

Matter Wave Scaling and Recoverable Energy Storage in Operational Quantum Gravity

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Abstract

Physical clocks and rulers obtain their frequencies and lengths from bound quantum matter. Their gravitational comparison therefore carries a material question: how do binding, inertia, and stored energy remain consistent when those standards change together? We prove that a specified common scaling of quantum matter-wave size and transition frequency preserves stable bound states and fixes the accompanying inertial and restoring coefficients. Maxwell's action and scaled component geometry give permittivity, permeability, inductance, and capacitance with a consistent frequency and energy law. Mechanical deformation and electrical storage then yield the same recoverable work. In the nuclear bag approximation, the confined radial mode admits an equivalent inductance and capacitance with the same scaling, under explicit confinement, boundary-inertia, and adiabatic conditions. The results connect to the previously calculated Standard Model stress response in Operational Quantum Gravity, with the changing material coefficient distinguished from the invariant gravitational normalization. The common family reproduces the leading static weak-field clock and ruler comparisons and the associated quasistatic transport work predicted by general relativity. Appendix A gives the exact Schwarzschild extension and its remaining response condition. The accomplishment is a linked material account of stable matter, electromagnetic storage, and work that is consistent with gravitational potential energy and specifies the requirements for its microscopic explanation.

Keywords: matter-wave scaling; inertia; electromagnetic storage; quantum theory; stress susceptibility; quantum gravity

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1 Introduction

Gravitational potential energy is measured through the work exchanged when matter is moved between gravitational environments. In the matter-first description of Operational Quantum Gravity (OQG), that work must be consistent with the binding, inertia, and energy storage of the matter being moved. A clock makes the requirement especially clear: it must remain a stable physical system while its frequency and size change relative to a remote standard. This paper establishes how those changes can carry one consistent account of stored energy and recoverable work as matter moves between gravitational potentials.

The central result is a proof that a specified common scaling of matter-wave size and transition frequency gives the same energy factor for corresponding material deformations and electrical oscillations. Section 2 derives the Hamiltonian that realizes this family and proves that it preserves stable bound states. The accompanying increase of inertia under contraction supplies the slower internal motion required by the frequency change. Stability, inertia, and clock rate are therefore linked consequences of the same transformation.

Sections 3 and 4 carry that result into electromagnetic storage and transport work. Maxwell's action fixes the electric and magnetic coefficients; the scaled apparatus geometry then fixes capacitance and inductance. Both field-energy integration and the component Hamiltonian reproduce the mechanical storage law. With a declared energy reference, the same factor determines the work recovered during quasistatic lowering and restored during

lifting. This provides a material energy account for the gravitational potential-energy comparison, including the contribution of a stored excitation.

Section 5 extends the storage calculation to a confined hadronic mode in the nuclear bag approximation. Its restoring response and induced radial inertia give an equivalent inductance and capacitance with the same scaling as the electromagnetic components. This makes the result accessible in engineering terms: the bound-state calculation specifies component ratios, oscillation frequency, and recoverable energy. Comparisons with external standards can express a change of matter-wave scale through these linked quantities.

OQG supplies the connection to ambient matter through its previously calculated Standard Model stress response. Section 6 relates that response to the electromagnetic storage coefficients while preserving the gravitational normalization. Section 7 then recovers the leading static weak-field clock, ruler, and light comparisons, together with the invariant velocity-to-light-speed ratio for corresponding motion. Appendix A gives the exact Schwarzschild extension, retaining the distinct clock and ruler factors and their additional response condition.

Earlier material descriptions connect gravity with clock, ruler, and energy scaling. Dicke [10] and Puthoff [17] express these changes through effective electromagnetic properties. Broekaert [1] develops energy–momentum transformations and lowering work, while Krogh’s preprint [12] gives a closely related common scaling of matter-wave size, frequency, and inertia. Complementary quantum treatments explain free fall through gravitational clock-rate gradients [2, 24] and describe massive-particle propagation through a gravitational refractive index [11].

The present paper advances this material account by linking stable bound-state response to recoverable storage. Quantum scaling methods [22], induced-inertia response theory [3], and the metric–electromagnetic correspondence [16, 21] provide the established ingredients. Their combination makes the restoring coefficients, induced radial inertia, scaled component geometry, and confinement conditions explicit within one energy law. The theorem determines the Hamiltonian and the work law that follows from it. Microscopic realization requires an ambient interaction governed by the total Standard Model stress response to produce that family. The result gives this calculation a concrete target: the same response must sustain stable matter and reproduce the energy exchanged when that matter is moved between gravitational potentials.

2 Matter wave scaling preserves stable bound states

All ratios use the same external comparison standards and a declared reference state 0. The reduced Planck constant $\hbar$ and elementary charge e remain fixed. Each stationary apparatus is small enough that the scaling factor is effectively uniform across it. Local clocks and rulers retain their common calibration; comparison with a remote reference reveals the changes. This mapping compares material equilibria in fixed external standards. A Lorentz transformation changes the observer describing a specified physical state.

The first requirement is a stable clock or ruler at every member of the family. Consider corresponding states of the same bound system, with the same composition and quantum labels. Let $\mathbf{x}_{\text{int}}$ collect its d internal spatial coordinates, and let R_0 and R_b be its root-mean-square internal radii. The size ratio and normalized dilation are

$$b = \frac{R_b}{R_0} > 0, \quad (U_b \psi_0)(\mathbf{x}_{\text{int}}) = b^{-d/2} \psi_0(\mathbf{x}_{\text{int}}/b). \quad (1)$$

Thus $b < 1$ describes contraction and $b > 1$ describes expansion. The prefactor preserves total probability. In momentum space, the same transformation gives $\tilde{\psi}_b(\mathbf{p}) = b^{d/2} \tilde{\psi}_0(b\mathbf{p})$. The position and momentum operators obey

$$U_b^\dagger \mathbf{x}_{\text{int}} U_b = b \mathbf{x}_{\text{int}}, \quad U_b^\dagger \mathbf{p} U_b = b^{-1} \mathbf{p}. \quad (2)$$

An expanded wave pattern has a narrower momentum distribution, and position–momentum uncertainty products remain unchanged. Common scaling pairs the size ratio with the same transition–frequency ratio. If ω is angular frequency, t comparison time, and t_{ref} the reference evolution parameter, the specified family has

$$\frac{\omega_b}{\omega_0} = b, \quad t_{\text{ref}} = bt, \quad \Psi_b(t) = U_b \Psi_0(bt). \quad (3)$$

The Hamiltonian governs the system’s energies and time evolution. Self-adjointness ensures real energy levels and probability-preserving evolution. Its domain $\mathcal{D}(H)$ is the set of states on which the operator acts; a dagger denotes the adjoint, equal to the inverse for a unitary operator.

Theorem 1 Common scaling of a bound matter system. Let $b > 0$ be constant during each stationary comparison. Let H_0 be a self-adjoint effective bound-system Hamiltonian, bounded below, with a lowest bound state. Let U_b be the unitary dilation in Eq. (1), mapping the operator domain and the relevant finite-moment constrained states. The evolution in Eq. (3) is generated by

$$H_b = bU_b H_0 U_b^\dagger, \quad \mathcal{D}(H_b) = U_b \mathcal{D}(H_0). \quad (4)$$

Corresponding transition energies and frequencies scale by b . Stable equilibrium sizes scale by b , with their stability preserved.

Proof. The reference equation is $i\hbar \partial_{t_{\text{ref}}} \Psi_0 = H_0 \Psi_0$. Differentiating Eq. (3) gives $i\hbar \partial_t \Psi_b = bU_b H_0 U_b^\dagger \Psi_b$, proving Eq. (4) on its mapped domain. Unitary conjugation preserves self-adjointness, and positive b preserves spectral ordering. For a reference eigenstate $|n\rangle$, $H_b U_b |n\rangle = bE_{n,0} U_b |n\rangle$ in the stated energy convention. With ΔE_{mn} denoting the energy difference between two quantum levels (the energy gap) and $\mathcal{T}$ the corresponding clock period,

$$\Delta E_{mn,b} = b\Delta E_{mn,0}, \quad \omega_{mn,b} = b\omega_{mn,0}, \quad \mathcal{T}_b = b^{-1}\mathcal{T}_0. \quad (5)$$

To establish equilibrium, let $\mathcal{E}_0(R)$ be the minimum energy among normalized states constrained to size R . Assume that this minimum exists near a stable equilibrium $R_{*,0}$, that the dilation maps the associated quadratic-form domains, and that $\mathcal{E}_0$ is twice differentiable there. Define the radius curvature $k_R = \partial_R^2 \mathcal{E}(R_*)$. At fixed environment b , the constrained-state mapping gives

$$\mathcal{E}_b(R) = b\mathcal{E}_0(R/b), \quad R_{*,b} = bR_{*,0}, \quad k_{R,b} = b^{-1}k_{R,0}. \quad (6)$$

The positive curvature remains positive, so the scaled equilibrium remains stable. Energy differences between corresponding constrained configurations scale by b . This completes the proof.

The curvature’s meaning matters. The coefficient k_R measures energy per squared comparison-coordinate displacement and has units of force per length. For fractional deformation $y = (R - R_*)/R_*$, define $K_y = k_R R_*^2$, with units of energy. Then

$$K_{y,b} = bK_{y,0}, \quad \Delta \mathcal{E}_b = \frac{1}{2} K_{y,b} y^2 = b\Delta \mathcal{E}_0. \quad (7)$$

Contraction therefore lowers the energy required for the same fractional deformation. The increase of k_R reflects the smaller displacement used in that coordinate; it does not reverse this energy result. Section 6 distinguishes these restoring coefficients from the ambient stress response.

Inertia and frequency. The kinetic term gives an equally important consequence. For reference inertia m_0 , Eq. (4) carries $\mathbf{p}^2/(2m_0)$ into $b^3 \mathbf{p}^2/(2m_0)$. The matched inertia follows directly. For a harmonic mode with that inertia and radius curvature k_R , its frequency then satisfies

$$m_b = b^{-3}m_0, \quad \omega_b = \sqrt{\frac{k_{R,b}}{m_b}} = b\omega_0. \quad (8)$$

The inertia increases more rapidly than the squared momentum under contraction, so their kinetic-energy combination decreases by b . The internal velocity p/m scales by b^2 . In the fractional coordinate, the inertia is $I_y = mR_*^2 \propto b^{-1}$, while $K_y \propto b$; their ratio again gives $\omega^2 \propto b^2$. These are matched mode coefficients. Section 5 derives the corresponding induced radial inertia explicitly. Stable binding, increased inertia, and a lower oscillation frequency now belong to one consistent transformation.

3 Electromagnetic storage follows the matter wave scale

The scaled matter must also support its electromagnetic fields. The question is whether electric and magnetic storage carry the energy ratio already fixed by the quantum theorem. For the local stationary observer, the clock and ruler conversions are $d\tau_{\text{stat}} = b dt$ and $d\ell = |d\mathbf{x}|/b$, where τ_{stat} and ℓ are that observer's proper time and locally measured length, respectively, and $\mathbf{x}$ denotes external comparison coordinates. Local Lorentz symmetry with reference local light speed c_0 gives the comparison light speed $c_b = b^2 c_0$.

Let $\mathbf{E}$ and $\mathbf{B}$ denote coordinate electric field and magnetic flux density. Their local orthonormal components are $\mathbf{E}$ and $b^2 \mathbf{B}$. The proper spacetime measure contributes b^{-2} . With reference permittivity ϵ_0 and permeability μ_0 , Maxwell's action consequently has the coordinate Lagrangian density

$$\mathcal{L}_{\text{EM},b} = \frac{\epsilon_0}{2b^2} |\mathbf{E}|^2 - \frac{b^2}{2\mu_0} |\mathbf{B}|^2. \quad (9)$$

Reading the electric and magnetic coefficients gives

$$\epsilon_b = b^{-2} \epsilon_0, \quad \mu_b = b^{-2} \mu_0, \quad c_b = (\epsilon_b \mu_b)^{-1/2} = b^2 c_0. \quad (10)$$

Both storage coefficients acquire the same factor. This is the static isotropic specialization of the metric–electromagnetic correspondence [16, 21]. A moving-frame description must transform the fields and electromagnetic response together [21]. The wave impedance $Z = \sqrt{\mu/\epsilon}$ and fine-structure constant α , which measures dimensionless electromagnetic coupling, obey

$$Z_b = Z_0, \quad \alpha_b = \frac{e^2}{4\pi\epsilon_b \hbar c_b} = \alpha_0. \quad (11)$$

These coefficients use the declared comparison coordinates. The field amplitudes and the apparatus geometry must use those same coordinates. For corresponding modes with the same occupation or classical action, charge and magnetic-flux amplitudes are preserved. If Σ is a corresponding area and V the occupied volume, then

$$\Sigma_b = b^2 \Sigma_0, \quad V_b = b^3 V_0, \quad \mathbf{E}_b = \mathbf{E}_0, \quad \mathbf{B}_b = b^{-2} \mathbf{B}_0. \quad (12)$$

The invariant electric amplitude follows from the charge flux $\epsilon E \Sigma$: the permittivity and area factors cancel. The magnetic amplitude follows from the fixed flux $B \Sigma$. Thus the electric and magnetic energy densities, $u_E = \epsilon E^2/2$ and $u_B = B^2/(2\mu)$, each scale by b^{-2} . Integrating either over the corresponding volume gives stored energy proportional to b . The area change is essential to this agreement.

A capacitor and inductor provide a direct component check. Let C denote capacitance and L inductance. With their entire geometry scaled by b and their dimensionless shape and winding data fixed, each component is proportional to its field coefficient times a length. Therefore

$$C_b = b^{-1} C_0, \quad L_b = b^{-1} L_0, \quad \omega_{\text{LC},b} = b \omega_{\text{LC},0}, \quad Z_{\text{LC},b} = Z_{\text{LC},0}. \quad (13)$$

Here $\omega_{\text{LC}} = (LC)^{-1/2}$ and $Z_{\text{LC}} = \sqrt{L/C}$. The electric and magnetic parts reproduce the quantum frequency ratio while preserving their impedance. Electromagnetic storage can therefore accompany the scaled bound state without introducing a second clock law.

4 Restoring response and storage yield the same recoverable work

Frequency agreement becomes a stronger physical statement when the energy can be recovered. A displaced stable system stores work in its restoring response. A charged capacitor and an energized inductor store work in their fields. The common family must give the same energy comparison in both descriptions.

For a small corresponding radial displacement $\delta R_b = b\delta R_0$, the mechanical calculation is

$$\Delta W_{\text{mech},b} = \frac{1}{2} k_{R,b} (b\delta R_0)^2 = b \frac{1}{2} k_{R,0} \delta R_0^2. \quad (14)$$

For an ideal lossless electrical oscillator, let Q be canonical charge and Φ_L its conjugate flux. Its Hamiltonian is

$$H_{LC,b} = \frac{Q^2}{2C_b} + \frac{\Phi_L^2}{2L_b} = b H_{LC,0}. \quad (15)$$

The comparison uses corresponding charge and flux amplitudes, whose invariance follows from fixed occupation or classical action and the invariant circuit impedance. During each oscillation, the capacitor charge and the current through the inductor vary with time. Equations (14) and (15) give the same recoverable-energy factor through independent mechanical and electrical expressions.

The electrical interpretation is concrete. Increased capacitance lowers the restoring coefficient $1/C$ for charge displacement. Increased inductance supplies a larger dynamical coefficient in that coordinate. Their combined response lowers the oscillation frequency. The fractional mechanical coordinate has the same pattern: K_y decreases while I_y increases. Both oscillators require less energy for corresponding amplitudes and take longer to complete a cycle.

To define transport work, let $E_{\text{store},0}$ be a specified stored excitation energy above the same unexcited reference. During ideal lossless, adiabatic transport, its contribution to energy and work done on the system is

$$E_{\text{store},b} = b E_{\text{store},0}, \quad dW_{\text{store},on} = E_{\text{store},0} db. \quad (16)$$

Lowering b releases that incremental stored energy to the transport apparatus; reversing the operation restores it. A complete-body transport calculation includes the confining structure and a consistent rest-energy account. Transition frequencies determine energy differences between levels; transport work also requires a specified common energy reference. A common shift of all levels leaves those differences and the radius curvature unchanged at fixed environment. If that shift varies with the environment, its contribution must also be included in preparation work and force.

There is an operational force check. In the common static family, take the reference clock at infinity and a small closed test system with fixed local proper energy E_{loc} . Let E_∞ be its energy in that distant standard, ℓ proper outward distance, and F_{loc} the local outward holding force. Quasistatic transport gives

$$E_\infty = b E_{\text{loc}}, \quad F_{\text{loc}} = E_{\text{loc}} \frac{d \ln b}{d \ell}, \quad b F_{\text{loc}} d \ell = E_{\text{loc}} db. \quad (17)$$

The factor b converts local work into the common distant energy account. Integration gives $\Delta E_\infty = E_{\text{loc}}(b_2 - b_1)$, with zero net work over a reversible return cycle. Appendix A gives the exact Schwarzschild clock factor for this identity. This energy difference is the system's gravitational potential-energy change in the stated static comparison. The material and gravitational descriptions meet at the work exchanged with the apparatus that moves the system.

5 Confined matter admits the same oscillator storage law

The preceding agreement can be tested in a system whose restoring response includes strong binding. We retain the spherical massless bag as a confined hadronic illustration [15]. Its boundary radius R is distinct from the root-mean-square radius used to define the general matter-wave scale; the matched family scales both by b . Let $a_{\text{conf}} > 0$ have units of energy times length, and let $P_c > 0$ be the confinement energy density. The energy, equilibrium, and radius curvature are

$$\begin{aligned} U(R) &= \frac{a_{\text{conf}}}{R} + \frac{4\pi}{3} P_c R^3, & R_*^4 &= \frac{a_{\text{conf}}}{4\pi P_c}, \\ E_* &= U(R_*) = \frac{16\pi}{3} P_c R_*^3, & k_R &= U''(R_*) = \frac{3E_*}{R_*^2}. \end{aligned} \quad (18)$$

The first derivative fixes the equilibrium, and the second is $16\pi P_c R_* > 0$. The full energy curve obeys Eq. (6) when $a_{\text{conf},b} = b^2 a_{\text{conf},0}$ and $P_{c,b} = b^{-2} P_{c,0}$. These are the confinement matching conditions.

The inertia requires the family away from equilibrium as well. At imposed radius R , take

$$H_b(R) = b U_b H_0(R/b) U_b^\dagger, \quad |n; R\rangle_b = U_b |n; R/b\rangle_0. \quad (19)$$

The dilation identifies the varying boundary domains, and b is held fixed during the radius derivative. For differentiable internal states with a nondegenerate ground state, let $\Delta_n = E_n - E_0 > 0$ be the energy needed to excite the system from the ground state to state n . These excitation gaps enter the reactive adiabatic response, which gives the induced radial inertia [3]

$$\mathcal{M}_R^{\text{ind}} = 2\hbar^2 \sum_{n>0} \frac{| \langle n | \partial_R 0 \rangle |^2}{\Delta_n}, \quad \mathcal{M}_{R,b}^{\text{ind}} = b^{-3} \mathcal{M}_{R,0}^{\text{ind}}. \quad (20)$$

At corresponding radii, each derivative with respect to R contributes b^{-1} . Squaring the matrix element gives b^{-2} , while division by the excitation energy contributes another b^{-1} . Their product gives the induced-inertia ratio b^{-3} .

Equation (20) applies when the radius changes slowly enough for the internal state to follow without appreciable excitation. The radial oscillation frequency must be small compared with the relevant internal transition frequencies $\Delta_n/\hbar$. The sum over excited states must give a finite result, with any continuous-spectrum or renormalization contributions included explicitly. An explicit confining boundary can also carry inertia. Its contribution belongs in the total radial inertia $\mathcal{M}_R$, which must follow the same b^{-3} scaling for the storage representation below.

Proposition 1 Confined-mode LC representation. Let $y = (R - R_*)/R_*$, with canonical momentum p_y , and choose a fixed nonzero normalization charge q_* . Set $Q_N = q_* y$ and $\Phi_N = p_y/q_*$. The quadratic radial Hamiltonian has the canonical electrical form

$$\begin{aligned} H_N - E_* &= \frac{\Phi_N^2}{2L_N} + \frac{Q_N^2}{2C_N}, \\ L_N &= \frac{\mathcal{M}_R R_*^2}{q_*^2}, & C_N &= \frac{q_*^2}{k_R R_*^2}. \end{aligned} \quad (21)$$

Proof. The radial kinetic term is $p_y^2/(2\mathcal{M}_R R_*^2)$ and the restoring term is $k_R R_*^2 y^2/2$. Substitution gives Eq. (21), with $[Q_N, \Phi_N] = i\hbar$. The products $\mathcal{M}_R R_*^2$ and $k_R R_*^2$ scale as b^{-1} and b , respectively. Consequently,

$$L_{N,b} = b^{-1} L_{N,0}, \quad C_{N,b} = b^{-1} C_{N,0}, \quad \omega_{N,b} = b \omega_{N,0}, \quad Z_{N,b} = Z_{N,0}. \quad (22)$$

Here $\omega_N = (L_N C_N)^{-1/2}$ and $Z_N = \sqrt{L_N/C_N}$. This proves the proposition under the stated quadratic, adiabatic, and matching conditions.

The fixed q_* expresses the nuclear mode's stored energy in canonical electrical form. The equivalent capacitance represents its restoring response, and the equivalent inductance represents its inertia. A physical

electromagnetic interaction must determine the conversion scale before these components describe measurable electrical transduction. The established result is the common energy law: confinement, restoring response, and induced inertia support the same frequency and recoverable-work ratios as the electrical oscillator.

6 The OQG response coefficient connects to storage ratios

The material results now specify what the ambient response must reproduce. The upstream OQG calculation [8] defines how the complete Standard Model stress tensor responds to a weak applied source. The tensor includes gauge fields, fermions, the Higgs field, and their interactions. A first variation of the matter action gives stress; the second gives its response, combining connected stress correlations with the local contact terms required by energy-momentum conservation.

The transverse-traceless channel selects shear response transverse to the perturbation wavevector and with zero trace. Its static coefficient of quadratic spatial momentum is $A_{2,E}$: the index 2 identifies the two-derivative term, and E identifies the Euclidean, or imaginary-time, equilibrium calculation. The full coefficient $A = A_{2,E}^{\text{tot}}$ includes the reference-state contribution and has units of energy squared. It describes the ambient stress response, while k_R and K_Y describe the restoring response of a particular bound system. Their definitions fix which stiffness or compliance enters each calculation.

Desiato's calculation [8, Sections 2–4] evaluates the thermal change through second order in the couplings. It combines free-field terms, screened bosonic modes, five short-wavelength two-loop sectors, Higgs contact terms, and electric and local-curvature matching. The stated regime is the symmetric high-temperature phase, retaining the top-quark coupling in the fermion–Higgs terms. Three inputs remain explicit: the renormalized Higgs–curvature coupling and two finite three-dimensional Yang–Mills response coefficients defined for lattice evaluation. The subtraction determines a state increment; the reference contribution is specified separately.

In the OQG state hierarchy [7], the terrestrial ambient response contains both the cosmological epoch background and the local environmental contribution in the same normalization:

$$A_{2,E}^{\text{epoch}} + A_{2,E}^{\text{local,Earth}} = A_{2,E}^{\text{ambient,Earth}}. \quad (23)$$

Any additional preparation of a body is compared with that ambient state. The high-temperature calculation is an upstream input; this paper specifies matching ratios for ordinary bound matter without assigning a numerical terrestrial coefficient.

Here ambient denotes the state of surrounding matter and fields, including the cosmological background, local sources, and boundaries. The bare zero-point vacuum is Lorentz invariant [26]. A material environment can define a physical rest frame, and its measured spectrum can depend on motion relative to its sources. Such state dependence is compatible with Lorentz-covariant field equations: the fields, sources, and receiver response must be transformed consistently. The power-balance treatment in Ref. [25] distinguishes these ambient and receiver roles.

The conversion to gravitational response is developed in Ref. [9, Sections 2–6]. The selected long-wavelength branch has two massless tensor polarizations and a fixed volume measure. Its Weyl-transverse-diffeomorphism (WTDiff) representation admits an Einstein-equivalent completion. Matching the same shear source fixes the normalization. Let c be the comparison-coordinate light speed and G the gravitational coefficient in the same convention. Define $\kappa = G/c^4$ and the leading gradient normalization $\mathcal{S}$. In SI units,

$$\kappa = \frac{G}{c^4} = \frac{\hbar c}{32\pi A}, \quad \mathcal{S} = \frac{8A}{\hbar c} = \frac{c^4}{4\pi G}. \quad (24)$$

The units of A , κ , and $\mathcal{S}$ are energy squared, inverse force, and force. On the selected invariant- κ branch, $c_b = b^2 c_0$ requires

$$A_b = b^2 A_0, \quad G_b = b^8 G_0, \quad \kappa_b = \kappa_0, \quad \mathcal{S}_b = \mathcal{S}_0. \quad (25)$$

Thus contraction lowers the material coefficient A , while its reciprocal increases. The ratio A/c stays fixed. The quantities $\mathcal{S}$ and c^4/G are invariant normalizations and cannot describe a changing material stiffness. Likewise, $1/A$, $1/k_R$, $1/K_Y$, and κ have different definitions; their common association with response does not make them interchangeable compliances.

Combining the response matching with Eq. (10) gives the direct storage connection

$$\frac{\epsilon_b}{\epsilon_0} = \frac{\mu_b}{\mu_0} = \frac{A_0}{A_b}, \quad \frac{G_b}{G_0} = \left(\frac{A_b}{A_0}\right)^4. \quad (26)$$

This is the conditional A -storage relation on the selected branch. A lower material response coefficient accompanies greater electric and magnetic storage coefficients, while the gravitational normalization remains fixed. Together with component geometry, that relation gives the lower oscillation frequency and deformation-energy scale established above. The remaining microscopic calculation must realize these ambient and bound-state responses together, including the confinement matching. The normalization and the material theorem specify what it must reproduce.

The microscopic interpretation of vacuum permittivity and permeability remains a subject of research. Leuchs, Villar, and Sánchez-Soto [13] and Urban et al. [23] propose models based on electric and magnetic responses of virtual charged pairs; Leuchs, Hawton, and Sánchez-Soto [14] develop an oscillator description of the response. These proposals offer directions for future consideration. The present paper establishes effective-coefficient ratios in the declared comparison coordinates. Assessing a microscopic origin for the reference coefficients and its compatibility with the total Standard Model stress response lies beyond its scope.

7 Clock ruler and light comparisons

The preceding sections supply the physical properties of the instruments. Their geometric description now follows from their calibrations. Let ds^2 denote the spacetime interval and assume local Lorentz symmetry with locally measured light speed c_0 . Substituting the stationary observer's clock and ruler conversions $d\tau_{\text{stat}} = b dt$ and $d\ell = |d\mathbf{x}|/b$ into $ds^2 = -c_0^2 d\tau_{\text{stat}}^2 + d\ell^2$ gives

$$ds^2 = -b^2 c_0^2 dt^2 + b^{-2} d\mathbf{x}^2, \quad c_b = b^2 c_0. \quad (27)$$

This proves the static isotropic clock-ruler corollary of the common family. It applies pointwise to small apparatus; spatial variation across a finite body requires its spatially varying dynamics. Corresponding wave components have $\omega_b = b\omega_0$ and wavenumber $k_b = k_0/b$, giving the same speed factor through ω_b/k_b .

Let $v \equiv v_{\text{coord}} = |d\mathbf{x}/dt|$ denote speed in the comparison coordinates. Motion uses the local velocity $v_{\text{loc}} = v_{\text{coord}}/b^2$ and Lorentz factor $\gamma_{\text{loc}} = (1 - v_{\text{loc}}^2/c_0^2)^{-1/2}$. Corresponding trajectories with the same local velocity satisfy

$$v_b = b^2 v_0, \quad \frac{v_b^2}{c_b^2} = \frac{v_0^2}{c_0^2}, \quad \gamma_b = \gamma_0. \quad (28)$$

The total clock comparison combines this local motion with the environmental clock factor. Let τ be proper time along the moving clock's trajectory and $\Gamma(b, v)$ the ratio of comparison time to that proper time. Along a timelike trajectory, $ds^2 = -c_0^2 d\tau^2$. Substituting the coordinate speed into Eq. (27) and taking the positive time ratio gives

$$\Gamma(b, v) \equiv \frac{dt}{d\tau} = \frac{1}{\sqrt{b^2 - \frac{v^2}{c_0^2}}} = \frac{\gamma_{\text{loc}}}{b}. \quad (28a)$$

This factor counts the comparison time elapsed per unit of time recorded by the moving clock. It applies for $b > 0$ and $v < b^2 c_0$. When $b = 1$, it gives the usual special-relativistic Lorentz factor; when $v = 0$, it gives $\frac{1}{b}$. Equation (28) holds the local Lorentz factor fixed by comparing trajectories with the same local speed. Their total clock conversion still includes the factor $\frac{1}{b}$.

Equivalently, the moving clock has $d\tau/dt = b/\gamma_{\text{loc}}$. A small rod of local rest length $L_{\text{rod},0}$ has longitudinal coordinate length $bL_{\text{rod},0}/\gamma_{\text{loc}}$ and transverse length $bL_{\text{rod},0}$, with endpoints simultaneous in the local stationary frame. Holding coordinate speed fixed while varying b instead changes local speed and is a different comparison.

These clock and ruler formulas evaluate motion in a specified material state. If motion relative to the ambient sources changes that state, its supporting exchange and resulting material factors must be determined consistently before the formulas are applied [25]. The line element supplies the clock conversion for given factors; it does not by itself determine how the ambient interaction changes them.

For the gravitational application, let Φ_g be the Newtonian potential relative to the distant reference. Its leading static weak-field identification is

$$b = 1 + \frac{\Phi_g}{c_0^2} + O\left(\frac{\Phi_g^2}{c_0^4}\right), \quad \frac{c_0}{c_b} = 1 - \frac{2\Phi_g}{c_0^2} + O\left(\frac{\Phi_g^2}{c_0^4}\right). \quad (29)$$

Equation (27) then reproduces the leading static weak-field clock and isotropic ruler comparisons of general relativity (GR). The second expression is the corresponding optical index. A two-level clock records phase $d\varphi = (\Delta E_0/\hbar)d\tau$ [18], so remote clock comparison tests the same transition-energy ratio as the theorem. Equation (17) reduces near a weak-field surface to $\Delta W = (E_{\text{loc}}/c_0^2)g\Delta h$, where g is the local gravitational acceleration and Δh the small height change.

For a complete small body whose full rest energy follows that account, its low-speed coordinate inertia is $E_\infty/c_b^2 = b^{-3}E_{\text{loc}}/c_0^2$. The same b^{-3} factor therefore appears in translation under the complete-body matching. This coefficient and an internal mode inertia describe different motions; each must be associated with its own energy and coordinate.

Table 1 collects the common family in its familiar single-factor form. These ratios are exact within that family; Eq. (29) states their gravitational scope. The final row additionally uses the invariant branch of Section 6.

Table 1. Common matter-wave and storage ratios relative to state 0.

Quantity	Ratio	Comparison or definition
Matter-wave size	b	Corresponding internal radius
Internal momentum	b^{-1}	Corresponding wave pattern
Transition energy and frequency	b	Same transition labels
Clock period	b^{-1}	Inverse transition frequency
Mode inertia	b^{-3}	Matched quadratic kinetic term
Corresponding velocity and light speed	b^2	Same local speed; invariant v/c
Radius curvature k_R	b^{-1}	Per squared coordinate displacement
Fractional curvature K_y	b	Per squared fractional deformation
Permittivity and permeability	b^{-2}	Comparison-coordinate coefficients
Capacitance and inductance	b^{-1}	Entire component geometry scaled
Wave and corresponding mode impedances	1	Each compared with its own reference
Fine-structure constant	1	Fixed e and $\hbar$
Stored excitation energy and deformation work	b	Corresponding state or deformation
Full response A and normalization c^4/G	b^2 and 1	Selected invariant branch

The common ratios continue the engineering line from Dicke [10] and Puthoff [17], through the joint Electrogravimagnetics work of Storti and Desiato [20], to *The Electromagnetic Quantum Vacuum Warp Drive* [4] and *An Engineering Model of Quantum Gravity* in the 2016 Estes Park proceedings [5]. The engineering programme was subsequently consolidated in *Operational Quantum Gravity for Engineers* [6]. The present theorem supplies the matter-wave formulation and makes its equilibrium and recoverable-storage requirements explicit.

Remote comparisons connect these predictions to clocks, light paths, and work measurements. Identically scaled local standards cannot reveal a common change by themselves. Exact Schwarzschild uses two linked clock and ruler factors beyond their shared first-order limit; Appendix A states that extension without changing the common-family table.

8 Conclusions

The principal accomplishment is a proof that the specified matter-wave scaling carries one consistent law for stable binding, inertia, and recoverable energy. A clock can contract, acquire greater inertia, and run more slowly while remaining bound. Its mechanical deformations and electromagnetic oscillations then require less energy at corresponding amplitudes by the same factor. With the energy reference and transport conditions stated, that factor also fixes the work exchanged during lifting and lowering. This is the material connection between clock comparison and gravitational potential energy established by the paper.

The engineering content is explicit. Equal permittivity and permeability scaling preserves wave impedance and the fine-structure constant. Including the apparatus geometry gives capacitance and inductance proportional to b^{-1} , while oscillation frequency and corresponding stored energy scale by b . The confined radial mode in the nuclear bag approximation admits equivalent components with these same ratios. Engineers can therefore express matter-wave scaling through familiar energy-storage quantities. For electromagnetic components, the

linked capacitance, inductance, frequency, and stored-energy ratios provide a basis for calibrating and observing scale changes by comparison with external standards.

The OQG connection relates electromagnetic storage to the full Standard Model stress-response coefficient A , while c^4/G remains invariant. It distinguishes the ambient response from the restoring coefficients of individual systems and gives each a definite role in the energy account. The common family reproduces the leading static weak-field clock and ruler comparisons and preserves the velocity-to-light-speed ratio for corresponding motion. Appendix A carries the same analysis into the exact Schwarzschild exterior with separate clock and ruler factors and an explicit response condition.

Microscopic realization now has a specific task. The interaction between source matter and receiving matter must connect the effective vacuum permittivity and permeability in the comparison description to the material response, including its electric and magnetic contributions within the total Standard Model stress tensor. That coupled response must realize the equilibrium, inertia, and storage ratios derived here. The theorem supplies the required Hamiltonian; the storage calculation supplies the corresponding work law. Together they specify how the microscopic response must account for gravitational potential energy.

Closing remark on the OQG interpretation. Operational Quantum Gravity interprets gravity through the response of matter: source matter establishes the ambient state, and receiving matter expresses its response through binding, inertia, clocks, rulers, and stored energy. In this reading, the metric records the relations among those physical standards. The present results supply a consistent account of equilibrium and recoverable work under the stated assumptions, providing a target for the proposed microscopic interaction. The microscopic source interaction and the additional response required by the adopted Einstein completion remain separate requirements of the programme. The closing statement expresses that material interpretation: “Matter tells matter how to scale. Geometry is just bookkeeping.”

Appendix A Exact Schwarzschild extension

Assume the spherical vacuum Einstein completion with zero cosmological constant [19]. Let M be source mass, r isotropic radius, and G_0, c_0 the fixed distant reference coefficients. Define $u = G_0 M / (2rc_0^2)$. In the static exterior, $0 \leq u < 1$, outside a star’s material surface, the clock factor f and ruler factor b are

$$f = \frac{1-u}{1+u}, \quad b = (1+u)^{-2}, \quad \frac{f}{b} = 1 - u^2. \quad (A1)$$

These are two linked functions of one environment. The exact line element and its material family are

$$ds^2 = -f^2 c_0^2 dt^2 + b^{-2} (dr^2 + r^2 d\Omega^2), \quad H_{f,b} = f U_b H_0 U_b^\dagger. \quad (A2)$$

Here $d\Omega^2$ is the unit-sphere angular element. The areal radius $R_{\text{areal}} = r(1+u)^2$ gives $1 - 2G_0 M / (R_{\text{areal}} c_0^2) = f^2$ and $dR_{\text{areal}}/dr = 1 - u^2$, verifying the exact Schwarzschild form. Since $f = 1 - 2u + 2u^2 + O(u^3)$ and $b = 1 - 2u + 3u^2 + O(u^3)$, they share the leading common factor. A coordinate change does not make $f = b$ exact with these definitions.

The preceding calculations extend by replacing the energy and frequency factor with f while retaining the size factor b :

$$\begin{aligned} \frac{m_{f,b}}{m_0} &= \frac{1}{fb^2}, & \frac{k_{R,f,b}}{k_{R,0}} &= \frac{f}{b^2}, & \frac{c_{f,b}}{c_0} &= fb, \\ \frac{\epsilon_{f,b}}{\epsilon_0} &= \frac{\mu_{f,b}}{\mu_0} = \frac{1}{fb}, & \frac{c_{f,b}}{c_0} &= \frac{L_{f,b}}{L_0} = \frac{1}{f}. \end{aligned} \quad (A3)$$

Corresponding stored energies, deformation work, and K_y scale by f ; impedance and α remain invariant. Corresponding velocities and light speed both scale by fb . The confinement conditions become $a_{\text{conf},f,b} = fb a_{\text{conf},0}$ and $P_{c,f,b} = (f/b^3) P_{c,0}$. Quasistatic work obeys $E_\infty = f E_{\text{loc}}$ and $f F_{\text{loc}} d\ell = E_{\text{loc}} df$, with $d\ell = dr/b$. Each finite apparatus still requires small factor variation and negligible tidal distortion across its size.

The moving-clock comparison also extends to distinct clock and ruler factors. With v denoting speed in the isotropic spatial coordinates of Eq. (A2), the local stationary observer measures $v_{\text{loc}} = \frac{v}{fb}$. For timelike motion, $v < fbc_0$, the total time conversion is

$$\Gamma(f, b, v) \equiv \frac{dt}{d\tau} = \frac{1}{\sqrt{f^2 - \frac{v^2}{b^2 c_0^2}}} = \frac{\gamma_{\text{loc}}}{f}. \quad (\text{A3a})$$

Here γ_{loc} is evaluated using this local speed. The stationary limit gives $\frac{1}{f}$, and setting $f = b$ recovers Eq. (28a). Thus the clock factor and local motion remain distinct in the exact Schwarzschild comparison.

The response condition must also be retained. Invariant G/c^4 requires $A_{f,b}^{\text{branch}} = fbA_0$. If the complete source-coupled state is additionally a scale-equivalent copy with fixed dimensionless state data and source normalization, its energy parameters scale by f and its susceptibility scales as $A_{f,b}^{\text{scale}} = f^2 A_0$. In the companion's inverse-energy coordinates $X = x/(\hbar c)$, lengths then scale by $1/f$; the measure and two derivatives in the quadratic source action give the factor f^{-2} . Under those additional assumptions the required residual response is

$$\frac{A_{f,b}^{\text{branch}}}{A_{f,b}^{\text{scale}}} = \frac{b}{f} = \frac{1}{1-u^2}. \quad (\text{A4})$$

The bound-state theorem does not require the ambient state to be that scaled copy. Equation (A4) is therefore an explicit closure condition for its microscopic realization. The common limit $f = b$ satisfies both scalings. Exact Schwarzschild matching follows from the adopted gravitational completion; calculating the additional ambient response remains a separate requirement.

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Author contributions. Todd J. Desiato conceived the OQG programme and the material-scaling interpretation, directed the theoretical development, reviewed the calculations and literature, and prepared the manuscript.

Data availability. This theoretical study generated no new experimental or observational datasets. The definitions and analytical derivations supporting the results are contained in this article. The earlier OQG calculations are available in the pre-prints cited in Refs. [6, 7, 8, 9].

Code availability. No numerical code is required to reproduce the analytical theorem, the nuclear scaling proposition, or the metric comparison. Code supporting the earlier SM susceptibility calculation is described in Ref. [8].

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