

Nuclear Linear Response Scaling and Gravitational Correspondence in Operational Quantum Gravity

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Abstract

Operational Quantum Gravity (OQG) proposes that matter–field exchange and the resulting energy storage produce the clock, ruler, and motion relationships described geometrically by general relativity (GR). This companion paper develops their nuclear linear-response consequences, using the earlier OQG constitutive construction and corresponding-state theorems. The transformation retains strong and electromagnetic interactions, discrete transitions, continuum strength, and charge–current correlations. Nuclear spectra determine component storage; matched electric and magnetic field profiles connect that storage to permittivity and permeability. With correspondingly mapped electromagnetic surroundings, the supporting exchange scales as the square of the clock-frequency factor. The complete strain-dependent energy supplies the relaxed stiffness. A static isotropic GR benchmark yields the same Hamiltonian in common comparison variables. Its metric-derived factors are benchmark inputs; the OQG source construction is cited with its conditions. Synthetic numerical checks reproduce the response identities to discrepancies below 7×10^{-14} , and an independent review reports exact reproduction of the archived results. The calculation establishes the response correspondence. OQG interprets geometry as a description of the underlying matter–field equilibrium. Agreement alone does not select that interpretation. A helium frequency comparison across gravitational environments tests the shared clock law; a single-environment spectrum cannot reveal common scaling of all local standards. The response theorem is complete within its stated assumptions and does not require another nuclear dataset to finish its proof.

Keywords: Operational Quantum Gravity; nuclear response; gravitational redshift; electromagnetic equilibrium

1 Physical purpose and accomplishment

The question addressed here is whether OQG’s clock and ruler scaling can be carried consistently through an interacting nucleus, including how that nucleus responds to an electromagnetic disturbance. A list of transition energies alone does not answer that question. The response also depends on how strongly each transition couples to the applied field and on correlations between charge and current patterns. We show that one specified transformation of the Hamiltonian determines the transformation of all these linear-response quantities. The result applies to any reference spectrum admitted by the construction, including a continuum of breakup states.

OQG proposes a material explanation of gravity through the equilibrium exchange between matter and its electromagnetic surroundings [1]. Quantum electrodynamics requires the vacuum-field contribution alongside radiation reaction in radiatively coupled dynamics; Milonni discusses their role in preserving canonical commutation relations [2]. Puthoff’s equilibrium calculations describe balanced particle–vacuum exchange in the systems considered [3]. OQG uses these established matter–field ingredients and proposes a further gravitational identification: scattering by surrounding matter changes the supporting exchange, the material equilibrium adjusts, and spatial variations of the resulting material energy produce gravitational motion. The present paper develops the nuclear response associated with the specified adjustment.

The local ambient field includes vacuum fluctuations together with fields shaped by surrounding matter, sources, and boundaries. The bare zero-point vacuum is Lorentz invariant; a specified material environment can have a physical rest frame. Its measured spectrum can therefore depend on motion relative to the matter and boundaries that establish it. The field and matter equations remain Lorentz covariant when all participating quantities are transformed consistently [1]. Supporting power refers to the balanced exchange of the specified receiver with this environment.

The physical question is whether the linked supporting-power ratios and stored-energy changes, expressed through permittivity ϵ and permeability μ , produce the relationships called gravitational, or whether those

constitutive relationships are a representation of matter and fields in a spacetime geometry. OQG adopts the former interpretation. In the standard GR formulation, geometry and matter obey coupled field equations [9]; a specified metric then determines the clock and ruler comparisons. The present work compares the resulting response laws. It does not infer a unique causal direction from their mathematical agreement.

A corresponding state means that the internal wavefunction, energy scale, and measured operators are transformed together according to a declared rule. This mapping compares material equilibria in fixed comparison standards. A Lorentz transformation changes the observer describing a given physical state. Write $f > 0$ for the energy and frequency ratio and $b > 0$ for the length ratio, expressed in stated comparison standards. Comparisons use material clocks and rulers. An unobservable common change of every local standard must be distinguished from a ratio between physical systems in different environments. This paper carries the earlier OQG transformation through the complete nuclear linear response. It then constructs a GR benchmark using the same comparison standards.

Existing observations support a prediction wherever it is recovered with the same measurement protocol. Optical clock comparisons, for example, have resolved the gravitational shift associated with a 33-centimetre elevation change [4]. If an OQG nuclear clock and a GR nuclear clock obey the same redshift relation, repeating that comparison with a nuclear transition does not become a distinguishing experiment. Agreement with GR is a valid accomplishment; a difference is not a prerequisite for recognizing it.

The result is a completed transformation theorem that builds on earlier OQG work [1]. Theorem 1 of that work, Section 4.1 and Eqs. (48)–(50), establishes the dilation family and preservation of bound-state stability. Theorem 2, Section 4.3 and Eqs. (55)–(56), establishes the supporting-power relation. Here P denotes balanced supporting exchange and P_0 its reference value, so the positive clock factor is $f = \sqrt{P/P_0}$; the common branch additionally has $b = f$. The nuclear response developed below states the corresponding linear-response consequences explicitly.

The earlier results retain their stated physical conditions when used here. A power ratio fixes f within the corresponding family; determining that ratio for a source environment uses the coupled equilibrium. The independent length factor requires the corresponding ruler response when the common branch is not used. The following table identifies the relevant dependencies. Equation and section numbers in its second column belong to Ref. [1].

Result used	Location in Ref. [1]	Role in this paper
Nuclear storage and field coefficients	Sections 3.1–3.2; Eqs. (27)–(34)	Connects the interacting spectrum and matched spatial profiles to capacitance, inductance, permittivity, and permeability.
Stable corresponding states	Theorem 1; Section 4.1	Supplies the Hamiltonian and operator mapping.
Constitutive scaling	Section 4.2; Eqs. (53)–(54)	Combines component storage and geometry to obtain the field-coefficient ratios.
Supporting-power relation	Theorem 2; Section 4.3	Relates exchange to clock scaling under the matched response conditions.
Material equilibrium	Section 3.5; Eqs. (43)–(47)	Specifies the coupled energy, stress, and power problem.
Gravitational applications	Sections 7 and 8	Supplies the calibrated weak spherical application and the exact exterior reconstruction under its additional response conditions.

These dependencies let a companion paper address one part of OQG without rederiving the whole framework. The source distribution, field state, and equilibrium solution enter through the cited application. They are not

inferred from the synthetic spectra used here. Section 2 proves the response identity; Section 3 states its storage, environmental, and mechanical conditions; Section 4 makes the GR benchmark identification; and Sections 5 and 6 explain the observable comparison and verification.

2 From a scaled Hamiltonian to the nuclear response

The first step is to describe the same internal pattern at a different length and energy scale. Let H_0 be the reference Hamiltonian, the operator that gives the nuclear energy and time evolution, with strong and electromagnetic interactions retained. It is self-adjoint on a specified domain and has a stable reference ground state. Let $\mathbf{x}$ contain the d independent internal spatial coordinates and let $\psi(\mathbf{x})$ be a normalized wavefunction. Removing the center of mass leaves $d = 9$ for four nucleons; spin and other internal labels are carried along unchanged. Subscript 0 denotes the chosen reference system. The reduced Planck constant $\hbar$ and electric charge unit remain fixed.

Define $f > 0$ as the energy and frequency ratio and $b > 0$ as the length ratio, both expressed in the declared comparison standards. The dilation operator U_b changes spatial scale while preserving probability. A dagger denotes the adjoint; for this unitary operator it is also the inverse. The transformed Hamiltonian is

$$(U_b\psi)(\mathbf{x}) = b^{-d/2}\psi(\mathbf{x}/b), \quad H_{f,b} = fU_bH_0U_b^\dagger. \quad (1)$$

The prefactor compensates for the change of integration volume. The domain of H_0 is transformed with U_b . Label the ground state by g and an excited state by n , with reference energies $E_{g,0}$ and $E_{n,0}$. Equation (1) gives the excitation gap $\Delta_{n,0} = E_{n,0} - E_{g,0}$ the value $\Delta_{n,f,b} = f\Delta_{n,0}$. Its transition frequency in cycles per second is $\nu_n = \Delta_n/(2\pi\hbar)$ and has the same factor. Radii scale by b . Breakup thresholds and bound levels scale together, so a bound level stays below its corresponding threshold. This establishes stable binding throughout the mapped family for finite positive factors.

For a kinetic term, let $\mathbf{p}$ be canonical momentum, m_0 the reference mass coefficient, and $V_0(\mathbf{x})$ the reference potential. Their transformed sum is $fb^2\mathbf{p}^2/(2m_0) + fV_0(\mathbf{x}/b)$. The equivalent mass coefficient is $m_{f,b} = m_0/(fb^2)$. This example explains the simultaneous scaling of motion and binding. It retains the complete nuclear interaction and does not replace that interaction with electrostatic attraction.

We next ask how much the nucleus responds to a weak applied disturbance. Let X be the Hermitian observable being read out and Y the Hermitian observable coupled to the applied source. Let g_X and g_Y be their real, nonzero amplitude factors: $X_{f,b} = g_XU_bX_0U_b^\dagger$, with the analogous rule for Y . The source amplitude $h_Y(t)$ enters the energy as $-h_Y(t)Y$, where t is time. The retarded susceptibility χ_{XY}^R is the impulse-response kernel. Its frequency-domain value gives the linear change in the expectation of X per unit source amplitude at that frequency. ‘‘Retarded’’ means that the response follows the applied disturbance.

In the Kubo convention [5], $\theta(t)$ is zero for $t < 0$ and one for $t > 0$, $i^2 = -1$, $[X, Y] = XY - YX$ is a commutator, and angle brackets denote an equilibrium expectation. The notation $X(t)$ means the observable evolved to time t , and superscript R denotes the retarded response. Angular frequency is ω , measured in radians per second. For an impulse at time zero, the time and frequency representations are

$$\chi_{XY}^R(t) = \frac{i}{\hbar}\theta(t)\langle[X(t), Y(0)]\rangle, \quad \chi_{XY}^R(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} \chi_{XY}^R(t) dt. \quad (2)$$

Use the reference ground state and its transformed counterpart. A thermal equilibrium state obeys the same argument if its temperature also scales by f , keeping level occupation probabilities unchanged. Equation (1) makes operator evolution obey $X_{f,b}(t) = g_XU_bX_0(ft)U_b^\dagger$. In Eq. (2), the operator amplitudes contribute g_Xg_Y , and changing the time integration variable to ft contributes $1/f$. Therefore

$$\chi_{XY,f,b}^R(\omega) = \frac{g_Xg_Y}{f} \chi_{XY,0}^R(\omega/f). \quad (3)$$

Response theorem. At the correspondingly shifted frequency, the response has the amplitude factor g_Xg_Y/f . Resonances move along the frequency axis by f , while their strengths transform with the measured operators. The identity includes cross responses between different observables and every discrete or continuum contribution retained in the Hamiltonian. Causality is preserved. A passive reference system remains passive: the transformed response matrix cannot supply net energy to a weak cyclic drive.

If the measured observable depends explicitly on its applied source, the physical response also contains an instantaneous term. The source amplitude must transform as $h_{Y,f,b} = (f/g_Y)h_{Y,0}$ so that its coupling energy shares the factor f . Differentiating the mapped observable with respect to this source gives the same factor $g_X g_Y / f$ for the instantaneous term. Thus it must be retained with the commutator response. This extends the correspondence to the full source-derived linear response under the same coupling prescription.

To make the continuum statement precise, let E denote excitation energy. The matrix element $\langle n|X|g \rangle$ measures the coupling through X from ground state g to excited state n . Let δ denote the Dirac delta distribution and $S_X(E)$ the squared transition matrix element per unit energy. For discrete states the strength density is the sum of $|\langle n|X|g \rangle|^2 \delta(E - \Delta_n)$; continuum states contribute an integral. Define the energy-weighted moment $M_k = \int_0^\infty E^k S_X(E) dE$, where k is a specified real exponent and the integral must exist. Then

$$S_{X,f,b}(E) = \frac{g_X^2}{f} S_{X,0}(E/f), \quad M_{k,f,b} = g_X^2 f^k M_{k,0}. \quad (4)$$

The strength density contains $1/f$ because an energy interval is stretched by f ; integrated strength scales by g_X^2 . This step completes the response transformation without requiring equal shell spacing or a particular helium-4 spectrum. It transforms a supplied response rather than calculating the reference spectrum from scratch.

3 Electrical storage supporting exchange and mechanical response

3.1 Charge redistribution and current conservation

Electromagnetic response depends on where charge moves, not only on the energies available. Let $\mathbf{r}$ denote position relative to the nuclear center, $\rho(\mathbf{r})$ charge density, and $\mathbf{j}(\mathbf{r})$ current density. The position argument and operator states must both be transformed:

$$\rho_{f,b}(\mathbf{r}) = b^{-3} U_b \rho_0(\mathbf{r}/b) U_b^\dagger, \quad \mathbf{j}_{f,b}(\mathbf{r}) = f b^{-2} U_b \mathbf{j}_0(\mathbf{r}/b) U_b^\dagger. \quad (5)$$

The charge-density factor compensates for volume. Current density is charge per area per time, giving f/b^2 . The continuity equation expresses local charge conservation: the time derivative of charge density plus the spatial divergence of current density is zero. Both terms acquire f/b^3 , so conservation is preserved. Currents involving more than one nucleon must be retained when required by the chosen nuclear interaction.

To describe an electrical response channel, let a label a spatial weighting function $w_a(\mathbf{r})$, which is dimensionless. Define its charge coordinate by $Q_a = \int w_a(\mathbf{r}) \rho(\mathbf{r}) d^3r$. The weighting pattern follows the dilation, so its transformed value at $\mathbf{r}$ equals its reference value at $\mathbf{r}/b$. Consequently Q_a has amplitude factor 1, while its current $I_a = dQ_a/dt$ has factor f . These coordinates measure charge redistribution. Uniform weighting would instead give conserved total charge, whose excitation response is zero and which is excluded from the responsive channels.

An electric dipole weights charge by position and therefore has factor b . A magnetic dipole contains one position factor and a volume integral of current density, giving $f b^2$. Substitution of these factors into Eq. (3) fixes the charge, current, electric-dipole, and magnetic-dipole responses without violating charge conservation.

3.2 Capacitance inductance and constitutive coefficients

The circuit representation gives an engineering interpretation of the low-frequency response. A voltage amplitude v_a couples to channel Q_a through energy $-v_a Q_a$. Collect the channel charges and voltages into vectors $\mathbf{Q}$ and $\mathbf{v}$. Their small-signal relation is $\delta\langle \mathbf{Q} \rangle = \chi_{QQ}(\omega) \mathbf{v}$, where the bold susceptibility is a matrix and $\delta\langle \mathbf{Q} \rangle$ is the induced change. For reciprocal channels with real transition amplitudes, define C as the static capacitance matrix and D_2 as the coefficient of squared frequency. Define L through the inverse response:

$$\chi_{QQ}(\omega) = C + \omega^2 D_2 + \dots, \quad \chi_{QQ}^{-1}(\omega) = C^{-1} - \omega^2 L + \dots. \quad (6)$$

The omitted terms are higher-order frequency contributions. This expansion applies below the excitation thresholds, where the stated coefficients exist. In SI units, C is in farads, D_2 in farad seconds squared, and L in henries. The inverse response therefore has the familiar capacitive restoring term and inertial or inductive term of an oscillator.

Let $\mathbf{q}_n$ contain the reference or transformed transition amplitudes $\langle n|Q_a|g\rangle$ for all retained channels, and let $\Delta_n > 0$ be the corresponding gap. A superscript T denotes matrix transpose. Expansion of the spectral response gives

$$C = 2 \sum_n \frac{\mathbf{q}_n \mathbf{q}_n^T}{\Delta_n}, \quad D_2 = 2 \hbar^2 \sum_n \frac{\mathbf{q}_n \mathbf{q}_n^T}{\Delta_n^3}, \quad L = C^{-1} D_2 C^{-1}. \quad (7)$$

The sums include continuum integration and require convergent inverse-energy moments. Inverses act only on the subspace of channels with nonzero response. Complex transition amplitudes require the full complex spectral expression; Eq. (3) remains valid, while Eq. (7) uses the stated real-channel restriction. Because the charge amplitudes remain unchanged and every gap has factor f ,

$$C_{f,b} = C_0/f, \quad D_{2,f,b} = D_{2,0}/f^3, \quad L_{f,b} = L_0/f. \quad (8)$$

The physical construction connecting these components to field coefficients is developed in Ref. [1], Sections 3.1–3.2, Eqs. (27)–(34). Let ε and μ be scalar constitutive coefficients in the comparison representation. They describe locally isotropic response in the material rest frame, expressed in the declared comparison standards. Their inverses measure the curvature of recoverable field energy density with respect to electric displacement and magnetic flux density. This gives the coefficients a storage meaning. The full field energy retains the reference Maxwell contribution.

For one channel with spatially constant isotropic coefficients, let C_E and L_B be the capacitance and magnetic inductance obtained by integrating the electric and magnetic field energies. Let G_D and G_B be the corresponding spatial-profile factors, each with units of inverse length. The earlier construction gives $\varepsilon = G_D C_E$ and $\mu = G_B L_B$. Matching the component and field descriptions requires the same state, charge and current patterns, and mechanical constraints. It gives $C_E = C$ for the matched charge storage. The identification $L_B = L$ additionally requires that the spectral inertia represent the same magnetic storage. The spectrum supplies the component values, and the profiles supply their geometric conversion. This is the physical constitutive construction used here, with its matching conditions retained.

For geometrically similar profiles, both geometry factors scale as $1/b$. Equivalently, capacitance is proportional to permittivity times length, and magnetic inductance to permeability times length. Thus the matched model gives

$$C_{f,b} = \frac{\varepsilon_{f,b}}{\varepsilon_0} b C_0, \quad L_{f,b} = \frac{\mu_{f,b}}{\mu_0} b L_0. \quad (9)$$

Equivalently, the electric energy coefficient per squared charge scales as $1/(\varepsilon b)$, while the magnetic energy coefficient per squared current scales as μb . They are different geometric scalings. Equating Eqs. (8) and (9) fixes the constitutive ratios. With propagation speed $c = 1/\sqrt{\varepsilon\mu}$ and wave impedance $Z = \sqrt{\mu/\varepsilon}$, the result is

$$\frac{\varepsilon_{f,b}}{\varepsilon_0} = \frac{\mu_{f,b}}{\mu_0} = \frac{1}{fb}, \quad \frac{c_{f,b}}{c_0} = fb, \quad \frac{Z_{f,b}}{Z_0} = 1. \quad (10)$$

The factors f and b need not be equal. The common branch means the special choice $f = b$, for which both constitutive ratios are b^{-2} . In general, the constitutive ratios determine the product fb ; a clock or length comparison is needed to separate the two factors. The spectral L represents the inertia of the complete nuclear response. Its identification with magnetic storage in Eq. (9) is an explicit matching condition, not a conclusion that the strong interaction has disappeared.

3.3 Balanced electromagnetic exchange

Storage alone does not specify exchanged power; the surrounding field and its losses must also be included. We evaluate the exchange in the rest frame of the equilibrium receiver, using the same comparison time for its response and environmental spectra. Let $\delta Q_a = Q_a - \langle Q_a \rangle$ denote the spontaneous charge fluctuation in channel a . Let $S_{Q,aa'}(\omega)$ be the two-sided symmetrized fluctuation spectrum for channel labels a and a' , where “two-sided” includes positive and negative frequencies and $\{X, Y\} = XY + YX$ denotes an anticommutator. Its normalization is

$$S_{Q,aa'}(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} \frac{\langle \{\delta Q_a(t), \delta Q_{a'}(0)\} \rangle}{2} dt. \quad (11)$$

The matrix S_Q collects these channel spectra, each with units of coulomb squared seconds. Let $\mathcal{R}(\omega)$ be the dissipative, Hermitian part of the environmental impedance matrix, in ohms. It measures losses seen by the

channel currents and is distinct from the propagation impedance Z in Eq. (10). In the matched environment, the charge spectrum obeys $S_{Q,f,b}(f\omega) = S_{Q,0}(\omega)/f$ and the dissipative impedance obeys $\mathcal{R}_{f,b}(f\omega) = \mathcal{R}_0(\omega)$. These conditions include the relevant surroundings in the correspondence.

Define the supporting-exchange quantity P by the current fluctuations dissipating into those channels. The current spectrum contains $\omega^2 S_Q$, and Tr sums the diagonal channel contributions after matrix multiplication. For a convergent integral, or a common finite cutoff expressed in correspondingly scaled frequency variables,

$$P = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \omega^2 \text{Tr}[\mathcal{R}(\omega) S_Q(\omega)], \quad P_{f,b} = f^2 P_0. \quad (12)$$

This is a frequency-resolved version of current squared times resistance and has units of watts. The integration measure, squared frequency, impedance, and fluctuation spectrum contribute factors f , f^2 , 1 , and $1/f$, respectively. Their product proves the power factor. At equilibrium the associated incoming and outgoing exchanges balance. The defined symmetrized quantity is not a net radiated power or an extractable source of vacuum energy. The accomplishment is the exchange scaling for the specified environment; equality of incoming and outgoing powers alone would not determine their common magnitude.

Equation (12) compares matched equilibria. Motion relative to the ambient sources requires the field spectrum and receiver response to be evaluated together in a specified frame, including their directional dependence. The equilibrium power law alone does not determine that moving-state exchange. This distinction preserves the stated power scaling while identifying the additional response calculation needed for a moving receiver [1].

3.4 Stiffness after internal relaxation

The mechanical calculation asks how the nucleus resists a small imposed strain after its internal coordinates have adjusted. Let $\mathbf{e}$ denote dimensionless strain components, $\mathbf{z}$ dimensionless internal relaxation coordinates, $\mathcal{E}(\mathbf{e}, \mathbf{z})$ the complete material energy, and V its comparison volume. Apply the correspondence to this energy before relaxation: $\mathcal{E}_{f,b} = f\mathcal{E}_0$ at corresponding $\mathbf{e}, \mathbf{z}$, and $V_{f,b} = b^3 V_0$. The environment and scale factors are held fixed during each local stiffness derivative.

Let α, β label components of the combined coordinate vector $\xi = (\mathbf{e}, \mathbf{z})$. The stiffness matrix is the energy Hessian, or matrix of second derivatives, divided by volume:

$$K_{\alpha\beta} = \frac{1}{V} \frac{\partial^2 \mathcal{E}}{\partial \xi_\alpha \partial \xi_\beta}. \quad (13)$$

Its blocks $K_{ee}, K_{ez}, K_{ze}, K_{zz}$ describe strain stiffness, strain–internal coupling, and internal stiffness. All have units of pressure. At a stable constrained equilibrium with positive definite K_{zz} , the internal change is $\delta \mathbf{z} = -K_{zz}^{-1} K_{ze} \delta \mathbf{e}$. Eliminating this relaxed motion gives the effective stiffness, denoted by superscript “rel”:

$$K^{\text{rel}} = K_{ee} - K_{ez} K_{zz}^{-1} K_{ze}, \quad K_{f,b}^{\text{rel}} = \frac{f}{b^3} K_0^{\text{rel}}. \quad (14)$$

The subtracted term accounts for the reduction in stiffness when internal motion is allowed. Every Hessian block has the same energy-per-volume factor, so relaxation preserves that factor. This result includes strong-interaction energy through $\mathcal{E}$; it is not inferred from electromagnetic storage alone.

3.5 Assumptions and their consequences

Each result uses a definite set of premises. The table separates the response identity from storage matching, the environmental exchange, and application to a gravitational source. Earlier results enter by reference with their original conditions.

Result	Required premises	Consequence and scope
Nuclear response	Finite positive f, b ; mapped Hamiltonian domain, state, operators, and source coupling	Eq. (3) transforms the complete retained linear response, including continuum and contact terms.

Result	Required premises	Consequence and scope
Constitutive coefficients	The same state and constraints; matched electric and magnetic storage profiles	Eqs. (9)–(10) convert spectral storage to field coefficients; spectral inertia must represent the matched magnetic storage.
Supporting exchange	Corresponding equilibrium occupations, causal passive impedance, noise, and frequency regularization	Eq. (12) gives the balanced symmetrized exchange ratio; it is not net extractable power.
Relaxed stiffness	Complete strain-dependent energy scaling; positive invertible internal stiffness	Eq. (14) includes internal relaxation once and preserves the energy-per-volume factor.
GR benchmark and OQG source application	Static isotropic comparison with negligible variation across each nucleus; source application conditions from Ref. [1]	Eq. (15) supplies metric benchmark factors. OQG assigns their physical origin to the cited coupled equilibrium and source response.

The theorem and its implementation can therefore be assessed without repeating every earlier derivation. A claim about a particular source must also satisfy the premises of that source application.

4 The GR comparison with physical reference standards

We now construct the gravitational benchmark independently of the response algebra. This benchmark identifies the factors needed for a common representation; it does not select them through a new OQG scattering calculation. Compare identically prepared local systems in two stationary laboratories, labelled 0 and 1, in a static isotropic field. Use the same isotope, charge state, and local preparation, with nongravitational disturbances controlled. Neglect variation of the field and tidal effects across either nucleus. Let t and $\mathbf{r}$ be the static time and spatial coordinates, ds^2 the spacetime interval, $N(\mathbf{r}) > 0$ the lapse relating proper time to coordinate time, and $A(\mathbf{r}) > 0$ the factor relating proper length to coordinate length. Subscripts 0 and 1 on N and A mean their values at the two laboratories. With c_0 the local reference light speed, the metric and benchmark identification are

$$ds^2 = -N(\mathbf{r})^2 c_0^2 dt^2 + A(\mathbf{r})^2 d\mathbf{r}^2, \quad f = \frac{N_1}{N_0}, \quad b = \frac{A_0}{A_1}. \quad (15)$$

Normalize comparison time to the clock at laboratory 0 and comparison distance to its coordinate scale. At laboratory 1, local proper time is f times comparison time; a fixed local size occupies b times its reference coordinate size. The local internal Hamiltonian remains H_0 . Let $H_{\text{GR,ref}}$ denote its GR generator in the comparison variables, and $H_{\text{OQG,corr}}$ the corresponding OQG Hamiltonian. Transforming the time and normalized wavefunction gives

$$H_{\text{GR,ref}} = f U_b H_0 U_b^\dagger = H_{\text{OQG,corr}}. \quad (16)$$

Multiplication by f converts the time standard; conjugation by U_b converts the spatial representation. Their equality is the reason the response identity applies to both descriptions. Local internal variables and external comparison variables must be distinguished for composite systems [6]. Equation (15) is the precise point at which the GR metric supplies the benchmark factors. For an OQG source calculation, the input at this same point is the clock and ruler response obtained from the cited material-equilibrium construction. That input must satisfy the assumptions of the application used in Ref. [1]. The calculation here establishes the common response law once those factors are specified.

Maxwell's equations in these static isotropic coordinates also have a constitutive representation in which the effective permittivity and permeability are proportional to A/N [7]. Their laboratory-1 to laboratory-0 ratios are $A_1 N_0 / (A_0 N_1) = 1/(fb)$, reproducing Eq. (10). Locally, GR observers using their own clocks and rulers retain the usual vacuum constants. The varying coefficients here refer to the common comparison representation. The GR calculation obtains them from the metric, while the OQG construction in Section 3.2 obtains their storage

meaning from the matched material and field response. These routes express the same coefficients within the correspondence; equality of the coefficients does not by itself establish which physical account is fundamental.

The following factors summarize the result. Every entry uses the same reference variables; response functions are compared at corresponding frequencies.

Quantity in the comparison	Factor in both OQG and GR
Transition energy or frequency	f
Nuclear radius in comparison coordinates	b
Ratio of two corresponding transition energies	1
Electric-dipole susceptibility at $f\omega$ compared with its reference value at ω	b^2/f
Decay rate with a correspondingly mapped environment	f
Supporting exchange defined by Eq. (12)	f^2
Relaxed stiffness per comparison volume	f/b^3

The decay-rate entry follows because the mapped evolution runs with time argument ft : a reference decay takes $1/f$ as long in comparison time. Environmental linewidths must share that correspondence. A local ruler also has coordinate factor b , and a local clock reference has frequency factor f , so their local dimensionless ratios are unchanged. A signal from clock 1 received by stationary observer 0 has the redshift factor $N_1/N_0 = f$ relative to an identical local clock, after other frequency shifts are controlled. A distant length measurement additionally requires a defined optical, radar, or transport procedure; coordinate radius alone is not that measurement. The present benchmark uses stationary clocks. For moving clocks, Ref. [1], Eq. (86a), separates the material clock factor from local special-relativistic time dilation; motion-dependent signal shifts also enter the chosen measurement protocol.

The exact Schwarzschild exterior illustrates why two scale factors are retained. Let ϱ be isotropic radius, $r_s = 2GM/c_0^2$ the Schwarzschild radius, G Newton's constant, and M the central source mass. Define $u = r_s/(4\varrho)$. In the exterior region $0 \leq u < 1$, the factors are $N = (1 - u)/(1 + u)$ and $A = (1 + u)^2$. Consequently $f/b = (1 - u_1^2)/(1 - u_0^2)$. The common branch $f = b$ reproduces this comparison to first order in the weak field; the general theorem retains the higher-order difference. This static construction applies where stationary laboratories and a positive lapse are defined.

5 Observable ratios and the allowed family

The correspondence must be expressed in observables that have stated physical references. Let $i = 0, 1$ label laboratories, and let $\Delta_{n,i}$ and $\Delta_{m,i}$ be two nonzero corresponding excitation gaps expressed in the common comparison variables. Labels n and m identify the transitions. Equation (1) gives

$$\frac{\Delta_{n,1}}{\Delta_{n,0}} = f, \quad \frac{\Delta_{n,1}/\Delta_{m,1}}{\Delta_{n,0}/\Delta_{m,0}} = 1. \quad (17)$$

The first quantity has the clock-comparison meaning established in Section 4. The second compares two internal energy ratios and tests whether the transitions share the same scale factor. A common scaling changes neither that internal ratio nor a nucleus-to-local-ruler size ratio. No reference outside the universe is introduced. Strengths, currents, and deformation remain distinct measurable properties, with transformations supplied by the preceding response and mechanical relations.

Within this theorem an allowed corresponding state satisfies Eq. (1), the stated operator mapping, conservation, and stability. Supporting exchange additionally uses the matched surroundings of Section 3.3. An incompatible

arbitrary change is outside this defined family. A physical criticism must identify an error in the derivation, a failed correspondence premise in the proposed application, or an observable prediction that does not follow.

The dilation exists for every finite $b > 0$ and introduces no minimum radius. Consecutive dilations multiply their length factors, preserving the same construction at every finite step where the description applies. The endpoint $b = 0$ is not an invertible transformation. This establishes admissibility of the finite mapped states; whether a physical evolution reaches an endpoint, or how the construction applies inside a black hole, is a separate dynamical question. An absence of absolute rulers does not eliminate changes in dimensionless internal relationships.

5.1 What helium transition measurements can establish

A helium spectrum measured in one environment can supply transition energies, coupling strengths, and continuum information. Those data help specify the reference response. They do not measure the change between gravitational environments. If the transitions and the local frequency reference all acquire the same factor, their local frequency ratios do not change. The corresponding statement applies to nuclear size relative to a ruler that shares the length scaling. There is no external absolute standard to reveal that common change.

To measure the common transition-frequency change, compare a transition with a physical reference at a different gravitational potential, using a defined signal or transport protocol. Whether implemented as a continuously operating clock or a spectroscopic frequency comparison, this is a gravitational clock comparison. A suitable helium transition, if interrogated with sufficient stability and precision, would provide a helium-based realization. An electronic helium clock and a nuclear transition probe address different internal excitations; they share the fractional clock law within the stated correspondence.

Let ν be the reference transition frequency, $\Delta\nu$ its increase in the comparison when the system is raised, g the near-surface gravitational acceleration, and Δh a small height increase. The weak-field benchmark gives $\Delta\nu/\nu \simeq g\Delta h/c_0^2$. For $g \simeq 9.8 \text{ m s}^{-2}$ and $\Delta h = 1 \text{ m}$, the fraction is approximately 1.1×10^{-16} . A hypothetical 100 MeV transition would therefore shift by approximately $1.1 \times 10^{-8} \text{ eV}$ per metre in that comparison; this is an energy-scale illustration, not the specification of a helium clock transition. Its large carrier energy does not relax the required fractional precision. On the common weak-field branch, the length comparison has the same leading fractional change, although a clock measurement directly measures the frequency factor.

Optical clocks have already tested gravitational frequency comparisons over centimetre [4] and millimetre [8] height differences. These observations establish the accessibility of the clock observable; they do not constitute measurements of helium nuclear response. A helium comparison agreeing with the common law would extend its tested material realization. It would not distinguish OQG from GR where the predictions coincide.

Accordingly, no additional helium dataset or distinguishing clock experiment is needed to complete this response theorem. A request for a distinguishing signature in a single-environment transition table cannot supply a test of universal common scaling. Other dimensionless relationships, response strengths, or departures from the admitted family are separate observables and should be specified as such. A practical limit on a particular measurement is not a proof that every possible microscopic test is impossible, and it does not turn a completed conditional theorem into an unfinished derivation.

6 Numerical verification and contributor credit

The accompanying script checks the identities by calculating both sides separately. Its reference response has three charge channels, three discrete transitions, and a smooth positive continuum represented by 96 numerical integration nodes. For each of 20 positive pairs of f, b , it reconstructs the mapped response from scaled energies and matrix elements, then compares that result with the reference response transformed by Eq. (3) at 160 frequencies. Unequal scale factors are included. The same spectra test the storage coefficients and weighted strength moments.

All inputs are declared synthetic values in internally consistent units with $\hbar = 1$; setting this unit does not remove a physical parameter from a measurement. The random seed is 20261017. Relative matrix discrepancies use the Frobenius norm, the square root of the sum of squared element magnitudes, divided by the corresponding

predicted norm. The maximum response discrepancy is 6.73×10^{-14} , and capacitance and inductance discrepancies are at most 2.60×10^{-15} . A separate positive energy Hessian has three strain and four internal coordinates. Computing its relaxed stiffness before and after scaling gives a discrepancy of 1.90×10^{-16} .

A second calculation uses a causal Drude impedance, meaning a dissipative environment with a finite response time and a corresponding reactive component. Its storage matrices do not commute, so their products cannot be reordered. The fluctuation–dissipation relation connects equilibrium noise to dissipative response; the matrix loss identity used in that construction is checked to 7.43×10^{-15} . Four separately integrated exchange cases reproduce the predicted power ratios to 3.34×10^{-16} ; the largest quadrature error estimate is below 1.7×10^{-11} in the chosen units. Thus the ratio agreement is an algebraic consistency check, not a claim of experimental or absolute integration accuracy at that level. A finite Hermitian matrix family also supplies a finite-difference check of a scale derivative; it is an algebraic check, not a nuclear model. Original results, reproduced results, tolerances, and software versions are retained in the archive.

For a GR example, the program uses an ideal spherical source with specified $GM = 3.986004418 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$ and $c_0 = 299792458 \text{ m s}^{-1}$. The two areal radii are 6371000 m and 6371001 m; an areal radius is defined by sphere area 4π times radius squared and differs from isotropic coordinate radius. The calculation gives $f - 1 = 1.0926500555068 \times 10^{-16}$ and $b - 1 = 1.0926500551265 \times 10^{-16}$. OQG’s corresponding family and GR assign the same factors in this benchmark. The quoted digits describe the specified ideal model, not the uncertainty of a terrestrial experiment.

These checks verify implementations of the stated transformations. They use no measured helium-4 spectrum and do not independently infer f or b from surrounding matter. The analytic proof establishes the identity; the numerical work demonstrates its consistent implementation for a spectrum, coupled channels, and a dissipative environment.

Ali Alhawarat’s critical discussions prompted the extension from level scaling to the complete response and the explicit GR comparison. His 17 October 2026 source-level V2.2 audit reports independent checks of the coupled-equilibrium and relaxed-stiffness relations. His 18 October 2026 review of V1.1 reports a fresh execution of this response package, exact agreement with the archived results, analytical checks of the conditional transformations, and a separate high-precision calculation of the GR benchmark. His review also identified the need to state the source of the benchmark factors more prominently. I thank him for his contributions to independent scientific review, mathematical verification, and critical analysis. His review concerns V1.1; final approval of this revision is not implied.

The verification record separates the origin of the inputs from the checks performed on them.

Material checked	Input or method	Verification status
Response and storage identities	Declared synthetic spectrum and matrices	Author implementation and Alhawarat’s reported independent rerun agree.
Transformation theorem	Operator and spectral derivation	Alhawarat reports analytical consistency under the stated conditions.
GR benchmark	Specified metric and source parameters	Author calculation and Alhawarat’s separate high-precision check agree.
Physical helium source environment	Measured nuclear response and a specified coupled environment	Not evaluated in this archive; the cited framework supplies the separate application problem.

6.1 Equilibrium state and numerical regularization

The spectral checks use a reference ground state and its mapped counterpart. The response samples replace the infinitesimal causal frequency prescription by a positive broadening η , with reference value 0.037 in the declared units and mapped value $f\eta$. Frequencies range from 0.01 to 12. The discrete gaps are 0.8, 1.7, and 2.5; the synthetic continuum spans excitation energies from 2.8 to 22.8, with both endpoints multiplied by f in the

transformed system. The storage moments are evaluated directly from the unbroadened spectrum. The broadening regularizes the sampled response and is not a calculated nuclear linewidth.

The exchange calculation uses stationary zero-temperature equilibrium noise, with no additional coherent driving field. Here zero temperature specifies ground-state fluctuations of the declared response model; the receiver rest frame is the one specified in Section 3.3. Define K_D , L_D , and R_D as its positive definite capacitive restoring, inductive, and resistance matrices, respectively. Subscript D identifies this Drude model; K_D is an electrical coefficient, distinct from the mechanical stiffness in Section 3.4. Let $\Omega_D > 0$ be its environmental relaxation frequency. The impedance is $Z_D(\omega) = R_D \Omega_D / (\Omega_D - i\omega)$, and the charge response is $\chi_D(\omega) = [K_D - \omega^2 L_D - i\omega Z_D(\omega)]^{-1}$. These expressions retain the reactive and dissipative parts together. The impedance pole lies below the real frequency axis, and its dissipative part $\mathcal{R}_D = (Z_D + Z_D^\dagger)/2$ is positive semidefinite. Thus the declared bath is causal and passive.

Define $\text{Im}_H \chi_D = (\chi_D - \chi_D^\dagger)/(2i)$ as the Hermitian dissipative part of the susceptibility. For positive frequency, the zero-temperature symmetrized spectrum is $S_Q(\omega) = \hbar \text{Im}_H \chi_D(\omega)$. The script checks $\text{Im}_H \chi_D = \omega \chi_D \mathcal{R}_D \chi_D^\dagger$ and evaluates Eq. (12) as an integral over positive frequencies with measure $d\omega/\pi$. This normalization includes the negative-frequency half of the two-sided spectrum.

The reference relaxation frequency is $\Omega_D = 3.7$. The mapped parameters are $K_D \mapsto f K_D$, $L_D \mapsto L_D/f$, $R_D \mapsto R_D$, and $\Omega_D \mapsto f \Omega_D$. The Drude frequency provides a smooth high-frequency cutoff; each convergent power integral extends to infinity. Absolute and relative quadrature tolerances are both 2×10^{-11} , with at most 500 subintervals. All matrix entries are supplied in the script.

The term $-i\omega Z_D$ is the model's environmental correction to the inverse response, often described as a bath self-energy. Its reactive contribution is included explicitly. This synthetic bath is not an independently computed nuclear radiative self-energy, and the calculation performs no separate nuclear renormalization. In a microscopic application, already incorporated radiative shifts and the explicitly retained environmental contribution must be identified consistently to count each interaction once. These choices specify the exchange calculation used to verify the scaling; they do not supply a new surrounding-source model.

7 Physical interpretation and conclusions

This companion paper establishes the complete nuclear linear-response transformation for the earlier OQG corresponding family. Discrete transitions, continuum strength, conserved charge and current response, and low-frequency storage obey the same mapping. Matched field profiles give the constitutive ratios, matched electromagnetic surroundings give the supporting-exchange ratio, and the complete strain-dependent energy gives the relaxed stiffness. The numerical package verifies these identities for its declared synthetic inputs.

The physical construction of permittivity and permeability is part of the argument through Ref. [1], Sections 3.1–3.2. Nuclear response supplies component storage, and spatial field profiles convert that storage into constitutive coefficients. The earlier scaling and power theorems then relate those coefficients to clocks, rulers, and supporting exchange. The present paper extends those results through the full linear response without repeating the complete source theory. Its GR benchmark supplies the comparison factors from the metric; its synthetic calculation does not independently select them from a microscopic source environment.

The central physical question is whether the linked power ratios and stored-energy changes, represented by variable ε and μ , generate gravitational behavior, or whether these constitutive relationships describe matter and fields in the metric formulation of GR. OQG assigns the physical origin to the coupled matter–field equilibrium. Its spatially varying stored energy supplies gravitational work and motion under the cited construction; geometry records the resulting clock and ruler relationships. A uniform common rescaling alone does not produce a force. In the GR benchmark, the metric determines the corresponding comparison coefficients, while local observers retain the usual vacuum constants. GR couples matter and geometry, so the comparison does not require a claim that curvature has an experimentally established one-way causal priority over matter.

The agreement establishes the shared response law within the stated domain. It supports the consistency of OQG's proposed material explanation, but does not alone establish that explanation uniquely. Assessment should address

the cited constitutive and source derivations and their physical premises. A specific objection can identify an error, a failed matching condition, or an observable conflict; the absence of repeated derivations in this companion paper does not make the cited results absent.

A single-environment helium spectrum cannot reveal common scaling of the system and all its local standards. Comparing the common frequency change between heights is operationally a clock comparison. Existing redshift measurements support that shared observable law in their tested systems and regimes; a helium realization would extend its tested material application. Where OQG and GR predict the same observable, a distinguishing result is not required to establish their correspondence. Additional nuclear data can specify a particular reference response or source calculation. They are unnecessary to finish the transformation theorem proved here.

Data and calculation availability

The accompanying reproducibility archive contains the verification script, its original and reproduced results, package versions, the manuscript source, the document builder, and execution instructions. The response inputs are synthetic and fully specified in the script. No new experimental nuclear dataset is reported. AI assistance was used in developing the derivation, manuscript, and numerical implementation. The author retains responsibility for the claims; the external credit above identifies the separate scope of Alhawarat's contribution.

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