab} = \int d^3x \boldsymbol{\beta}_a^T \mu^{-1} \boldsymbol{\beta}_b. \quad (32)$$

For one mode with spatially constant isotropic coefficients, all geometric dependence is contained in  $\mathcal{G}_D = \int |\mathbf{d}|^2 d^3x$  and  $\mathcal{G}_B = \int |\boldsymbol{\beta}|^2 d^3x$ , each with units of inverse length. Thus

$$\epsilon = \mathcal{G}_D C_E, \quad \mu = \mathcal{G}_B L_B. \quad (33)$$

The component and field calculations describe the same system when they use the same charge pattern, quantum state, and mechanical constraints. Under this matching, the electric capacitance is the susceptibility in Eq. (28). The nuclear inductance likewise equals  $L_B$  when its current pattern represents the same magnetic storage. The scalar spectral expressions then give

$$\epsilon = 2\mathcal{G}_D \sum_{n>0} \frac{|q_n|^2}{4_n},$$

$$\mu = \frac{2\hbar^2 \mathcal{G}_B}{c^2} \sum_{n>0} \frac{|q_n|^2}{4_n^3}, \quad C = 2 \sum_{n>0} \frac{|q_n|^2}{4_n}. \quad (34)$$

This establishes the conversion from nuclear response to field coefficients. The excitation spectrum determines the component storage, and the spatial profiles supply the geometric conversion. Both calculations must describe the same material state; otherwise they would assign different energies to the same motion.

These constitutive coefficients compare settled material states in the declared frame. Describing the same response from a relatively moving frame requires transforming the fields, currents, and constitutive law

together; electric–magnetic cross-couplings can then appear [29]. The state scalings derived below apply in their stated rest-frame comparison.

### 3.3 A passive model of contraction and restoration

The next question is how field coupling loads this storage and whether equal electric and magnetic changes resist imbalance. Consider one material oscillator coupled to two environmental modes of the shared field. Use dimensionless canonical coordinates with the reference material frequency and  $\hbar$  set to one; restore physical  $\hbar$  in energy shifts. Write the logarithmic coefficient changes as  $q_e = \ln(\epsilon/\epsilon_0)$  and  $q_m = \ln(\mu/\mu_0)$ . Their mean  $s = (q_e + q_m)/2$  describes their common change, while  $d = (q_e - q_m)/2$  measures their difference. Equal changes have  $d = 0$ ; on the common branch,  $b = e^{-s/2}$ .

Let  $(Q, \Phi)$  describe the material oscillator, with  $(X_e, P_e)$  and  $(X_m, P_m)$  the environmental coordinate–momentum pairs. In this model  $P_e, P_m$  are momenta. The environmental modes have common frequency  $\Omega > 0$  and dimensionless couplings  $\eta_e, \eta_m$ . Take the positive storage Hamiltonian

$$H = \frac{1}{2} e^{-(s+d)/2} Q^2 + \frac{1}{2} e^{-(s-d)/2} \Phi^2 + \frac{\Omega}{2} [(X_e - \eta_e Q)^2 + P_e^2 + X_m^2 + (P_m - \eta_m \Phi)^2]. \quad (35)$$

Every square is nonnegative. Expanding the squares keeps both the interaction terms and the associated quadratic storage terms, so the model remains positive. To find the load on the receiver, differentiate its deformation while holding the environmental frequencies and couplings fixed.

Let  $\Delta E(s, d)$  be the ground-state energy change relative to the uncoupled material and environmental oscillators. At material frequency  $\omega = e^{-s/2}$ , second-order perturbation theory gives the leading coupling contribution. The remainder  $O(\eta^4)$  contains fourth and higher total orders in  $\eta_e, \eta_m$ :

$$\Delta E(s, d) = \frac{\hbar \Omega \omega}{4(\Omega + \omega)} (\eta_e^2 e^{d/2} + \eta_m^2 e^{-d/2}) + O(\eta^4). \quad (36)$$

For equal couplings  $\eta_e = \eta_m = \eta$ , the bracket is  $2\eta^2 \cosh(d/2)$ , with  $\cosh$  denoting the hyperbolic cosine. We need two energy derivatives. The curvature  $K_d$  measures resistance to electric–magnetic imbalance, while  $J_s$  is the load toward increasing common response  $s$ . At  $d = 0$ ,

$$K_d = \partial_d^2 \Delta E|_{d=0} = \frac{\hbar \eta^2 \Omega \omega}{8(\Omega + \omega)} + O(\eta^4) > 0, \quad J_s = -\partial_s \Delta E|_{d=0} = \frac{\hbar \eta^2 \Omega^2 \omega}{4(\Omega + \omega)^2} + O(\eta^4) > 0. \quad (37)$$

Because increasing  $s$  decreases size  $b$ , positive  $J_s$  loads the receiver toward contraction. Positive  $K_d$  resists unequal electric and magnetic changes. Unequal couplings move the weak-coupling optimum to  $d_* = 2\ln(\eta_m/\eta_e)$ . Section 3.4 adds the reference material's restoring energy to locate the final equilibrium. Figure 4 checks the perturbative result against the independent oscillations, or normal modes, of the full quadratic system. It also shows the interaction-energy shift  $\Delta E_N$  when  $N$  material oscillators share the environmental modes.

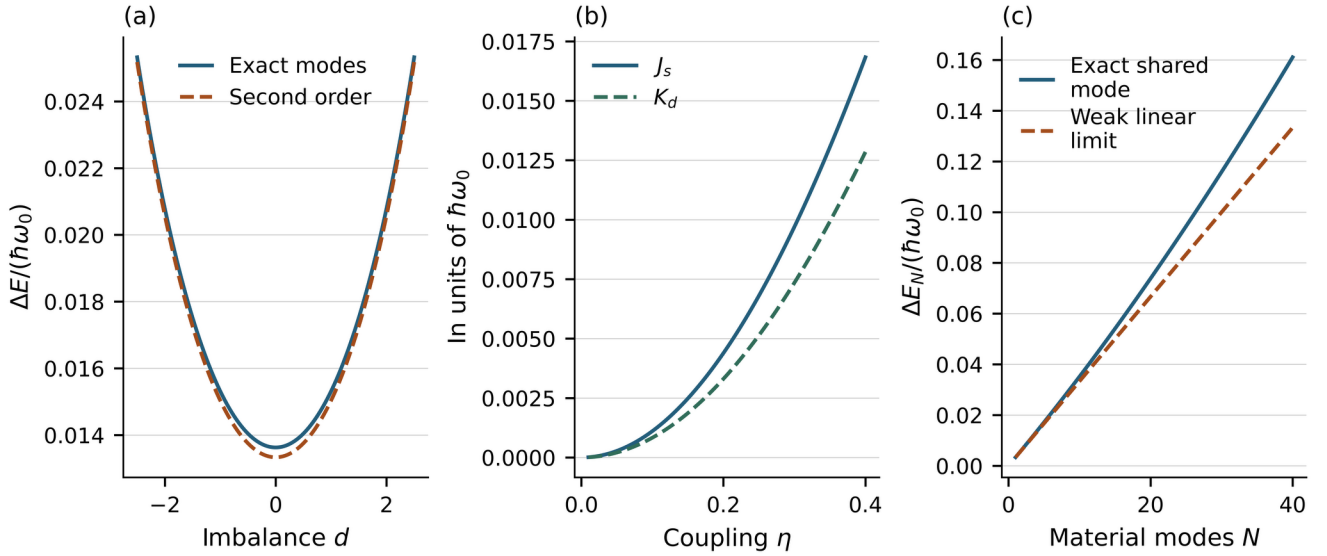


**Fig. 4** Passive quantum storage in units  $\hbar = \omega_0 = 1$ , where  $\omega_0$  is reference material frequency, with environmental frequency  $\Omega = 2$ . Panel (a) varies imbalance  $d$  at  $s = 0$  and matched coupling  $\eta = 0.2$ . Panel (b) varies  $\eta$  at  $s = d = 0$ , comparing loading  $J_s$  and stiffness  $K_d$ . Panel (c) compares the exact shared-mode interaction energy with the weak linear limit for  $N$  material oscillators at  $s = d = 0$ ,  $\eta = 0.1$ .

For a positive integer  $N$  identical material oscillators coupled with equal phase to the shared modes, let  $\Delta E_N$  be their total interaction-energy shift, and  $\Delta E_1$  the shift for one oscillator. The normalized symmetric combination carries the interaction with effective coupling  $\sqrt{N}\eta$ ; the remaining  $N - 1$  combinations decouple in this geometry. Orthogonal transformation into this collective basis proves

$$\Delta E_N(\eta) = \Delta E_1(\sqrt{N}\eta). \quad (38)$$

At small  $N\eta^2$ , the energy shift grows approximately in proportion to oscillator count. At larger coupling, the exact collective modes determine the departure from that limit. In extended matter, the radiation matrix retains the propagation phases and spatial form factors needed to calculate how contributions accumulate.

The restoring response must be calculated from the full energy, allowing the quantum state to adjust. Its Hessian is the matrix of second derivatives. For deformation parameters  $\theta_a$ , write  $\partial_a = \partial / \partial \theta_a$ . If the unique ground state has energy  $E_0$  and excited states have energies  $E_n$ , differentiation gives

$$\partial_a \partial_b E_0 = \langle \partial_a \partial_b H \rangle - 2\text{Re} \sum_{n>0} \frac{\langle 0 | \partial_a H | n \rangle \langle n | \partial_b H | 0 \rangle}{E_n - E_0}. \quad (39)$$

The first term differentiates the Hamiltonian directly; the second includes rearrangement of the quantum state. Together they give the relaxed curvature. For the numerical check, the ground-state energy is half the sum of positive normal-mode frequencies, with the uncoupled value subtracted. At  $s = d = 0$ ,  $\eta_e = \eta_m = 0.2$ , and  $\Omega = 2$ , the interaction energy is 0.01362777956, the contraction load is 0.004385153827, and the imbalance stiffness is 0.003303516608. The collective identity agrees to  $1.33 \times 10^{-15}$ . The model therefore establishes contraction-directed loading and restoration against imbalance, with the signs checked independently of the perturbative expansion.

### 3.4 Material compliance determines the equilibrium

A load gives a tendency to deform; the material's restoring energy determines the displacement. Let  $x$  be a deformation coordinate,  $F_{\text{ref}}(x)$  the reference material energy including isolated internal storage, and  $\Delta F_{\text{EM}}(x)$  the environmental interaction-energy change. These scalar  $F$  quantities are energies. Their sum must be stationary at equilibrium and have positive restoring curvature:

$$\partial_x[F_{\text{ref}} + \Delta F_{\text{EM}}] = 0, \quad \partial_x^2[F_{\text{ref}} + \Delta F_{\text{EM}}] > 0. \quad (40)$$

The first condition locates the adjusted state. The second ensures that a small displacement costs energy. For several deformations, positivity is required on the allowed physical subspace.

For a volume coordinate  $V$ , let  $V_0$  be reference equilibrium volume and  $\delta V = V - V_0$  its change. Primes denote derivatives with respect to  $V$ . Choose  $F_{\text{ref}}'(V_0) = 0$  and define bulk modulus  $K_{\text{mat}} = V_0 F_{\text{ref}}''(V_0) > 0$ , the restoring pressure scale. Let  $\Delta p_{\text{EM}} = -\partial_V \Delta F_{\text{EM}}$  be outward-positive interaction pressure, distinct from the dimensionless supporting-power ratio  $p$ . Linearization gives

$$\frac{\delta V}{V_0} = \frac{\Delta p_{\text{EM}}(V_0)}{K_{\text{mat}}}. \quad (41)$$

Negative outward pressure therefore contracts the material, against its nuclear restoring stress. Tensor compliance extends the result to direction-dependent deformation. Internal coordinates can also relax while charge is displaced. To include a stable internal coordinate  $x$ , partition the energy Hessian into charge block  $K_{QQ}$ , internal block  $K_{xx}$ , and mixed blocks  $K_{Qx}, K_{xQ}$ . Eliminating  $x$  through its equilibrium condition gives the Schur complement:

$$K_{QQ}^{\text{relaxed}} = K_{QQ} - K_{Qx} K_{xx}^{-1} K_{xQ}. \quad (42)$$

This is the charge stiffness after internal adjustment. The clamped stiffness describes the same response with that coordinate held fixed. Using the relaxed expression incorporates the deformation once in the energy used for the field and charge calculations.

The material response is now expressed as an equilibrium calculation. The changed exchange and field coupling provide the load, and the complete energy determines the stable adjustment. The next subsection combines this adjustment with the electrical response and balanced supporting exchange.

### 3.5 Coupled stress and supporting power equilibrium

The equilibrium condition can now be assembled with the electrical response and supporting exchange. An applied load changes the material state; the adjusted state changes the currents and field correlations that supply its load and balanced power. These quantities must therefore be evaluated together. We consider static loads and quasistatic changes between stable equilibria.

Let  $q_A$  collect dimensionless deformation and internal coordinates, with  $A$  labeling the allowed coordinates, and let  $\lambda$  specify the external source settings and boundary conditions. Write  $\mathcal{F}_{\text{tot}}(q; \lambda)$  for the equilibrium energy potential, in joules, of the coupled matter and field. It includes the reference material energy, environmental interaction, and source work appropriate to the imposed electrical

constraints, with each contribution counted once. At zero temperature it is obtained from the constrained ground-state energy; at finite equilibrium temperature it includes the corresponding entropy term. The quantum state is allowed to adjust at each  $q$ . Denote the settled coordinates by  $q_*$  and the common supporting exchange by  $P_{\text{bal}}$ . The coupled conditions are

$$\begin{aligned} \frac{\partial \mathcal{F}_{\text{tot}}}{\partial q_A}(q_*; \lambda) &= 0, \\ P_{\text{in}}(q_*; \lambda) &= P_{\text{out}}(q_*; \lambda) = P_{\text{bal}}. \end{aligned} \quad (43)$$

The energy Hessian must be positive on the allowed physical deformations at this solution. Its quantum contribution includes the state rearrangement in Eq. (39). The power equality is evaluated with the same causal response and compensating fluctuations as Eqs. (17) and (18). Stationarity fixes the configuration, while the exchange calculation fixes the common power in watts.

To make the stress response explicit, let  $e_{ij}$  be a small symmetric strain about a reference equilibrium of volume  $V_0$ . Its trace is the fractional volume change to first order. Cartesian indices  $i, j, k, l$  run over the three spatial directions, and repeated indices are summed. Let  $z$  collect the remaining dimensionless internal coordinates. Define the blocks  $\mathbf{K}_{ee}$ ,  $\mathbf{K}_{ez}$ ,  $\mathbf{K}_{ze}$ , and  $\mathbf{K}_{zz}$  by the second derivatives of the reference equilibrium potential divided by  $V_0$ ; they therefore have units of pressure. For a stable invertible internal block, relaxing  $z$  gives the strain stiffness

$$\mathbf{K}^{\text{rel}} = \mathbf{K}_{ee} - \mathbf{K}_{ez} \mathbf{K}_{zz}^{-1} \mathbf{K}_{ze}. \quad (44)$$

This is Eq. (42) applied to the strain coordinates. It incorporates each retained internal adjustment once; coordinates already eliminated in the quantum energy are not eliminated a second time. Let  $\Delta \bar{\mathcal{F}}_{\text{load}}$  be the change of the energy potential caused by a small added load, after internal relaxation. Define its deformation-driving stress  $\tau_{ij}^{\text{load}}$  as the negative strain derivative per reference volume. Linearization of Eq. (43) then gives

$$\begin{aligned} \tau_{ij}^{\text{load}} &= -\frac{1}{V_0} \frac{\partial \Delta \bar{\mathcal{F}}_{\text{load}}}{\partial e_{ij}}(0), \\ K_{ij\,kl}^{\text{rel}} e_{kl} &= \tau_{ij}^{\text{load}}. \end{aligned} \quad (45)$$

Both the stiffness and the applied stress are measured in pascals. Equation (45) determines the first-order deformation; finite changes use the full stationarity condition with the response updated at each state. For spatially varying deformation, the corresponding variational statement gives local total-stress balance and the imposed boundary tractions. It does not require the total stress to vanish pointwise.

The electrical coefficients are evaluated from this same relaxed state. Equations (26)–(34) connect its interacting spectrum and spatial current profiles to charge response, storage, permittivity, and permeability. The strong-interaction contributions remain in the nuclear Hamiltonian. Near resolved resonances the full frequency-dependent response is used in the exchange integrals, within the response domains stated in Section 3.1. For a corresponding directional deformation, the ruler matrix  $\mathbf{R}$  and clock factor  $f$  give the matched permittivity and permeability tensors through Eq. (66). Thus deformation, electrical storage, and field exchange enter one self-consistent calculation.

**Corollary Small isotropic load on the common branch.** Suppose the settled states satisfy the corresponding-state assumptions of Theorems 1 and 2 and the common relation  $f = b$ . At finite temperature use the corresponding temperatures required by Theorem 2. With  $\mathbf{R} = b\mathbf{I}$ , the volume is  $V = b^3 V_0$ . Define  $p = P_{\text{bal}}/P_0$ , where  $P_0$  is the reference supporting power. The power theorem gives

$$p = b^2 = \left(\frac{V}{V_0}\right)^{2/3}. \quad (46)$$

For a small isotropic interaction load, use the outward-positive pressure  $\Delta p_{\text{EM}}$  and reference bulk modulus  $K_{\text{mat}}$  of Section 3.4, evaluated for the same relaxed state. Let  $\Delta P = P_{\text{bal}} - P_0$  and  $\Delta V = V - V_0$ . Expanding Eq. (46) and applying Eq. (41) proves

$$\begin{aligned} \frac{\Delta P}{P_0} &= \frac{2}{3} \frac{\Delta V}{V_0} + O\left(\left(\frac{\Delta V}{V_0}\right)^2\right), \\ \frac{\Delta P}{P_0} &= \frac{2\Delta p_{\text{EM}}}{3K_{\text{mat}}} + O\left(\left(\frac{\Delta p_{\text{EM}}}{K_{\text{mat}}}\right)^2\right). \end{aligned} \quad (47)$$

Negative outward pressure therefore produces contraction and a lower supporting exchange on this branch. The corollary relates neighboring matched equilibria; it does not assign a common spectral scale to an arbitrary shape-changing load.

The relaxed stiffness in Eq. (45) includes the bulk, shear, and mixed response relevant to the chosen material coordinates. The coefficient  $A_{2,E}^{\text{tot}}$  introduced in Section 4.4 is instead the transverse, traceless gradient coefficient of the completed stress response. It does not replace this full material stiffness matrix.

For a nuclear implementation, solve the coupled equilibrium and evaluate its balanced power from Eq. (18). Compare the clock factors inferred from transition gaps, the size factor inferred from dimensions, and the independently calculated power ratio. This determines whether the realized response belongs to the corresponding family. The following theorems establish the stable binding, storage, and power relations for that family. The present construction supplies the equilibrium problem in which those relations can be evaluated together.

## 4 Stable matter and the supporting-power theorem

The material must remain bound as its supporting balance changes. The factors  $f$  and  $b$ , introduced in Section 2.1, track the clock-energy change and the size change separately. We first prove the transformation of a corresponding family of states, then calculate its supporting power. On the common branch, a single power ratio determines both factors.

### 4.1 The matter-wave scaling theorem

A bound matter wave has a spatial distribution, energy levels, and a restoring energy against deformation. A corresponding family compares the same quantum states after rescaling their lengths and their rate of evolution. The theorem below states the assumptions for that family and proves that stable binding is preserved.

Let  $\mathbf{x}_{\text{int}}$  collect  $d_{\text{int}}$  internal coordinates,  $\psi$  a normalized wavefunction, and  $\mathbf{p}$  the corresponding canonical momentum. Let  $H_0$  be a self-adjoint reference Hamiltonian, so it generates unitary time evolution, bounded below and possessing a lowest bound state. For positive size factor  $b$ , define the probability-preserving dilation operator  $U_b$ :

$$\begin{aligned} (U_b \psi)(\mathbf{x}_{\text{int}}) &= b^{-d_{\text{int}}/2} \psi(\mathbf{x}_{\text{int}}/b), \\ U_b^\dagger \mathbf{x}_{\text{int}} U_b &= b \mathbf{x}_{\text{int}}, \quad U_b^\dagger \mathbf{p} U_b = b^{-1} \mathbf{p}. \end{aligned} \tag{48}$$

The factor  $b^{-d_{\text{int}}/2}$  keeps total probability unchanged. The dilation maps the Hamiltonian domain and the constrained states used to define size. Treat  $f, b$  as constant across each stationary apparatus. In a slowly varying environment the same relations apply locally and adiabatically, so the internal state has time to follow its equilibrium.

This dilation compares distinct material equilibria using one set of reference standards. A Lorentz transformation instead changes the observer describing a given physical state. The two operations have separate roles in the construction.

**Theorem 1 Corresponding scaling preserves stable bound states.** Let  $H_0$  be self-adjoint, bounded below, and possess a lowest bound state. Take positive  $f, b$  and a common energy reference. Map the evolving state  $\Psi_0(t)$  to  $\Psi_{f,b}(t) = U_b \Psi_0(ft)$ . Assume the constrained reference energy is twice differentiable near its stable minimum. Denote the mapped Hamiltonian by  $H_{f,b}$ , its domain by  $\mathcal{D}(H_{f,b})$ , its level energies by  $E_{n,f,b}$ , and transition gaps by  $\Delta E_{mn,f,b} = E_{m,f,b} - E_{n,f,b}$ . Then

$$\begin{aligned} H_{f,b} &= f U_b H_0 U_b^\dagger, & \mathcal{D}(H_{f,b}) &= U_b \mathcal{D}(H_0), \\ E_{n,f,b} &= f E_{n,0}, & \Delta E_{mn,f,b} &= f \Delta E_{mn,0}. \end{aligned} \tag{49}$$

The result assigns radius the factor  $b$ , transition energy and frequency the factor  $f$ , and restoring curvature the positive factor  $f/b^2$ . Thus the common family  $f = b$  permits contraction and slower clocks together while retaining a stable bound state.

**Proof.** Changing integration variables to  $\mathbf{x}_{\text{int}}/b$  preserves the norm and establishes unitarity. Differentiating the mapped evolution gives  $i\hbar \partial_t \Psi_{f,b} = f U_b H_0 U_b^\dagger \Psi_{f,b}$ . Unitary conjugation preserves self-adjointness, while positive  $f$  preserves the spectral ordering and lower bound. Every reference eigenvector maps to  $U_b |n\rangle$  with energy  $f E_{n,0}$ , which proves the energy and transition-frequency statements.

Let  $\mathcal{E}_0(R)$  be the minimum energy among normalized states with root-mean-square radius  $R$ , and let  $R_0$  be a stable minimum. Dilation maps the complete constrained set and its quadratic-form domain. Consequently,

$$\begin{aligned} \mathcal{E}_{f,b}(R) &= f \mathcal{E}_0(R/b), & \mathcal{E}'_{f,b}(bR_0) &= 0, \\ \mathcal{E}''_{f,b}(bR_0) &= \frac{f}{b^2} \mathcal{E}''_0(R_0) > 0. \end{aligned} \tag{50}$$

The mapped minimum is stable. Subtracting the energies of corresponding constrained configurations also proves that recoverable deformation work scales by  $f$ . This completes the proof.

The theorem fixes inertia as well as binding. Let  $m_0$  be reference inertia and  $m_{f,b}$  its mapped value. Since  $U_b \mathbf{p} U_b^\dagger = b \mathbf{p}$ , the reference kinetic term becomes  $f b^2 \mathbf{p}^2 / (2m_0)$ . At the mapped minimum  $R_{\min}$ , define radial stiffness  $k_R = \mathcal{E}''(R_{\min})$ , fractional displacement  $y = (R - R_{\min})/R_{\min}$ , its stiffness  $K_y = k_R R_{\min}^2$ , and its inertia  $I_y = m R_{\min}^2$ . Write  $\Delta W$  for the small-displacement restoring work and  $\omega$  for the resulting oscillation frequency. Then

$$\begin{aligned} \frac{m_{f,b}}{m_0} &= \frac{1}{f b^2}, & \frac{k_{R,f,b}}{k_{R,0}} &= \frac{f}{b^2}, & \frac{K_{y,f,b}}{K_{y,0}} &= f, & \frac{I_{y,f,b}}{I_{y,0}} &= \frac{1}{f}, \\ \frac{\omega_{f,b}}{\omega_0} &= f, & \Delta W_{f,b} &= 1/2 K_{y,f,b} y^2 = f \Delta W_0. \end{aligned} \quad (51)$$

On the common branch,  $m_b = b^{-3} m_0$ ,  $k_{R,b} = b^{-1} k_{R,0}$ , and  $\omega_b = b \omega_0$ . A smaller state can oscillate more slowly because its inertia also changes. This linked adjustment is essential to the material scaling. For a translating body the same construction applies to its complete energy, including binding and reference contributions.

The proof makes the preservation of binding and recoverable work explicit within the power-balance argument.

For clarity, the common-branch constrained energy curve  $\mathcal{E}_b(R)$  and its stable minimum obey

$$\mathcal{E}_b(R) = b \mathcal{E}_0(R/b), \quad R_{\min,b} = b R_0, \quad \mathcal{E}_b''(b R_0) = \frac{\mathcal{E}_0''(R_0)}{b} > 0. \quad (52)$$

Every positive  $b$  therefore gives a stable corresponding minimum. Figure 5 illustrates the change using a quadratic reference energy. The power theorem below supplies  $b = \sqrt{P_b/P_0}$ , connecting the required exchange to the member of this family. The complete equilibrium calculation determines whether the specified environment realizes that member.

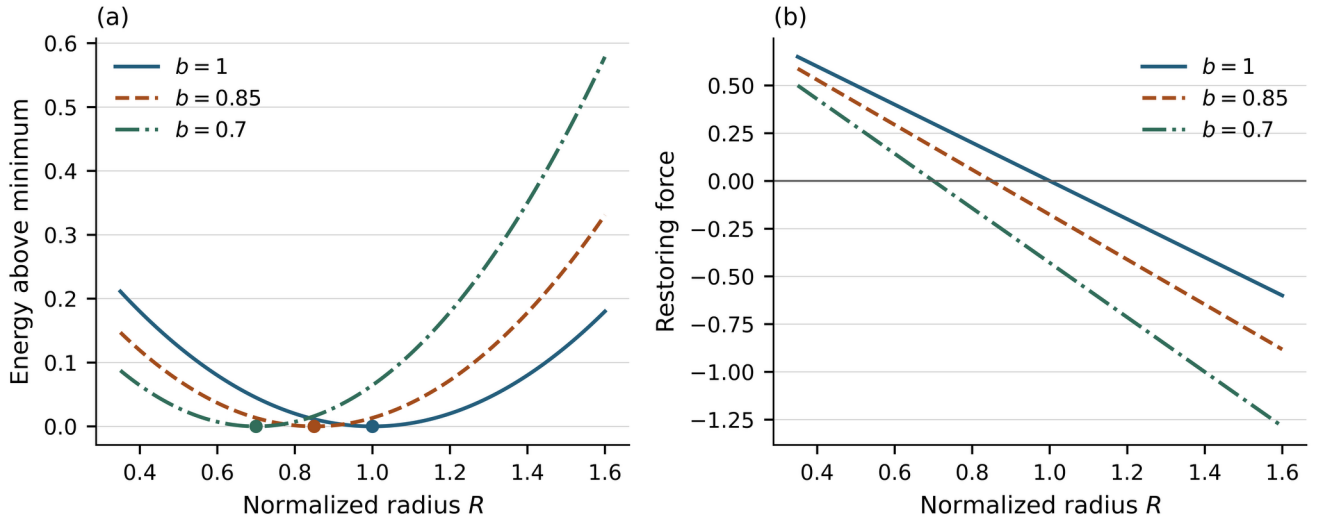


**Fig. 5** Stable binding under common dilation. The reference curve  $\mathcal{E}_0(R) = (R - 1)^2/2$  maps to  $\mathcal{E}_b(R) = (R - b)^2/(2b)$  in normalized units. The minima occur at  $R = b$ ; their positive curvature is  $1/b$ . The restoring force changes sign at each corresponding equilibrium.

## 4.2 The constitutive scaling laws

The same transformation must apply to the electrical modes. Label the mapped state by  $f, b$ . Fixed charge matrix elements and excitation gaps scaled by  $f$  give  $\chi_{f,b}^R(\nu) = f^{-1}\chi_0^R(\nu/f)$  and  $C, L \propto f^{-1}$ . Spatial dilation gives  $\mathbf{d}_b(\mathbf{x}) = b^{-2}\mathbf{d}_0(\mathbf{x}/b)$ , with the same rule for magnetic profiles, so the geometry factors scale by  $b^{-1}$ . Let  $Z = \sqrt{\mu/\epsilon}$  be wave impedance, the traveling-wave electric-to-magnetic field ratio, distinguished from the mutual mode impedance  $Z_{ab}$ . Combining component response and geometry gives

$$\begin{aligned} \frac{c_{f,b}}{c_0} &= \frac{L_{f,b}}{L_0} = f^{-1}, & \frac{g_{D,b}}{g_{D,0}} &= \frac{g_{B,b}}{g_{B,0}} = b^{-1}, \\ \frac{\epsilon_{f,b}}{\epsilon_0} &= \frac{\mu_{f,b}}{\mu_0} = \frac{1}{fb}, & \frac{c_{f,b}}{c_0} &= fb, & \frac{Z_{f,b}}{Z_0} &= 1. \end{aligned} \quad (53)$$

The component storage contributes  $f^{-1}$ , and geometry contributes  $b^{-1}$ . They can be separated through  $f = C_0/C$  and  $b = (C/C_0)/(\epsilon/\epsilon_0)$ ; the field-coefficient pair alone fixes their product  $fb$ . On the common branch  $f = b$ , the complete ratios become

$$\begin{aligned} \omega_b &= b\omega_0, & L_b &= b^{-1}L_0, & C_b &= b^{-1}C_0, \\ \epsilon_b &= b^{-2}\epsilon_0, & \mu_b &= b^{-2}\mu_0, & c_b &= b^2c_0, & m_b &= b^{-3}m_0. \end{aligned} \quad (54)$$

Permittivity and permeability rise together as the common size and frequency fall, leaving wave impedance  $Z = \sqrt{\mu/\epsilon}$  unchanged. The passive example in Section 3.3 supplies an explicit restoring response against unequal changes. We can now use these same current profiles and coefficients to find the required supporting power.

## 4.3 The supporting-power theorem

**Theorem 2 Supporting power scales as the square of the clock factor.** Assume the corresponding family of Theorem 1, fixed charge, matched electromagnetic current profiles, and the same dimensionless equilibrium response. At finite temperature take the corresponding state  $T = fT_0$ ; at zero temperature use the mapped ground state. Retain the dissipative channels, cutoffs, and correlations in that mapping. Then  $P_{\text{bal}}(f, b)/P_0 = f^2$ .

**Proof.** The normalized current profiles obey  $\mathbf{j}_b(\mathbf{x}) = b^{-2}\mathbf{j}_0(\mathbf{x}/b)$ . At corresponding frequency  $f\nu$ , the propagation phase is invariant because  $c = fbc_0$ . Changing integration variables in the form factor and using the radiation matrix gives

$$\mathbf{g}_{f,b}(\mathbf{n}, f\nu) = b\mathbf{g}_0(\mathbf{n}, \nu), \quad R_{f,b}^{\text{rad}}(f\nu) = R_0^{\text{rad}}(\nu). \quad (55)$$

The prefactor  $\nu^2/(\epsilon c^3)$  acquires  $b^{-2}$ , while the form-factor product acquires  $b^2$ . Radiation resistance is therefore unchanged under this corresponding-state scaling, including the interference phases of corresponding configurations. Since  $L\omega = L_0\omega_0$ , the single-mode coupling  $g_{\text{rad}}$  is unchanged under this scaling as well.

The fluctuation relation gives  $S_{Q,f,b}(f\nu) = f^{-1}S_{Q,0}(\nu)$ . In the dissipative integral, the frequency square contributes  $f^2$ , the spectrum contributes  $f^{-1}$ , and the integration measure contributes  $f$ . The factors combine to prove

$$\frac{P_{\text{bal}}(f,b)}{P_0} = f^2 = \frac{\omega_{f,b}^2}{\omega_0^2}, \quad \left. \frac{P_b}{P_0} \right|_{f=b} = b^2. \quad (56)$$

The input integral has the same value by the power-balance identity, completing the proof. An independent Maxwell check gives mapped  $\mathbf{E}, \mathbf{H} \propto f/b$ , hence Poynting flux proportional to  $f^2/b^2$ ; multiplying by the corresponding area, proportional to  $b^2$ , again gives  $f^2$ . Numerical tests with a finite current-profile radiation matrix and a convergent two-port oscillator reproduce the impedance and power transformations to approximately  $10^{-15}$  in normalized units.

The theorem makes the power comparison especially simple on the common contraction branch  $f = b \leq 1$ . Define a nonnegative parameter  $\omega_{\text{loss}}$  that expresses the power difference in squared-frequency units:

$$\delta_P = \frac{P_{\text{bal},0} - P_{\text{bal},b}}{P_{\text{bal},0}} = 1 - b^2 = \frac{\omega_{\text{loss}}^2}{\omega_0^2}, \quad \hbar\omega_b^2 = \hbar(\omega_0^2 - \omega_{\text{loss}}^2). \quad (57)$$

For one mode the reduction in watts is  $\Delta P_{\text{scat}} = g_{\text{rad},0} \hbar\omega_{\text{loss}}^2/2$ , while the stored-energy change is  $\hbar\omega_0(b - 1)/2$ . The distinction follows from the theorem: balanced power scales with frequency squared, whereas stored energy scales with frequency. Actual powers still subtract linearly as in Eq. (1).

The theorem concerns the balanced exchanges of the mapped equilibria. It supplies the clock factor once that exchange ratio is known. A change of observer, or motion of a receiver relative to the ambient sources, requires its own consistently transformed response calculation; the squared-frequency law alone does not determine that moving-state exchange.

#### 4.4 From supporting power to the material response

To connect these ratios with gravity, use the common branch  $f = b$ , matched wave impedance  $Z = Z_0$ , and fixed normalization  $\kappa = G/c^4$ , where  $G$  is the gravitational coupling in the comparison standards. Write  $P = P_b$ . The full ambient stress stiffness is  $A = A_{2,E}^{\text{tot}}$ , with units of energy squared. It measures the quadratic gradient cost of a transverse, traceless shear response: a shear perpendicular to the perturbation wave vector with no volume-changing trace. In its label, 2 indicates second spatial-derivative order, E the Euclidean or imaginary-time response convention, and “tot” the inclusion of the reference ambient term and matched stress contributions.

The response can be defined directly from the matter system. Let  $h$  be a dimensionless external shear source and  $W_E[h] = -\ln Z_E[h]$  the equilibrium generating functional, where  $Z_E$  is the partition function. Its second variation is the completed stress Hessian: it includes connected stress fluctuations together with the local contact terms generated by varying the stress operator itself [13]. After removing the zero-gradient equilibrium term, the coefficient of the quadratic wave-number dependence defines the derivative response. This is the collective stress counterpart of the charge-response expansion used to define nuclear storage in Section 3.1.

For the gravitational matching, take the local long-wavelength branch with two massless tensor polarizations and propagation speed  $c$ . Use inverse-energy coordinates  $X^\mu = \frac{x^\mu}{\hbar c}$ , with  $x^0 = ct$  before continuation to imaginary time. Let  $h_{\mu\nu}^{\text{TT}}$  denote the transverse, traceless source,  $\partial_\alpha = \partial/\partial X^\alpha$ , and repeated Greek indices denote Euclidean contraction. The positive inverse-response convention defines  $A$  through the dimensionless quadratic functional  $\Gamma_{\text{mat,E}}^{(2)}$ :

$$\Gamma_{\text{mat,E}}^{(2)} = \frac{A}{2} \int d^4 X \partial_\alpha h_{\mu\nu}^{\text{TT}} \partial_\alpha h_{\mu\nu}^{\text{TT}}. \quad (58)$$

The published massless spin-two constructions admit both full diffeomorphism symmetry (Diff) and Weyl symmetry combined with transverse diffeomorphisms (WTDiff) [1, 2]. Both admit the classical Einstein equations for conserved matter, with the cosmological constant entering as an integration constant in the unimodular formulation. Their common transverse, traceless quadratic form supplies the normalization needed here; the distinction between their gauge representations does not change that matching. Section 8 separately constructs the Schwarzschild exterior through material clocks and rulers.

Let  $C_E$  multiply the curvature term in the dimensionless gravitational action. In length coordinates the Einstein-normalized action has coefficient  $\frac{c^3}{16\pi G}$ . Dividing the action by  $\hbar$  and converting its curvature integral to  $X^\mu$  coordinates introduces the factor  $(\hbar c)^2$ . In either representation, the transverse, traceless quadratic term has coefficient  $\frac{C_E}{4}$  multiplying the integral in Eq. (58). Matching it to  $\frac{A}{2}$ , with the same symmetric shear source and the same tensor contraction, therefore gives

$$C_E = \frac{\hbar c^5}{16\pi G} = 2A, \quad A = \frac{\hbar c^5}{32\pi G}. \quad (59)$$

The numerical factor is fixed by matching these two quadratic forms. The material coefficient is then expressed in the gravitational comparison standards. The reference state carries subscript 0, and the constitutive result  $c/c_0 = fb$  gives

$$\kappa = \frac{G}{c^4} = \frac{\hbar c}{32\pi A}, \quad A_0 = \frac{\hbar c_0^5}{32\pi G_0}, \quad \frac{A}{A_0} = fb. \quad (60)$$

The measured reference value  $G_0/c_0^4$  calibrates the full coefficient. A microscopic calculation of a change in the stress Hessian determines a state-dependent increment; the reference ambient contribution remains part of the total  $A$  being matched. Because  $c/c_0 = fb$ , holding the gravitational normalization fixed requires  $A/A_0 = fb$ . On the common branch this is  $b^2 = p$ : the relative supporting power also gives the relative ambient stiffness. Thus the same power ratio controls the electromagnetic storage response and the gravitationally normalized stress response.

Eliminating the size factor  $b$  makes the dependence on available supporting power explicit.

For the mechanical entries below, let  $E_0 = m_0 c_0^2$  be the complete reference energy of a test body,  $E_{\text{rest}}$  its rest energy, and  $\mathbf{F}_{\text{rest}}$  its force in fixed comparison coordinates. The gradient  $\nabla$  differentiates position in those coordinates. The combined relations are

$$\begin{aligned} p &= \frac{P_{\text{bal}}}{P_{\text{bal},0}}, & \frac{\epsilon}{\epsilon_0} &= \frac{\mu}{\mu_0} = p^{-1}, \\ \frac{A_{2,E}^{\text{tot}}}{A_{2,E,0}^{\text{tot}}} &= \frac{c}{c_0} = p, & \frac{G}{G_0} &= p^4, \\ \frac{E_{\text{rest}}}{E_0} &= \sqrt{p}, & \mathbf{F}_{\text{rest}} &= -E_0 \nabla \sqrt{p}. \end{aligned} \quad (61)$$

The first line converts power to electromagnetic response. The second includes the calibrated stress stiffness and propagation speed, with  $G/G_0 = (c/c_0)^4$ . The final line gives the complete-body energy and its spatial force, proved in Section 5. A single supporting-power ratio thus controls the response that determines storage, material standards, and motion.

For  $p = 1 - \delta_P$ , the coefficient change can also be written directly as

$$\begin{aligned} \frac{\epsilon}{\epsilon_0} &= \frac{\mu}{\mu_0} = \frac{P_0}{P} = \frac{1}{1 - \delta_P} = \frac{\omega_0^2}{\omega_0^2 - \omega_{\text{loss}}^2}, \\ \frac{\epsilon - \epsilon_0}{\epsilon_0} &= \frac{\mu - \mu_0}{\mu_0} = \frac{\delta_P}{1 - \delta_P}. \end{aligned} \quad (62)$$

To first order, a one-part-per-million power deficit increases each coefficient by one part per million. Their ratio remains fixed. The corresponding component, size, and frequency changes are

$$\begin{aligned} \frac{c}{c_0} &= \frac{L}{L_0} = (1 - \delta_P)^{-1/2}, & b &= \frac{\omega}{\omega_0} = \sqrt{1 - \delta_P}, \\ \frac{c}{c_0} &= 1 - \delta_P, & \frac{Z}{Z_0} &= 1, & 0 \leq \delta_P < 1. \end{aligned} \quad (63)$$

The different powers also explain how contraction affects energy density. Let  $U$  be total stored energy and  $V$  volume, with reference values  $U_0, V_0$ . The density  $U/V$  then obeys

$$\frac{U}{U_0} = b, \quad \frac{V}{V_0} = b^3, \quad \frac{U/V}{U_0/V_0} = b^{-2} = \frac{P_0}{P}. \quad (64)$$

The volume falls faster than the total energy. A smaller corresponding state can therefore have lower total storage and lower supporting power, yet higher storage density. These are different measures of the same adjusted state.

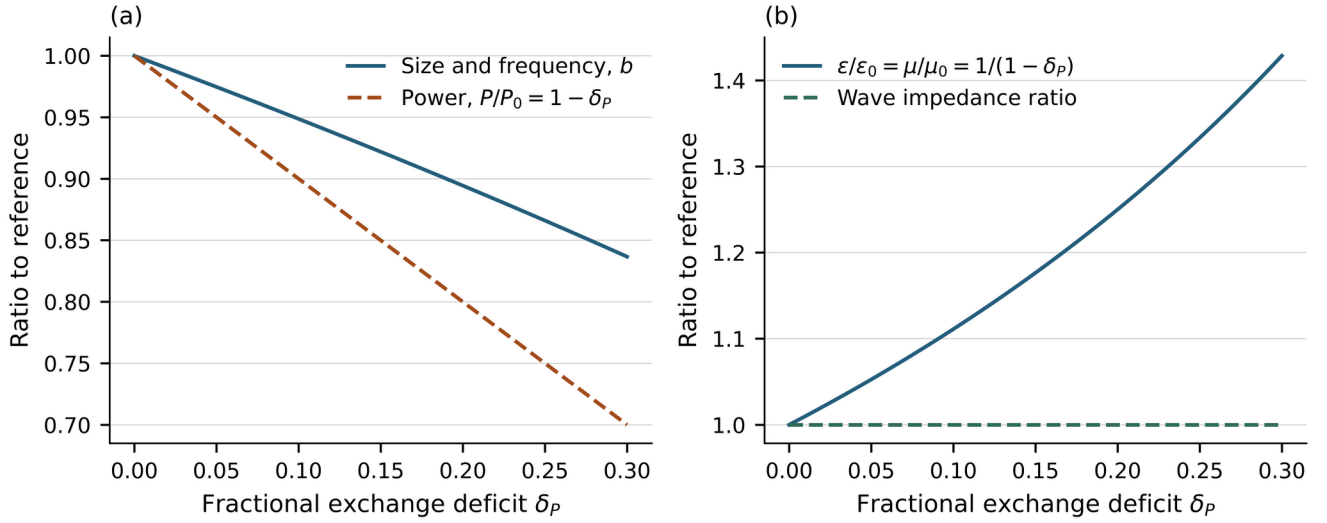


**Fig. 6** Power and material response on the common branch. The left panel shows  $P/P_0 = 1 - \delta_P$  and  $b = \sqrt{1 - \delta_P}$ . The right panel shows  $\epsilon/\epsilon_0 = \mu/\mu_0 = 1/(1 - \delta_P)$ . The wave impedance is constant. The displayed deficits illustrate the exact ratios over an enlarged range.

#### 4.5 Spin-2 gravitational waves in the material response

The preceding relations describe how a settled supporting balance changes material storage and standards. We now make the propagation of a small disturbance explicit. The calculation uses the local, conservative, long-wavelength tensor dynamics specified in Section 4.4. Its purpose is to identify the electromagnetic variables that carry this response, derive their wave equation, and show how their two polarizations enter the WTDiff description. A uniform equilibrium is used as the reference state, with coefficients  $\epsilon_0, \mu_0$ , signal speed  $c_0$ , and stress stiffness  $A_0$ .

The scalar ruler factor  $b$  must first be extended to a directional deformation. Let  $\mathbf{R}$  be a positive-definite symmetric, dimensionless local ruler matrix, so that a small reference displacement  $d\mathbf{\xi}$  maps to  $d\mathbf{x} = \mathbf{R} d\mathbf{\xi}$ . Its determinant  $J = \det \mathbf{R} > 0$  is the volume ratio. The isotropic case is  $\mathbf{R} = b\mathbb{I}$ , where  $\mathbb{I}$  is the identity matrix. Retain the corresponding-state energy scaling  $E = f E_{\text{loc}}$  and the conserved charge and magnetic flux used in Section 4.2. The area transformation then gives the reference field components and the energy-density conversion:

$$\mathbf{D}_{\text{loc}} = J \mathbf{R}^{-1} \mathbf{D}, \quad \mathbf{B}_{\text{loc}} = J \mathbf{R}^{-1} \mathbf{B}, \quad u = \frac{f}{J} u_{\text{loc}}. \quad (65)$$

Here the subscript loc labels the undeformed reference quantities associated with the same mapped state. The factor  $J$  accounts for the changed area and volume, while  $\mathbf{R}^{-1}$  resolves their directions. Insert these fields into the reference quadratic storage energy of Eq. (31). With  $\mathbf{R}^{-T}$  denoting the transpose of the inverse matrix, comparison with the energy in the deformed state gives

$$u = \frac{fJ}{2} \left[ \frac{\mathbf{D}^T \mathbf{R}^{-T} \mathbf{R}^{-1} \mathbf{D}}{\epsilon_0} + \frac{\mathbf{B}^T \mathbf{R}^{-T} \mathbf{R}^{-1} \mathbf{B}}{\mu_0} \right], \quad (66)$$

$$\frac{\epsilon}{\epsilon_0} = \frac{\mu}{\mu_0} = \frac{fJ}{fJ}.$$

This is the directional extension of the same storage transformation. Setting  $R = b\mathbb{I}$  gives  $J = b^3$  and recovers  $\epsilon/\epsilon_0 = \mu/\mu_0 = 1/(fb)$  exactly. The electric and magnetic tensors therefore remain matched under this deformation. Their directional changes describe the response of the common material state.

Let  $h_{ij}$  be the spatial transverse, traceless shear appearing in Eq. (58), with the TT superscript suppressed in this subsection. Cartesian indices  $i, j, k$  run over the three spatial directions and repeated indices are summed. To first order about the uniform reference, write  $R = \mathbb{I} - h/2$  and retain  $f = 1$  in this radiative representation. Transverse means that the shear has no component along its wave vector; traceless means that its three diagonal changes sum to zero. Equation (66) then yields

$$\begin{aligned} \partial_i h_{ij} &= 0, & h_{ii} &= 0, & J &= 1 + O(h^2), \\ \frac{\delta \epsilon^{ij}}{\epsilon_0} &= \frac{\delta \mu^{ij}}{\mu_0} &= -h_{ij} &+ O(h^2). \end{aligned} \tag{67}$$

The sign follows from the ruler convention: the squared-distance comparison matrix is  $R^{-T}R^{-1} = \mathbb{I} + h + O(h^2)$ , whereas the constitutive tensors involve its inverse. Thus the same shear changes material length comparisons and electromagnetic storage. The isotropic factor  $b$  describes the common size; the radiative shear describes directional differences with no first-order volume change. The static equality  $f = b$  continues to apply to the common background and does not identify these distinct shear components with a single scalar modulation.

The propagation law follows from the collective stress functional already normalized in Eq. (58). Continue its stated causal tensor dynamics to real time and restore length and time units using  $X^\mu = x^\mu/(\hbar c_0)$  and  $x^0 = c_0 t$ . Define  $\mathcal{S}_0 = A_0/(\hbar c_0)$ , a positive gradient stiffness with units of joules per meter. The real-time quadratic action  $S_{\text{TT}}^{(2)}$  is

$$\begin{aligned} \mathcal{S}_0 &= \frac{A_0}{\hbar c_0} = \frac{c_0^4}{32\pi G_0}, \\ S_{\text{TT}}^{(2)} &= \frac{\mathcal{S}_0}{2} \int dt d^3x \left[ \frac{\dot{h}_{ij}\dot{h}_{ij}}{c_0^2} - \partial_k h_{ij} \partial_k h_{ij} \right]. \end{aligned} \tag{68}$$

A dot denotes a time derivative. The first term measures time variation of the response; the second measures its spatial variation. Their relative coefficient is the causal propagation normalization already specified in Section 4.4. The overall coefficient follows from Eq. (60), with the same symmetric-tensor contraction. Let  $\pi_{ij}$  be the momentum density conjugate to  $h_{ij}$ . A Legendre transformation gives the collective field Hamiltonian  $H_{\text{TT}}$ :

$$\begin{aligned} \pi_{ij} &= \frac{\mathcal{S}_0}{c_0^2} \dot{h}_{ij}, \\ H_{\text{TT}} &= \int d^3x \left[ \frac{c_0^2}{2\mathcal{S}_0} \pi_{ij} \pi_{ij} + \frac{\mathcal{S}_0}{2} \partial_k h_{ij} \partial_k h_{ij} \right]. \end{aligned} \tag{69}$$

This Hamiltonian describes energy carried by the collective disturbance. The matter-wave Hamiltonian in Section 6 describes a body moving through the response. Variations of Eq. (69) stay within the TT sector.

Hamilton's equations give the first-order evolution laws; eliminating the conjugate momentum gives the second-order wave equation:

$$\begin{aligned} \dot{h}_{ij} &= \frac{c_0^2}{s_0} \pi_{ij}, & \dot{\pi}_{ij} &= s_0 \nabla^2 h_{ij}, \\ \frac{1}{c_0^2} \partial_t^2 h_{ij} - \nabla^2 h_{ij} &= 0. \end{aligned} \quad (70)$$

The Laplacian  $\nabla^2$  sums second spatial derivatives. Equation (70) shows that a disturbance propagates at the reference signal speed  $c_0$ . Through Eq. (67), the matched TT perturbations of permittivity and permeability obey the same wave equation. A weak disturbance can therefore be described either through its directional electromagnetic coefficients or through its material shear.

For propagation along the positive  $z$  direction, the transverse and traceless conditions leave two independent amplitudes. Call them  $h_+$  and  $h_\times$ , corresponding to the plus and cross patterns. With constant complex amplitudes  $a_+, a_\times$ , wave number  $k$ , and angular frequency  $\omega$ , the real plane waves are

$$\begin{aligned} \mathbf{h} &= \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ h_a &= \text{Re}[a_a e^{i(kz - \omega t)}], \quad a \in \{+, \times\}, \quad \omega^2 = c_0^2 k^2. \end{aligned} \quad (71)$$

Here  $\omega$  denotes the angular frequency of the collective modulation. The plus pattern increases one transverse length comparison while decreasing the other; the cross pattern is the same deformation rotated by 45 degrees. In the ruler convention above, a pure plus wave gives  $b_x = 1 - h_+/2$  and  $b_y = 1 + h_+/2$  to first order. The wave has no frequency gap at zero wave number, which is the massless dispersion relation in this effective description. To identify its spin, define the circular combinations  $h_{\pm 2} = h_+ \pm i h_\times$ . Under an active rotation of the pattern through angle  $\theta$  about its propagation direction,

$$h_{\pm 2} \mapsto e^{\pm 2i\theta} h_{\pm 2}. \quad (72)$$

The factor of two in the rotation phase identifies a spin-2 wave with two helicities,  $+2$  and  $-2$ . Helicity describes how a propagating pattern transforms when rotated about its direction of travel. Here it belongs to the collective constitutive shear. The two patterns are the physical tensor polarizations shared by the Diff and WTDiff descriptions.

Energy transport follows from the same action. Let  $u_{\text{TT}}$  be the wave energy per unit volume and  $\mathcal{J}_k$  its energy flux in direction  $k$ . Their local expressions and conservation law are

$$\begin{aligned} u_{\text{TT}} &= \frac{s_0}{2} \left[ \frac{\dot{h}_{ij} \dot{h}_{ij}}{c_0^2} + \partial_k h_{ij} \partial_k h_{ij} \right], & \mathcal{J}_k &= -s_0 \dot{h}_{ij} \partial_k h_{ij}, \\ \partial_t u_{\text{TT}} + \partial_k \mathcal{J}_k &= 0. \end{aligned} \quad (73)$$

The energy is nonnegative because  $s_0 > 0$  and both terms are sums of squares. For a wave depending on  $z - c_0 t$ , Eq. (73) gives  $\mathcal{J}_z = c_0 u_{\text{TT}}$ : the disturbance carries its stored energy in the propagation direction. This supplies the energetic meaning of the wave, in addition to its shape and speed.

Erkul and Leonhardt [9] give a related electromagnetic representation by translating Einstein evolution into conditions on permittivity and permeability. Here the corresponding-state storage transformation identifies the material constitutive shear, and the collective dynamics of Eq. (58) determines its propagation. Together these results establish the linear radiative link from the OQG response to the physical tensor sector of WTDiff. The full Diff and WTDiff representations retain the same two propagating tensor components [1, 2]; their different gauge descriptions do not change this wave equation or its normalization.

#### 4.6 Quantization of the spin-2 material response

The positive quadratic Hamiltonian in Eq. (69) permits the standard canonical quantization of free gravitational waves [10, 19]. We apply that procedure to the two physical tensor polarizations already identified in Section 4.5, retaining its conservative, long-wavelength regime and uniform reference state. This identifies the individual energy quanta carried by the material response.

Use a periodic normalization volume  $V$ , with nonzero wave vectors  $\mathbf{k}$  in this regime and angular frequencies  $\omega_{\mathbf{k}} = c_0 |\mathbf{k}|$ . The two real polarization tensors  $e_{ij}^\lambda(\mathbf{k})$ , labeled by  $\lambda \in \{+, \times\}$ , obey  $e_{ij}^\lambda e_{ij}^{\lambda'} = 2\delta_{\lambda\lambda'}$  at each wave vector. Here  $\delta$  is the Kronecker delta, equal to one for matching labels and zero otherwise. A hat denotes an operator;  $\hat{a}_{\mathbf{k}\lambda}$  removes one quantum and its adjoint  $\hat{a}_{\mathbf{k}\lambda}^\dagger$  adds one. The field expansion is

$$\begin{aligned} \hat{h}_{ij}(\mathbf{x}, t) &= \sum_{\mathbf{k}, \lambda} \mathcal{N}_{\mathbf{k}} e_{ij}^\lambda(\mathbf{k}) [\hat{a}_{\mathbf{k}\lambda} e^{i(\mathbf{k} \cdot \mathbf{x} - \omega_{\mathbf{k}} t)} + \hat{a}_{\mathbf{k}\lambda}^\dagger e^{-i(\mathbf{k} \cdot \mathbf{x} - \omega_{\mathbf{k}} t)}], \\ \mathcal{N}_{\mathbf{k}} &= \sqrt{\frac{\hbar c_0^2}{4\mathcal{S}_0 V \omega_{\mathbf{k}}}} = \sqrt{\frac{8\pi G_0 \hbar}{c_0^2 V \omega_{\mathbf{k}}}}. \end{aligned} \quad (74)$$

The amplitude  $\mathcal{N}_{\mathbf{k}}$  follows from the stiffness in Eq. (68) and the stated polarization normalization. Imposing the bosonic commutators gives independent harmonic oscillators. With  $[A, B] = AB - BA$ , and all annihilation–annihilation and creation–creation commutators zero, substitution into Eq. (69) gives

$$\begin{aligned} [\hat{a}_{\mathbf{k}\lambda}, \hat{a}_{\mathbf{k}'\lambda'}^\dagger] &= \delta_{\mathbf{k}\mathbf{k}'} \delta_{\lambda\lambda'}, \\ \hat{H}_{\text{TT}} &= \sum_{\mathbf{k}, \lambda} \hbar \omega_{\mathbf{k}} \left( \hat{a}_{\mathbf{k}\lambda}^\dagger \hat{a}_{\mathbf{k}\lambda} + \frac{1}{2} \right). \end{aligned} \quad (75)$$

Let  $|0\rangle$  be the collective vacuum, annihilated by every  $\hat{a}_{\mathbf{k}\lambda}$ . The state  $|1_{\mathbf{k}\lambda}\rangle = \hat{a}_{\mathbf{k}\lambda}^\dagger |0\rangle$  carries energy  $E = \hbar \omega_{\mathbf{k}}$  above that vacuum and momentum  $\mathbf{p} = \hbar \mathbf{k}$ . Hence  $E^2 - c_0^2 |\mathbf{p}|^2 = 0$ . Its circular polarizations have helicities  $+2$  and  $-2$ , following Eq. (72). A graviton in this effective description is therefore a massless spin-2 quantum of the collective material response.

Equation (67) supplies its electromagnetic meaning. Promoting the same linear relation to operators gives

$$\frac{\delta \hat{\epsilon}^{ij}}{\epsilon_0} = \frac{\delta \hat{\mu}^{ij}}{\mu_0} = -\hat{h}_{ij}. \quad (76)$$

The permittivity and permeability disturbances are thus two descriptions of the same quantum shear. In a coherent state, the expectation value of Eq. (74) recovers the classical wave of Section 4.5. The quantization preserves the existing wave speed, spin-2 polarizations, and stiffness normalization.

## 5 Gravitational potential energy as recoverable material storage

The energy change has a direct mechanical interpretation. If a body is moved slowly between positions with different supporting balances, external work changes its recoverable storage. The same energy gradient determines its force when it is released.

### 5.1 The recoverable-storage and work theorem

**Theorem 3 Gravitational potential energy is recoverable storage within the hypothesis.** Take a complete test body with reference energy  $E_0 = m_0 c_0^2$ , including binding, confinement, and assigned reference contributions. Assume Theorems 1 and 2 and reversible adiabatic transport through a static environment, so that endpoint energy changes can be recovered as work. Let  $W_{\text{on},1 \rightarrow 2}$  be work done on the body in moving from position 1 to position 2, where the clock factors are  $f_1$  and  $f_2$ . Its rest energy, work, and force are

$$\begin{aligned} E_{\text{rest}}(\mathbf{x}) &= E_0 f(\mathbf{x}) = E_0 \sqrt{P_b(\mathbf{x})/P_0}, \\ W_{\text{on},1 \rightarrow 2} &= E_0 (f_2 - f_1), \quad \mathbf{F}_{\text{rest}} = -E_0 \nabla f. \end{aligned} \tag{77}$$

**Proof.** Unitary dilation preserves the eigenvalue before multiplication by  $f$ , so the complete corresponding energy is  $fE_0$ . Reversible adiabatic work equals the endpoint energy change, and the force is the negative position derivative of that energy. Theorem 2 replaces  $f$  by  $\sqrt{P_b/P_0}$ . In the matched electromagnetic realization,  $C, L \propto f^{-1}$ , so the storage Hamiltonian at corresponding canonical charge and flux amplitudes carries the same energy factor. The material and electrical work accounts therefore agree, completing the proof.

With reference power fixed, the force is  $\mathbf{F}_{\text{rest}} = E_0 \nabla \delta_P / (2\sqrt{1 - \delta_P})$ . A larger deficit means lower rest energy. Motion toward smaller deficit therefore requires supplied work. Let  $U_g = E_0(f - 1)$  be gravitational storage energy relative to the reference state,  $\phi$  the weak gravitational potential per unit reference mass, and  $\Delta$  an endpoint difference. In the weak-field normalization  $f \simeq 1 + \phi/c_0^2$ . For a small upward height change  $\Delta z$  in approximately uniform gravitational acceleration of magnitude  $g$ , the lifting work  $W_{\text{lift}}$  is

$$\Delta U_g = m_0 c_0^2 \Delta f \simeq m_0 \Delta \phi, \quad W_{\text{lift}} \simeq m_0 g \Delta z. \tag{78}$$

Within the OQG hypothesis, the work of lifting remains in the body's changed material state. A crane supplies the increase; reversible lowering returns it as work. At the spherical Earth benchmark, raising 1 kg by 1 m adds approximately 9.80 J. Returning around a reversible closed path restores the initial storage and gives zero net work. This identifies gravitational potential-energy change with recoverable material energy.

For a test body, the source environment is held fixed and its effect is included in the body's position-dependent Hamiltonian. Count its potential-energy change once as  $E_0(f - 1)$ . If source and receiver both move, the complete account includes their shared interaction once, together with changes in the source and field.

Incoming and outgoing powers balance at each endpoint. During transport, the applied work and transient exchange account for the storage change. The stationary power deficit is therefore compatible with zero net heat production.

## 5.2 A mechanical mode and its electrical representation

To express this work in electrical variables, consider a stable nuclear size mode. Let  $R$  be its size coordinate near  $R_*$ , with energy  $E_*$  and quadratic Hamiltonian  $H_N$ . Define fractional displacement  $y = (R - R_*)/R_*$ , conjugate momentum  $p_y$ , radial inertia  $\mathcal{M}_R$ , and energy curvature  $k_R$ . For a charge scale  $q_*$ , introduce charge  $Q_N = q_* y$  and flux  $\Phi_N = p_y/q_*$ . The equivalent inductance and capacitance are  $L_N, C_N$ .

**Proposition 3** A stable quadratic material mode has an LC representation. The canonical commutator is  $[Q_N, \Phi_N] = i\hbar$ . Substitution into the deformation Hamiltonian proves

$$\begin{aligned} H_N - E_* &= \frac{p_y^2}{2\mathcal{M}_R R_*^2} + \frac{k_R R_*^2 y^2}{2} = \frac{\Phi_N^2}{2L_N} + \frac{Q_N^2}{2C_N}, \\ L_N &= \frac{\mathcal{M}_R R_*^2}{q_*^2}, \quad C_N = \frac{q_*^2}{k_R R_*^2}. \end{aligned} \tag{79}$$

This change of variables preserves the energy and quantum commutator. Inertia becomes inductance, and restoring compliance becomes capacitance. It therefore expresses the mechanical mode in the same electrical quantities used to couple it to the field.

The inertia can itself include the adjustment of internal quantum states as the radius changes. For a smooth unique ground state  $|0; R\rangle$ , excited states  $|n; R\rangle$  at the same radius, and positive gaps  $\Delta_n = E_n(R) - E_0(R) > 0$ , assume adiabatic motion below those gaps and a finite, consistently renormalized spectral sum. The induced radial inertia  $\mathcal{M}_R^{\text{ind}}$  is

$$\mathcal{M}_R^{\text{ind}} = 2\hbar^2 \sum_{n>0} \frac{|\langle n; R | \partial_R 0; R \rangle|^2}{\Delta_n}, \quad \mathcal{M}_{R,b}^{\text{ind}} = b^{-3} \mathcal{M}_{R,0}^{\text{ind}}. \tag{80}$$

The excitation gaps and the change of state with radius determine this contribution. Under common scaling it has the same factor  $b^{-3}$  as the kinetic inertia in Theorem 1. The mechanical and electrical descriptions therefore retain the same work and frequency laws.

## 5.3 Variable components exert the material force

The force can now be calculated directly from variable capacitance and inductance. Let  $x$  be a slow deformation,  $F_x^{\text{st}}$  its storage force, and  $v_C = C^{-1}Q$  the capacitor voltage column. Differentiate Eq. (29) at fixed canonical charge  $Q$  and flux  $\Phi$ , using symmetrized quantum products:

$$\begin{aligned} F_x^{\text{st}} &= -\langle \partial_x H_{\text{st}} \rangle = \frac{1}{2} \langle I^T (\partial_x L) I \rangle + \frac{1}{2} \langle v_C^T (\partial_x C) v_C \rangle, \\ v_C &= C^{-1}Q. \end{aligned} \tag{81}$$

Both electric and magnetic storage contribute. For an isolated harmonic ground state the variances give  $-\partial_x(\hbar\omega/2)$ , checking the sign and normalization. For coupled matter the joint state supplies those averages. This is the same energy derivative used in Theorem 3.

The circuit power balance displays where the mechanical work enters. Let  $P_{\text{ports}}$  be power supplied through the electrical terminals and  $U_{\text{st}} = I^T L I / 2 + Q^T C^{-1} Q / 2$  stored component energy. Dots denote time derivatives, including changes of  $L$  and  $C$  during deformation. Then

$$P_{\text{ports}} = \dot{U}_{\text{st}} + \frac{1}{2} I^T \dot{L} I + \frac{1}{2} v_C^T \dot{C} v_C. \quad (82)$$

The last two terms transfer power into material deformation. Radiation and its compensating fluctuations complete the exchange, and Poynting's theorem gives the field version of the same account. Changes of inductance and capacitance thus carry the mechanical work.

A uniform supporting-power change selects a different material equilibrium. A spatial variation gives different stored energies at different positions and produces a force. This energy account now allows us to follow the motion of a released body.

## 6 From the storage force to geodesic motion

We now follow a body moving through the storage gradient. Two uses of dispersion must be distinguished. A dispersion force is the interaction due to correlated electromagnetic fluctuations. Matter-wave dispersion is the relation between a body's energy and momentum. The first changes the stored energy; the second governs propagation through that energy landscape. We show that the resulting phase paths are geodesics of the body's material metric.

### 6.1 From the storage gradient to momentum change

For a small complete body, let  $E_{\text{loc}}$  and  $\mathbf{p}_{\text{loc}}$  be energy and momentum measured by a local observer at rest in the static comparison coordinates, using local clocks and rulers. The quantities  $E$  and  $\mathbf{p}$  express them in the fixed reference standards. Bold momentum is distinct from the scalar supporting-power ratio. Assume the local Lorentz relation  $E_{\text{loc}}^2 = m_0^2 c_0^4 + c_0^2 \mathbf{p}_{\text{loc}}^2$ . On the common branch the clock and ruler conversions are  $E = b E_{\text{loc}}$  and  $\mathbf{p}_{\text{loc}} = b \mathbf{p}$ . They give the energy–momentum relation

$$E(\mathbf{x}, \mathbf{p}) = \sqrt{m_0^2 c_0^4 b(\mathbf{x})^2 + c_0^2 b(\mathbf{x})^4 \mathbf{p}^2}. \quad (83)$$

The rest energy carries factor  $b$  and the inertia factor  $b^{-3}$ , consistently with  $m_b c_b^2 = b m_0 c_0^2$  and  $c_b = b^2 c_0$ . For comparison-coordinate velocity  $\mathbf{v} = d\mathbf{x}/dt$ , the small-momentum expansion gives the Hamiltonian and its motion equations:

$$E = b m_0 c_0^2 + \frac{b^3 \mathbf{p}^2}{2 m_0} + O(|\mathbf{p}|^4), \quad \mathbf{v} = \partial_{\mathbf{p}} E, \quad \dot{\mathbf{p}} = -\partial_{\mathbf{x}} E. \quad (84)$$

In a slowly varying environment, a wave packet follows the phase paths of this Hamiltonian. This is the eikonal approximation: phase changes rapidly while amplitude and environment change slowly. Hamilton's equations then track the packet center. At instantaneous rest,  $\dot{\mathbf{p}} = -m_0 c_0^2 \nabla b$ , which reduces to  $-m_0 \nabla \phi$  in the weak field. The stored-energy gradient has therefore become an explicit momentum change.

The same Hamiltonian also fixes the coupling of a moving body to the material response. Write that response in the logarithmic coordinate  $q_c = -2\ln b$ . Differentiation gives

$$-\frac{\partial E}{\partial q_c} = \frac{1}{2}(E + \mathbf{p} \cdot \mathbf{v}). \quad (85)$$

Here  $q_c$  is dimensionless and is not a charge. The right side contains both energy and momentum transport. The moving-body coupling therefore follows from the same dispersion relation as the rest force.

Here  $b(\mathbf{x})$  is a prescribed static material profile, independent of the test body's canonical momentum. The local Lorentz dispersion is an input to this propagation calculation. If relative motion changes the receiver's supporting equilibrium, that response must be included as an additional dynamical state or in the full momentum dependence of the Hamiltonian before derivatives are taken.

## 6.2 Material clocks and rulers follow the same response

The clocks and rulers used to describe this motion must follow the same material response. Hydrogen provides a familiar example. Let  $e$  be elementary charge,  $m_r$  electron–proton reduced mass,  $a_B$  Bohr radius, and  $n$  the principal quantum number of energy level  $E_n$ . The reduced mass describes relative electron–proton motion. With charge and  $\hbar$  fixed, and with  $m_r \propto b^{-3}$  and  $\epsilon \propto b^{-2}$ , the nonrelativistic Coulomb formulas give

$$a_B = \frac{4\pi\epsilon\hbar^2}{m_r e^2} \propto b, \quad E_n = -\frac{m_r e^4}{2(4\pi\epsilon)^2 \hbar^2 n^2} \propto b. \quad (86)$$

The atomic radius and transition energy acquire the same common factor as the nuclear family, while the fine-structure coupling  $e^2/(4\pi\epsilon\hbar c)$  remains unchanged. Atomic clocks and size standards therefore give consistent comparisons. Figure 7 places these changes beside the complete body's energy–momentum relation.

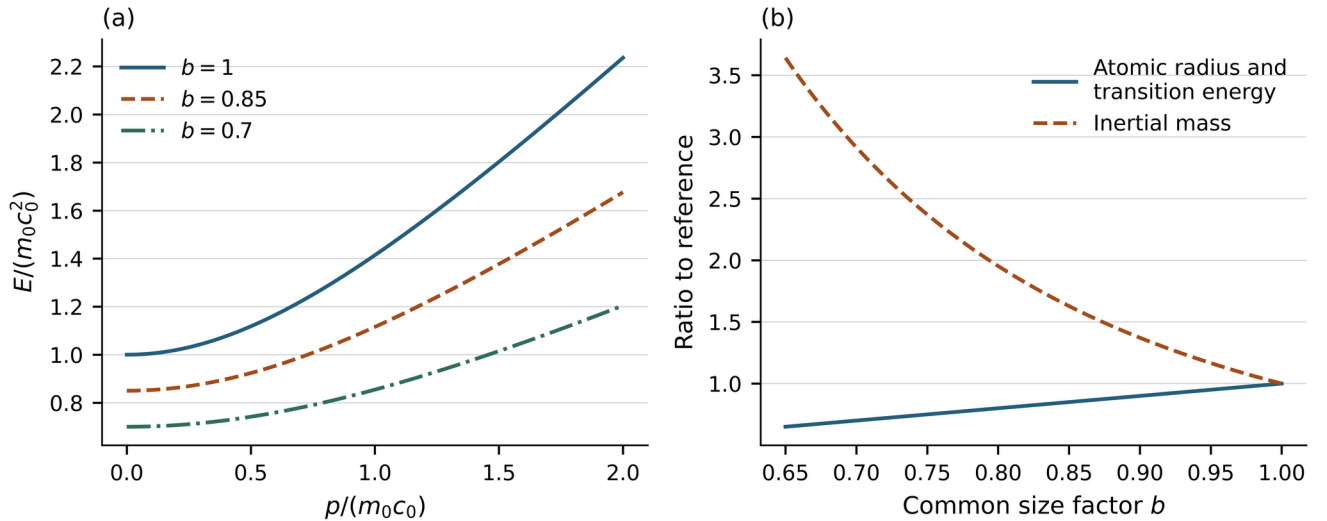


Fig. 7 Matter-wave dispersion and material standards. In this figure only,  $p = |\mathbf{p}|$  denotes momentum magnitude, rather than the supporting-power ratio. The left panel plots  $E/(m_0 c_0^2) = \sqrt{b^2 + b^4[p/(m_0 c_0)]^2}$  for three common-scale values. The right panel compares atomic radius, transition energy, and inertia in fixed reference standards.

The same standards also determine the rate of a moving clock. Let  $v = |\mathbf{v}|$  be comparison-coordinate speed,  $v_{\text{loc}}$  speed measured by the local static observer,  $\tau$  the moving clock's proper time, and  $\gamma_{\text{loc}}$  its Lorentz factor. On the common branch, the static observer's clock increment is  $b dt$  and a local ruler measures distance  $|d\mathbf{x}|/b$ . Their ratio gives the local speed. Combining that speed with local time dilation yields

$$v_{\text{loc}} = \frac{v}{b^2}, \quad \gamma_{\text{loc}} = \frac{1}{\sqrt{1-v_{\text{loc}}^2/c_0^2}}, \quad \frac{d\tau}{dt} = \frac{b}{\gamma_{\text{loc}}}. \quad (86a)$$

The factor  $b$  gives the clock-rate change between stationary material states. The additional factor  $1/\gamma_{\text{loc}}$  accounts for motion through the same local state. Thus a stationary clock has rate  $b$ , and at ambient support  $b = 1$  the expression reduces to the usual special-relativistic rate. Integrating this rate along a trajectory gives elapsed proper time. Equation (87) below reproduces the same result directly from the material interval.

### 6.3 Proof of geodesic motion

To establish the geometric description, express the body's motion through its quantum phase. Write the wavefunction as  $\psi = a_\psi \exp(iS/\hbar)$ , with slowly varying amplitude  $a_\psi$  and phase action  $S$ . Define momentum  $p_i = \partial_i S$ , energy  $E = -\partial_t S$ , and time coordinate  $x^0 = c_0 t$ . Latin indices label space and Greek indices spacetime; repeated upper and lower indices are summed. The Hamilton–Jacobi phase equation is  $\partial_t S + E(\mathbf{x}, \nabla S) = 0$ .

Let  $g_{\mu\nu}^{(m)}$  be the matter metric, the matrix expressing the material clock and distance comparisons, with inverse  $g_{(m)}^{\mu\nu}$ . The label  $(m)$  denotes matter, and diag lists diagonal entries with the time entry negative. The dispersion relation can then be written as

$$g_{\mu\nu}^{(m)} = \text{diag}(-b^2, b^{-2}, b^{-2}, b^{-2}), \quad g_{(m)}^{\mu\nu} \partial_\mu S \partial_\nu S + m_0^2 c_0^2 = 0. \quad (87)$$

Multiplying the phase equation by  $b^2 c_0^2$  recovers Eq. (83). The metric is thus a representation of the material dispersion already derived. Its phase paths can be obtained without adding a separate force law.

**Proposition 4 The dispersion-force Hamiltonian generates geodesic motion.** Take a smooth static common scale, a freely propagating small test body, and leading eikonal propagation. Define phase four-momentum  $p_\mu = \partial_\mu S$ , with  $p_0 = -E/c_0$ . Let  $\lambda$  be an affine path parameter, proportional to proper time measured along a massive ray. The characteristic Hamiltonian  $\mathcal{H} = 1/2 [g_{(m)}^{\mu\nu} p_\mu p_\nu + m_0^2 c_0^2]$  generates the phase paths. Hamilton's equations give  $dx^\mu/d\lambda = g_{(m)}^{\mu\nu} p_\nu$  and  $dp_\mu/d\lambda = -1/2 \partial_\mu g_{(m)}^{\alpha\beta} p_\alpha p_\beta$ . The connection coefficients  $\Gamma_{\alpha\beta}^\mu$ , or Christoffel symbols, describe how the metric's clocks and rulers vary with position. Substituting  $p_\mu = g_{\mu\nu}^{(m)} dx^\nu/d\lambda$  proves

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = 0, \quad (88)$$

$$\Gamma_{\alpha\beta}^\mu = \frac{1}{2} g_{(m)}^{\mu\nu} \left( \partial_\alpha g_{\nu\beta}^{(m)} + \partial_\beta g_{\nu\alpha}^{(m)} - \partial_\nu g_{\alpha\beta}^{(m)} \right).$$

The phase paths satisfy the geodesic equation, completing the proof. The position-dependent material Hamiltonian and the clock-and-ruler metric generate the same motion. The geometric description therefore records the effect of the storage interaction on propagation.

This equivalence can be checked by integrating the two trajectory descriptions independently. For comparison time, define  $\mathbf{s}_b = \nabla \ln b$ , with units of inverse length, and velocity  $\mathbf{v} = d\mathbf{x}/dt$ . The connection of Eq. (87) gives

$$\dot{\mathbf{v}} = 4(\mathbf{s}_b \cdot \mathbf{v})\mathbf{v} - (c_0^2 b^4 + \mathbf{v}^2)\mathbf{s}_b. \quad (89)$$

Integrating this equation and Hamilton's equations from the same initial state gives a maximum position difference of  $9.27 \times 10^{-11}$  in reference-length units and relative energy drift of  $9.25 \times 10^{-12}$ . Figure 8 shows the agreement for the specified profile. It checks the conversion between the material and geodesic descriptions.

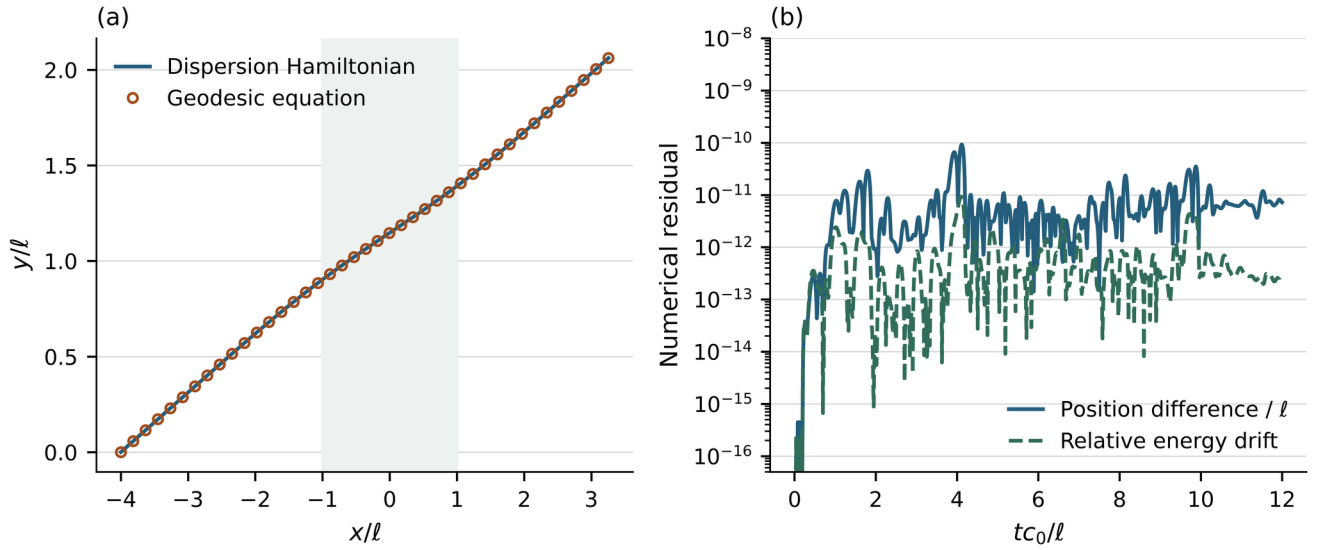


**Fig. 8** Independent integrations of one trajectory in Cartesian comparison coordinates  $x, y$ . Let  $\ell$  be the spatial width of the imposed profile  $b(x) = 1 - 0.08\exp[-x^2/(2\ell^2)]$ . In units  $c_0 = m_0 = \ell = 1$ , the initial position is  $(x, y) = (-4, 0)$  and its canonical momentum components are  $(p_x, p_y) = (0.8, 0.25)$ . The left panel compares Hamiltonian and geodesic paths; shading marks  $|x| \leq \ell$ . The right panel shows position difference and energy drift at solver relative tolerance  $10^{-11}$ .

Let  $\mathbf{F} = \dot{\mathbf{p}}$  denote force as the rate of momentum change in the comparison coordinates. The Hamiltonian gives

$$\mathbf{F} = \dot{\mathbf{p}} = -\frac{m_0^2 c_0^4 b + 2c_0^2 b^3 \mathbf{p}^2}{E} \nabla b, \quad \mathbf{F}_{\mathbf{p}=0} = -m_0 c_0^2 \nabla b. \quad (90)$$

The rest force reduces to  $-m_0 \nabla \phi$  at leading weak-field order. The full dispersion fixes its momentum dependence. A comoving accelerometer can read zero while comparison-coordinate momentum changes along this energy gradient. Momentum conservation includes source, field, and receiver together.

## 6.4 Free fall and the meaning of force

In general relativity (GR), the statement that “gravity is not a force” refers to ideal free fall: the body has zero proper acceleration and requires no nongravitational four-force to follow its geodesic. A comoving accelerometer reads zero, although neighboring trajectories can have tidal relative acceleration [20]. The statement specifies a local measurement and a representation of motion.

An accelerometer makes the distinction concrete. It measures the support load between its proof mass and casing. When both fall together, there is no local support load. Holding the casing at a fixed height makes it resist the free motion, and the instrument registers that load. The zero free-fall reading therefore does not establish the absence of an interaction or a spatial variation of stored energy.

Within OQG, the interaction is the dispersion force produced by the scattering-induced change of supporting power. Its mechanical expression is  $-E_0 \nabla \sqrt{p}$ ; its propagation expression is the Hamiltonian and geodesic equation derived above. Lifting changes recoverable storage, release permits motion along its gradient, and variation of the gradient produces tidal relative motion.

The illusion is to interpret the disappearance of a local force term as the disappearance of the physical interaction. Freely falling standards incorporate the common motion into their trajectory description. In the OQG interpretation, the underlying material interaction remains and its phase paths are geodesics.

The comparison force  $d\mathbf{p}/dt$  and proper four-force are different quantities; one can be nonzero while the other vanishes. The material interaction is counted once, through either the Hamiltonian or its equivalent metric.

## 6.5 Phase shifts and scattering of the moving matter wave

The ray description follows motion through a smooth environment. The same wave equation also describes phase shifts and diffraction when spatial structure is resolved. Here it is the receiving body's matter wave that scatters. Neutron crystal interferometers demonstrate coherent splitting and redirection [11], while gravitationally induced neutron interference demonstrates phase sensitivity to potential energy [5].

At leading weak-field and low-speed order, remove the constant rest-energy phase from Eq. (84). The wave equation is  $i\hbar \partial_t \psi = [-\hbar^2 \nabla^2 / (2m_0) + m_0 \phi] \psi$ , with the Laplacian  $\nabla^2$  supplying the kinetic term. The full matched operator fixes higher-order momentum and gradient terms. For background Hamiltonian  $H_{\text{ref}}$  and localized potential change  $\delta\phi$ , the added interaction is  $V = m_0 \delta\phi$ .

Let  $\mathcal{E}$  be the fixed scattering energy,  $|\psi_{\text{in}}\rangle$  the incident wave, and  $|\psi^{(+)}\rangle$  the full wave with an outgoing scattered contribution. The infinitesimal  $i0^+$  imposes the outgoing boundary condition on the inverse operator. The stationary wave then obeys

$$|\psi^{(+)}\rangle = |\psi_{\text{in}}\rangle + (\mathcal{E} - H_{\text{ref}} + i0^+)^{-1} V |\psi^{(+)}\rangle. \quad (91)$$

The inverse operator propagates the disturbance, with the long-range interaction retained in the background Hamiltonian. Smooth variations refract the phase paths and recover the ray equations. Resolved structure permits diffraction and reflection.

A static Hermitian interaction scatters elastically, preserving energy while redistributing momentum and amplitude. Inelastic channels exchange energy with internal or environmental modes and belong in the complete power account. Thus the material energy used for work and free fall also enters the body's quantum phase and scattering response.

## 7 Quantitative gravitational predictions

The derived relations connect power, storage, and motion. To attach a gravitational scale to them, we now calibrate the inverse-distance interaction to the measured strength. This yields a common set of quantitative comparisons for Earth, using the same reference matter and standards throughout.

### 7.1 Calibrating the source strength and power profile

**Corollary Inverse-distance storage gives the leading power deficit.** Let  $M$  be the source mass and  $r$  the source–receiver separation. Identify the leading conservative interaction  $U_{\text{scat}} = -\mathcal{K}_{\text{EM}}/r$  with the test body's storage change  $E_0(f - 1)$ . Then  $f = 1 - \mathcal{K}_{\text{EM}}/(E_0 r) + O(r^{-2})$ , and the power theorem gives  $\delta_P = 2\mathcal{K}_{\text{EM}}/(E_0 r) + O(r^{-2})$ . With  $E_0 = m_0 c_0^2$  and the calibrated coefficient  $\mathcal{K}_{\text{EM}} = G_0 M m_0$ , this is  $\delta_P = 2G_0 M/(r c_0^2) + O(r^{-2})$ . The factor of two follows from squaring the clock-energy factor in the power relation.

For the weak spherical gravitational comparison, fix the strength with measured  $G_0/c_0^4$ . Write the potential as  $\phi = -G_0 M/r$ , with  $|\phi| \ll c_0^2$ . The remainder  $O(\phi^2/c_0^4)$  collects second and higher orders. The resulting clock, common-size, and power relations are

$$f = b + O(\phi^2/c_0^4) = 1 + \frac{\phi}{c_0^2} + O(\phi^2/c_0^4), \quad \delta_P = \frac{2G_0 M}{r c_0^2} + O(\phi^2/c_0^4). \quad (92)$$

The factor of two in the deficit comes from the squared-frequency power law. With the strength calibrated, the same profile can now be used for material standards and gravitational work.

### 7.2 The Earth deficit and the force-producing gradient

To pass from a pair interaction to an extended spherical source, take the weak additive limit. Let  $\mathbf{R}$  be the receiver position,  $\mathbf{r}'$  a source position,  $\rho_M(\mathbf{r}')$  the mass density, and  $R = |\mathbf{R}|$  the distance from the source center. A radius  $R_{\text{source}}$  encloses total mass  $M$ . For a receiver of mass  $m_0$ , assume the inverse-distance coefficient is mass-proportional and normalized by  $G_0$ . The potential energy  $U(\mathbf{R})$  is

$$U(\mathbf{R}) = -G_0 m_0 \int \frac{\rho_M(\mathbf{r}')}{|\mathbf{R} - \mathbf{r}'|} d^3 r', \quad U(R) = -\frac{G_0 M m_0}{R} \quad (R > R_{\text{source}}). \quad (93)$$

Outside each spherical shell the angular integral is  $4\pi/R$ . Summing the shells leaves the inverse-distance expression with total mass  $M$ . Let  $M_\oplus, R_\oplus$  be Earth's mass and reference surface radius, and  $\delta_{P,\oplus}$  the Earth-induced deficit. The supporting-power profile is therefore

$$\delta_{P,\oplus}(R) \simeq \frac{2G_0 M_\oplus}{R c_0^2}, \quad \frac{d(P/P_0)}{dR} \simeq \frac{2G_0 M_\oplus}{c_0^2 R^2}. \quad (94)$$

Using the nominal inputs  $G_0 M_\oplus = 3.986004 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$ ,  $R_\oplus = 6.3781 \times 10^6 \text{ m}$ , and  $c_0 = 299792458 \text{ m s}^{-1}$  gives Table 2. The benchmark treats Earth as spherical and omits rotation. The reference is the same ambient state with the Earth's source contribution removed.

**Table 2** Calibrated spherical Earth benchmark in fixed reference standards

| Quantity                                     | Surface value                             |
|----------------------------------------------|-------------------------------------------|
| Gravitational acceleration                   | $9.798398 \text{ m s}^{-2}$               |
| Supporting-power deficit $\delta_{P,\oplus}$ | $1.390705 \times 10^{-9}$                 |
| Common size and frequency deficit $1 - b$    | $6.953525 \times 10^{-10}$                |
| Fractional rise of $\epsilon$ and $\mu$      | $1.390705 \times 10^{-9}$                 |
| Outward gradient $d(P/P_0)/dR$               | $2.180438 \times 10^{-16} \text{ m}^{-1}$ |
| Work to raise 1 kg by 1 m                    | approximately 9.7984 J                    |

At the surface, the required reduction is 1.39 parts per billion of the reference supporting power, with an outward fractional gradient of approximately  $2.18 \times 10^{-16}$  per meter. A uniform deficit changes the material state. Its spatial gradient changes the energy between positions and produces the force. If the reference exchange is known in watts, the surface reduction is  $\Delta P_\oplus = (1.390705 \times 10^{-9})P_0$ .

### 7.3 Clocks, work, and free fall from the same profile

Raising a clock samples the change of supporting power with height. Let  $\Delta z$  be a small elevation change near Earth's surface,  $g$  the acceleration magnitude,  $\Delta\omega$  the frequency increase in reference standards, and  $\Delta p$  the change of power ratio. The same potential determines the acceleration  $\mathbf{a}$  of a released body. For neighboring bodies separated by  $\xi$ , their acceleration difference  $\Delta a_i$  depends on the spatial curvature  $\partial_i \partial_j \phi$ ; repeated index  $j$  is summed. These comparisons give

$$\begin{aligned} \frac{\Delta\omega}{\omega_0} &\simeq \frac{g\Delta z}{c_0^2}, & \Delta p &\simeq \frac{2g\Delta z}{c_0^2}, \\ \mathbf{a} &\simeq -\nabla\phi, & \Delta a_i &\simeq -(\partial_i \partial_j \phi) \xi^j. \end{aligned} \tag{95}$$

The Earth benchmark gives a frequency increase of about  $1.09022 \times 10^{-16}$  per meter and power-ratio increase of  $2.18044 \times 10^{-16}$  per meter. The associated lifting work is  $\Delta E \simeq m_0 g \Delta z$ . Equation (61) supplies the accompanying common-branch changes of dimensions and electrical coefficients.

**Corollary Universal leading free fall on the matched branch.** Assume the same local power profile for complete receiving bodies and the matched energy and inertia laws. The leading rest force is  $-m_0 \nabla\phi$ , and the leading inertial coefficient is  $m_0$ . Their ratio gives  $\mathbf{a} = -\nabla\phi$  independently of  $m_0$ . Differentiating this acceleration across a separation gives the last expression in Eq. (95). The result includes both common free fall and tidal relative motion.

Composition enters the nuclear response that realizes the equilibrium. Bodies satisfying the same complete-energy and inertia correspondence share the leading free-fall acceleration. A systematic composition-dependent failure would appear as a departure from that result. Comparing clocks, sizes, and recoverable work tests whether the same power profile describes these responses together.

The Earth number is a fractional change in each receiver's own reference exchange. The source determines its spatial profile, and the complete receiver energy determines the force and work. This is why clocks, lifting, free fall, and tides can be compared through one calibrated power response.

## 8 Spherical geometry from material clocks and rulers

The common branch gives clocks and lengths the same factor. The exact spherical exterior requires their independent response, even though their leading weak-field changes agree. We now retain both factors, specify the additional spherical conditions, and solve for the material comparisons before identifying their metric.

### 8.1 The material metric and optical propagation

A metric records how clocks, rulers, and phase propagation compare between positions. The scalar  $b$  describes the static isotropic common family. More general electromagnetic and stress responses can depend on direction and include propagating tensor disturbances. A metric representation is useful when these responses admit a common local propagation structure.

Light propagation first determines which paths are lightlike. For a local reciprocal response without birefringence, meaning without polarization-dependent propagation speeds, a metric-type constitutive law gives an optical metric up to a positive scalar multiplier [18]. This conformal factor leaves the lightlike paths unchanged. Let  $g_{\text{EM}}$  and  $g_m$  be representatives of the optical and matter metrics. On the common family,

$$\begin{aligned} g_{\text{EM}} &= \text{diag}(-b^3, b^{-1}, b^{-1}, b^{-1}), \\ g_m &= \text{diag}(-b^2, b^{-2}, b^{-2}, b^{-2}), \quad g_{\text{EM}} = b g_m. \end{aligned} \tag{96}$$

Both have the same light cones and comparison speed  $b^2 c_0$ . The optical representative has determinant  $-1$  and gives the common coefficients in Maxwell's equations. Its overall conformal factor cancels from four-dimensional electromagnetic propagation. The massive body's clock and energy relation fix the additional scale needed for the matter metric.

### 8.2 The clock and ruler conditions for the spherical exterior

For independent  $f, b$ , write the body's comparison-coordinate Hamiltonian as  $H$ . Let  $\rho$  be isotropic radius, so radial and angular spatial intervals carry the same coefficient. The angles  $\theta, \varphi$  give the angular element  $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ . With squared interval  $ds^2$  and reference time  $t$ , the local Lorentz relation and material conversions give

$$\begin{aligned} H &= f \sqrt{E_0^2 + c_0^2 b^2 \mathbf{p}^2}, \quad p = f^2, \\ ds^2 &= -f^2 c_0^2 dt^2 + b^{-2} (d\rho^2 + \rho^2 d\Omega^2). \end{aligned} \tag{97}$$

For this independent clock and ruler family, the local speed becomes  $v_{\text{loc}} = v/(fb)$  and the moving clock rate is  $d\tau/dt = f/\gamma_{\text{loc}}$ , with  $\gamma_{\text{loc}}$  defined in Eq. (86a). The clock factor  $f$  and the Lorentz factor retain their separate meanings. Setting  $f = b$  recovers the common-branch result.

A second radial coordinate describes physical spherical area. Define the areal radius  $R = \rho/b$ , for which that area is  $4\pi R^2$ . The calibrated source length  $r_s = 2G_0M/c_0^2$  is the Schwarzschild radius. Distinguishing these radii is necessary when reconstructing the complete exterior.

**Proposition 5 Conditional reconstruction of the static spherical exterior.** Take positive  $f, b$  in the exterior  $R > r_s$ , with  $f, b \rightarrow 1$  at large radius. Specify two spherical material-response conditions: the exact continuation  $p(R) = 1 - r_s/R$  of the calibrated leading power profile, and the radial ruler response  $dR/d\rho = f/b$ . Then

$$R = \frac{\rho}{b}, \quad p(R) = 1 - \frac{r_s}{R}, \quad \frac{dR}{d\rho} = \frac{f}{b}, \quad r_s = \frac{2G_0M}{c_0^2}. \quad (98)$$

The power condition determines the clock factor through Theorem 2. The radial condition supplies the independent ruler response. Together they determine the Schwarzschild exterior; conversely, that exterior in isotropic coordinates satisfies both conditions.

**Proof.** The positive clock factor is  $f = \sqrt{1 - r_s/R}$ . Substituting  $b = \rho/R$  in the radial condition separates the variables. Integration with the reference normalization gives

$$\frac{d\rho}{\rho} = \frac{dR}{R\sqrt{1 - r_s/R}}, \quad \rho = \frac{R}{4} \left(1 + \sqrt{1 - r_s/R}\right)^2. \quad (99)$$

To verify the integration, put  $z = \sqrt{1 - r_s/R}$ . The right-hand differential becomes  $2 dz/(1 - z^2)$ , whose integral is  $\ln[(1 + z)/(1 - z)]$ . Fixing  $\rho/R \rightarrow 1$  supplies the integration constant and the displayed expression. Its inverse is  $R = \rho(1 + r_s/(4\rho))^2$ .

Define  $u = r_s/(4\rho)$  as a dimensionless inverse-radius variable and  $c_{\text{iso}}$  as radial light-propagation speed in the isotropic comparison coordinates. The solved material profiles are

$$f = \frac{1-u}{1+u}, \quad b = (1+u)^{-2}, \quad R = \rho(1+u)^2, \quad \frac{c_{\text{iso}}}{c_0} = fb. \quad (100)$$

Substitution into the material line element uses  $b^{-2}\rho^2 = R^2$  for the angular part and  $b^{-2}d\rho^2 = f^{-2}dR^2$  for the radial part. Thus

$$\begin{aligned} \delta_p &= 1 - f^2 = \frac{4u}{(1+u)^2} = \frac{r_s}{R}, \\ ds^2 &= -\left(1 - \frac{r_s}{R}\right) c_0^2 dt^2 + \left(1 - \frac{r_s}{R}\right)^{-1} dR^2 + R^2 d\Omega^2. \end{aligned} \quad (101)$$

This is the Schwarzschild exterior line element with the same calibrated mass parameter. Conversely, its lapse, the clock factor  $f$ , fixes  $f^2 = 1 - r_s/R$  and its radial coefficient fixes  $dR/d\rho = f/b$  on the outward-oriented exterior branch. This proves both directions of the correspondence.

The same solution gives recoverable storage  $U_g(R) = E_0[\sqrt{1 - r_s/R} - 1]$ , with expansion  $-G_0 M m_0 / R - G_0^2 M^2 m_0 / (2c_0^2 R^2) + O(R^{-3})$ . Its leading term is the gravitational potential energy already obtained. The higher terms belong to the specified spherical continuation.

The clock factor  $f$  and ruler factor  $b$  now differ by  $f/b = 1 - u^2$ , although their first-order weak-field changes agree. Consequently  $p = f^2$  still controls the clock, while fixed  $\kappa$  gives stiffness  $A/A_0 = fb$  and coefficients  $\epsilon/\epsilon_0 = \mu/\mu_0 = (fb)^{-1}$ . The common-branch identity  $A/A_0 = p$  applies when  $f = b$ . Keeping the two factors separate prevents the common-branch power map from being applied beyond its stated scope.

The reconstruction identifies the material content of the exact exterior: supporting power fixes the clock and stored energy, radial response fixes the rulers, and their Hamiltonian generates the trajectories. General covariance expresses these same relations in different coordinates. The proposed physical cause remains the electromagnetic exchange and material response.

## 9 Discussion

The calculations address successive parts of one physical question: how can surrounding matter change the supporting balance and thereby change storage and motion? The dipole example resolves the interference mechanism. The passive model resolves a load and restoring response. The theorems establish stable scaling, the supporting-power law, and recoverable work for the stated family. Their connection to gravity is the OQG storage hypothesis.

The examples have distinct roles. The prescribed narrow-band dipole field gives a received-absorption ratio, whereas the nuclear application requires a complete equilibrium exchange. In that example the far-field absorption deficit begins at inverse-square order, while the leading force integrates to an inverse-distance potential. Connecting these observables to nuclear storage requires the same nuclear spectra, spatial currents, and compensating field fluctuations in the coupled calculation.

The Earth deficit uses measured  $G_0/c_0^4$  to calibrate the weak spherical strength. The relative power, clock, and work comparisons follow from that calibration. The supporting power in watts additionally requires the mode spectrum and normalization. An independent evaluation of the gravitational strength from constituent parameters is outside the results obtained here.

The exact spherical result also has a stated scope. Its  $p(R)$  continuation and radial ruler condition are inputs beyond the leading scattering example. Solving them establishes the Schwarzschild exterior correspondence that demonstrates the Einstein equivalent metric interpretation.

Likewise, the geodesic proof establishes the propagation implied by the material Hamiltonian. It shows how a storage force and zero proper acceleration in free fall coexist, while leaving the microscopic origin to the coupled-response calculation. The interaction is counted once in its mechanical or geometric representation.

Throughout, strong-interaction physics remains in the nuclear states and complete energy. Each response is used within its specified frequency domain. Stationary supporting powers balance, and adjustment energy is included in storage, radiation, and work. Lossless mixing of equally occupied channels alone

does not create the proposed deficit; the required change concerns the coupled response and its equilibrium.

The case for OQG rests on the connection between electromagnetic exchange, nuclear storage, material scaling, and gravitational motion. These steps answer three objections that otherwise obscure what the construction establishes.

The first objection concerns the physical status of the proposed exchange. Nuclear charge and current responses, electromagnetic scattering, interference, and radiation reaction already have established descriptions. The nuclear storage coefficients derived here express that response through the interacting excitation spectrum and charge-transition matrix elements. Strong-interaction binding remains present in those states; the electrical representation does not replace nuclear matter with an assumed mechanical container. The worked scattering calculation further demonstrates that interference can reduce received power. The proposed mechanism therefore has a concrete electromagnetic basis.

The equilibrium involved is between matter and its local electromagnetic environment, as defined in Section 2.1. Surrounding matter modifies that shared field through scattering, with its source motion and boundary conditions included; the construction does not require a separate, directly prescribed transfer of supporting power between distant nucleons. Puthoff's semiclassical equilibrium calculation [21] illustrates how absorbed and radiated powers can balance while sustaining stationary motion. Equality of those powers establishes zero net heating, but does not require their common value to remain unchanged between environments.

Consequently, asking where the electromagnetic effect appears is a legitimate physical question. Its contributions to exchange, storage, and motion must be included in the account. OQG identifies the component associated with the derived material scaling as the interaction observed gravitationally. That identification rests on the linked consequences developed here, beyond the existence of electromagnetic coupling alone. Within the demonstrated Hamiltonian–geodesic correspondence, the material interaction and its geometric representation describe the same motion and must be counted once. Introducing an additional full-strength gravitational interaction would double-count that contribution. Likewise, a zero accelerometer reading in free fall is compatible with the derived motion: it measures the absence of a local support load, while the material Hamiltonian determines the changing comparison-coordinate momentum.

The second objection is that the supporting-power relation remains an independently adjustable assumption. Within the stated covariant family, it does not. Let  $f$  denote the transition-frequency ratio and  $b$  the corresponding length ratio. With fixed charge and the matched response specified by the theorems, the nuclear spectral scaling and supporting-power theorem give

$$\frac{C}{C_0} = \frac{L}{L_0} = \frac{1}{f}, \quad \frac{P}{P_0} = f^2.$$

The capacitance and inductance transformations follow from the nuclear spectrum, while the power theorem retains the current profiles, radiation reaction, fluctuations, and correlations of the corresponding states. Nuclear substructure and strong-interaction contributions do not become exceptions to the transformation merely because they occur at a deeper material level.

This result also permits inference from observation toward the mechanism. Once a measured gravitational clock ratio is represented by the corresponding-state family, its supporting-power ratio is fixed by  $p = f^2$ .

It is not necessary to introduce another scaling conjecture for power. The same construction connects that ratio to storage, inertia, dimensions, and motion. The separate electromagnetic coefficients follow with the ruler and impedance relations in the declared comparison standards. Thus the observations constrain a connected material response, rather than a collection of independently fitted changes.

The Earth benchmark makes this connection quantitative. After calibrating the gravitational strength, the inferred surface supporting-power deficit is approximately 1.39 parts per billion, and its spatial gradient supplies the observed acceleration. This is an inference through the proved correspondence, not a separate measurement of supporting power in watts. Its significance is that the power, clock, and work comparisons must agree under one normalization. A microscopic evaluation would determine whether the proposed nuclear-field realization supplies that response; it would not make the already-derived relationships optional.

The third objection concerns whether independent clock and ruler factors leave the spherical exterior arbitrary. The answer depends on retaining the complete conditions of the problem. Within the adopted Einstein-equivalent branch [1, 2], a spherically symmetric vacuum exterior with zero cosmological constant is Schwarzschild, up to coordinates and its mass parameter. Birkhoff's theorem supplies that uniqueness [23]. An alternative with exterior charge, additional stress, or different field equations changes the physical problem.

Allowing  $f$  and  $b$  to differ removes the common-factor restriction and permits the material description to represent this exterior. Section 8 establishes that correspondence under its explicit power and radial-response conditions. Energy conditions alone are insufficient to establish uniqueness, but the Einstein-equivalent vacuum equations provide the stronger restriction. The familiar exponential metric is not another solution of that same vacuum problem; interpreted through Einstein's equations, it requires an effective source violating the null energy condition and admits a traversable wormhole geometry [3]. The resulting distinction is precise: Schwarzschild selection is settled within the adopted macroscopic branch.

These answers also clarify the role of experimental equivalence. Wherever the material and geometric descriptions predict identical observables, a Cavendish experiment, clock comparison, or trajectory measurement cannot select between their physical interpretations. Such measurements constrain their shared predictions. A distinguishing experiment would therefore have to probe a regime or observable beyond the demonstrated equivalence.

The resulting challenge is therefore specific: identify an error in the derivations, a failure of their physical premises, or an incompatibility between the proposed nuclear response and the linked observations. Reproducing the same gravitational motion through geometry does not supply that refutation.

## 10 Conclusions

The starting point of this work is the power balance sustaining material equilibrium. Scattering changes the field received by matter and can lower its share of that exchange. The resonant-dipole calculation establishes this mechanism explicitly: interference exceeds the added scattered intensity and reduces receiver absorption by 0.07546395% at the specified parameters. The cross section and incident energy density convert that fractional result to watts.

The material consequences are expressed through nuclear storage. The excitation spectrum and current geometry determine capacitance, inductance, and their field coefficients. The passive model exhibits contraction-directed loading and restoration against electric–magnetic imbalance. The corresponding-state theorems then establish that stable binding survives the linked changes of dimensions, inertia, and clock frequency, and that supporting power scales as the square of the clock factor.

This power law connects the exchange to recoverable energy and work. On the common branch, the relative power fixes the electromagnetic coefficients and ambient stress stiffness, while complete material energy follows its square root. Within the OQG hypothesis, lifting supplies the increase in that storage, lowering can recover it, and the spatial energy gradient produces the dispersion force. The matter-wave Hamiltonian carries that interaction into geodesic motion, consistently with zero proper acceleration in ideal free fall.

The collective response also carries gravitational radiation. Extending the matched constitutive scaling to directional deformation identifies the transverse, traceless changes of permittivity and permeability with the shear used in the stress functional. Its stated long-wavelength dynamics yields the massless wave equation, the plus and cross polarizations, helicities  $\pm 2$ , and positive wave energy. This establishes the linear radiative correspondence between the material response and the physical tensor sector of WTDiff.

Calibration to the observed strength makes these relations quantitative. The spherical Earth benchmark requires a supporting-power deficit of 1.39 parts per billion and gives consistent clock shifts, lifting work, free fall, and tidal comparisons. Retaining independent clock and ruler factors also reconstructs the Schwarzschild exterior under the two explicit spherical response conditions.

The remaining microscopic step is to evaluate the collective nuclear supporting exchange with the same response used for storage and force. The results here specify what that calculation must connect. In the proposed mechanism, the loss of available supporting power is the source of gravity; its material response carries the changes measured by clocks, rulers, and mechanical work.

Operational Quantum Gravity places the physical explanation of gravity in the response of matter to its shared electromagnetic environment. The supporting-power balance determines the material equilibrium, and spatial variations of its stored energy govern motion. Because clocks and rulers are made of the same responding matter, their comparisons carry these changes into the metric description. Geometry thus records the relationships established by electromagnetic exchange, material scaling, and recoverable work. This is the physical meaning of the OQG principle: Matter tells matter how to scale. Geometry is just bookkeeping.

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**Author contributions.** Todd J. Desiato conceived the OQG program and the material-scaling interpretation, directed the theoretical development, reviewed the calculations and literature, and prepared the manuscript.

**Data availability.** This theoretical study generated no new experimental or observational datasets. The definitions and analytical derivations supporting the results are contained in this article. Online Resource 1 contains the numerical inputs, sampled outputs, and benchmark results for the passive-storage model and the Hamiltonian–geodesic comparison.

**Code availability.** Online Resource 1 supplies a standalone Python program, dependency versions, and execution instructions for the numerical checks in Sections 3.3 and 6.3.

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**Supplementary information.** Online Resource 1 contains the numerical reproduction program and results for quantum storage and matter-wave trajectories.

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