

Rotational Scattering and Frame Dragging in Operational Quantum Gravity

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Abstract

A rotating gravitating body changes how nearby matter moves relative to distant nonrotating standards. This paper calculates that response in Operational Quantum Gravity (OQG) using the same total-energy and stress formulation that describes static gravity. Supporting exchange establishes the material state and its stored energy. Rotation adds circulating momentum, which couples through the existing Einstein-equivalent WTDiff dynamics. Integrating a spherical source gives the exterior dragging rate, including its factor of two, with the strength fixed by the established stress stiffness. The stationary exterior equation retains this result at first order in spin without requiring weak static gravity. The receiver Hamiltonian acquires an orbital angular-momentum term that changes the relation between momentum and motion. It therefore explains how a receiver with zero canonical angular momentum can rotate relative to distant coordinates, and it gives the reference-frequency splitting of locally compared opposite azimuthal matter-wave patterns. The exact Kerr black-hole exterior is then expressed through a clock factor, directional ruler factors, and an angular shift. These variables also specify electromagnetic storage and its rotational electric-magnetic coupling. Numerical checks reproduce the source integral, receiver dispersion, and Kerr limits. The calculation extends OQG's energy-storage description to frame dragging with no additional rotational coupling: the directional response changes at first order in spin, while the scalar lapse correction begins at second order.

Keywords: Operational Quantum Gravity; frame dragging; rotational scattering; energy storage; stress response; Kerr spacetime

1 Introduction

A spherical body at rest affects nearby matter equally in opposite directions around its center. Rotation introduces a preferred sense of circulation. Nearby trajectories and matter-wave phases then depend on their direction relative to the source's rotation. Frame dragging describes this change in the local inertial response. One operational example is a freely moving test body whose conserved angular momentum is zero: it can still turn around the source relative to distant nonrotating coordinates. Explaining how this happens is the central objective of this paper.

Operational Quantum Gravity (OQG) begins with the response of matter. In the power-balance companion [1], the charge and current responses of surrounding matter scatter a shared electromagnetic field. Incident and scattered contributions interfere, changing the exchange available to support a stationary material state. The receiving body adjusts its internal storage and dimensions until incoming and outgoing powers balance again. Changes in the complete body's energy account for recoverable gravitational work, and its matter-wave energy function determines its motion. The clock and ruler factors used below express comparisons between these adjusted material states and a fixed reference standard.

The extension to rotation follows from retaining the momentum carried by that energy. The complete stress-energy response includes energy density, momentum density, and the transport of momentum through stresses. A rotating source has a circulating momentum density even though its center remains at rest. This supplies the directional part of the interaction. The calculation retains the internal energy and binding contributions of the whole body, with its internal degrees of freedom allowed to adjust.

Here, rotational scattering denotes the rotation-dependent part of this shared matter-field response as represented by OQG's collective stress functional. A functional assigns an energy or action to an entire field configuration; its derivatives describe how that configuration responds to a change. The long-wavelength description averages over the microscopic structure of matter while retaining its total stress. The stress-response companion [2] supplies the coefficient of this collective response and its WTDiff completion. WTDiff combines Weyl rescaling symmetry with coordinate transformations that preserve a fixed volume measure. With conserved total stress, the specified dynamics is Einstein-equivalent [4, 5]. This tensor structure couples source energy, momentum, and stress with a common gravitational normalization.

The calculation follows a source-to-receiver sequence. Section 2 identifies the common stress normalization already established by the storage framework. Section 3 uses it to calculate the response of a rotating source. Section 4 then asks what this response does to a receiving body: first to its energy, next to its motion, and finally to its matter-wave phase. Section 5 translates the exact Kerr exterior into clock and directional ruler comparisons, so that the same interpretation can be followed beyond the small-spin limit. In each case, the angular shift is the link between source circulation and receiver response.

The inputs are the calibrated total stress stiffness and the conserved-source, local two-derivative WTDiff dynamics of [2]. The rotational coefficient is derived from these inputs. The spherical source calculation establishes its

normalization; the finite-spin Kerr discussion uses the nonlinear branch and black-hole boundary conditions stated in Section 5. Keeping these two steps distinct makes clear which result is calculated from the source integral and which exact exterior is subsequently represented in operational variables.

2 Energy storage and the common stress normalization

2.1 Material storage and comparison standards

Consider corresponding stationary states of the same material system. Let P_{bal} be the common incoming and outgoing power supporting one such state, and let P_0 be its reference value. Define their ratio by $p=P_{\text{bal}}/P_0$. The clock factor f is the corresponding transition-energy or frequency ratio, expressed in fixed reference standards. The isotropic ruler factor b is the corresponding material length ratio. A subscript 0 denotes the reference state. Let ϵ denote permittivity, μ permeability, and c the electromagnetic propagation speed expressed in the same comparison standards. For the corresponding-state family of [1, 3],

$$p=f^2, \quad \frac{\epsilon}{\epsilon_0} = \frac{\mu}{\mu_0} = \frac{1}{fb}, \quad \frac{c}{c_0} = fb, \quad c_0^2 = \frac{1}{\epsilon_0 \mu_0}. \quad (1)$$

Permittivity and permeability specify the electric and magnetic storage response. Their matched variation preserves the reference wave impedance while changing the propagation speed in the comparison description. Isotropy means that the same ruler factor applies in every spatial direction. Equality between the clock and ruler factors is a further condition: on the common-scaling branch, $f=b$, both electromagnetic coefficients vary as p^{-1} and $c/c_0=p$. The exact spherical exterior will require separate clock and ruler factors even when the spatial coordinates are isotropic.

The storage variables make the energy accounting familiar. Let $\mathbf{D}$ be electric displacement, $\mathbf{B}$ magnetic flux density, and u_{EM} the electromagnetic Hamiltonian density per comparison-coordinate volume. In the local, conservative quadratic description, with real effective coefficients, its isotropic part is

$$u_{\text{EM}} = \frac{\mathbf{D} \cdot \mathbf{D}}{2\epsilon} + \frac{\mathbf{B} \cdot \mathbf{B}}{2\mu}. \quad (2)$$

For a capacitor, let Q be stored charge and C capacitance; for an inductor, let Φ_B be magnetic flux linkage and L inductance. Their energies are $Q^2/(2C)$ and $\Phi_B^2/(2L)$, respectively. At fixed charge or flux, increasing the corresponding storage coefficient lowers the energy. Holding voltage or current fixed describes a different comparison. Equation (2) is the field counterpart and makes its choice explicit through the variables $\mathbf{D}$ and $\mathbf{B}$.

The complete Hamiltonian also accounts for the retained matter degrees of freedom and their interactions. A consistent effective description counts each contribution once: a response already incorporated into an effective coefficient is not added again as the same internal energy. Nuclear and other binding contributions remain in the

complete material response. The quadratic field expression is used in its conservative effective regime; a general absorbing or frequency-dependent material requires its internal dynamical energy to be retained consistently.

There are two levels of description here. The capacitance and inductance picture explains how a change in storage response changes energy at fixed charge or flux. The complete material response also retains nuclear binding and the other internal contributions that establish the body's rest energy. Nuclear storage compliance and inertia belong to that internal response; they are not replaced by the simple electromagnetic field expression. The gravitational calculation needs the relaxed stress response of the complete system.

2.2 The stiffness that fixes the gravitational coupling

The stress response is evaluated after the internal state has adjusted to the imposed change. This is the meaning of a relaxed response. The second variation of the underlying energy or action is the stress Hessian; it describes how stress changes under a further perturbation. Let $A_0 = A_{2,E,0}^{\text{tot}}$ denote the full positive coefficient of the transverse, traceless stress-gradient response defined in [2]. Transverse, traceless components describe shear perpendicular to the perturbation wave vector with no trace component. The coefficient uses the inverse-energy-coordinate convention of that calculation and has dimensions of energy squared. It includes the reference-state contribution as well as any state-dependent increment.

The role of A_0 is therefore different from that of a capacitance, a permeability, or a nuclear bulk modulus. It is the coefficient of a particular collective stress-gradient response after the internal variables have been accounted for. Its value sets how strongly total energy and momentum source the long-wavelength gravitational response. This is the quantity needed to compare static and rotational gravity on the same footing.

Let $\hbar$ be the reduced Planck constant, G_0 the reference gravitational coupling, and κ_0 the coefficient that fixes the strength of the gravitational response to stress-energy. Matching the same shear source in the matter response and WTDiff action gives [2]

$$A_0 = \frac{\hbar c_0^5}{32\pi G_0}, \quad \kappa_0 \equiv \frac{G_0}{c_0^4} = \frac{\hbar c_0}{32\pi A_0}. \quad (3)$$

The reference value of κ_0 is calibrated to measurement in [1]. The thermal subtraction in [2] calculates a change in stiffness; Eq. (3) uses the total stiffness. The selected causal branch has two massless tensor-wave patterns and reference propagation speed c_0 . Its local two-derivative dynamics retains the leading differential response on scales large compared with microscopic structure.

Let $G_{\mu\nu}$ denote the Einstein curvature tensor and $T_{\mu\nu}$ the total conserved stress-energy tensor. Greek indices label spacetime components; repeated indices are summed. The WTDiff variation and stress conservation give $G_{\mu\nu} = 8\pi\kappa_0 T_{\mu\nu}$ when the cosmological integration constant is zero, as appropriate to the asymptotically flat exterior considered here [2]. Thus the energy density of a stationary body and the momentum density of a rotating body

enter the same response equations. The stiffness sets the overall strength; the tensor structure fixes the relative couplings.

The coefficient that will occur in the rotational result can consequently be written in gravitational or storage variables:

$$\frac{2G_0}{c_0^2} = \frac{\hbar c_0^3}{16\pi A_0} = \frac{\hbar}{16\pi A_0 (\epsilon_0 \mu_0)^{3/2}}. \quad (4)$$

This identity expresses the established normalization in different variables. The electromagnetic coefficients specify reference propagation, and A_0 specifies the gravitational stiffness. The following source calculation determines why this particular coefficient, including the factor of two, multiplies angular momentum.

3 The rotating source and its exterior response

The source calculation has two tasks. First, a weak-field integral fixes the strength and sign of the rotational response. Second, the stationary exterior equation tests whether that radial law survives when the static gravitational field is strong. The first task determines the coefficient; the second retains the full static compactness while still treating spin to first order.

3.1 Circulating momentum fixes the exterior coefficient

Begin with a stationary, weakly gravitating source whose center is at rest. Use Cartesian comparison coordinates $\mathbf{x}$ and the time coordinate $x^0 = c_0 t$, where t is the reference time. A prime labels a source position $\mathbf{x}'$. Let $\mathbf{r}$ be the observation position measured from the source center, and let $r = |\mathbf{r}|$. Latin indices label spatial components. Define the source momentum density by $j_i = T^{0i}/c_0$, using the same total stress-energy tensor as above. Its angular momentum $\mathbf{J}$ is

$$\mathbf{J} = \int \mathbf{x}' \times \mathbf{j}(\mathbf{x}') d^3 x'. \quad (5)$$

Let h_{0i} denote the time-space perturbation from the flat comparison metric, with signature $(-, +, +, +)$. Define $w_i = -c_0 h_{0i}$ to this order. The vector $\mathbf{w}$ has units of velocity and will enter the receiver's momentum-dependent energy. In the stationary transverse gauge, the mixed field equation is $\nabla^2 h_{0i} = 16\pi G_0 j_i / c_0^3$, where ∇^2 is the spatial Laplacian. It is a momentum constraint: the stationary directional response must be consistent with the source momentum. Multiplying by $-c_0$ and requiring the response to vanish at infinity gives

$$\nabla^2 \mathbf{w} = -\frac{16\pi G_0}{c_0^2} \mathbf{j}, \quad \mathbf{w}(\mathbf{r}) = \frac{4G_0}{c_0^2} \int \frac{\mathbf{j}(\mathbf{x}')}{|\mathbf{r} - \mathbf{x}'|} d^3 x'. \quad (6)$$

The inverse-separation factor is the static Green function, the response to a localized source, of the collective massless dynamics. The integral adds the contributions from all source elements. It uses the same gravitational strength as the static energy-density response, with circulating momentum supplying its directional content.

For an explicit evaluation, take a spherical source of radius R_s , radial mass density $\rho_m(s)$, and slow rigid angular velocity $\mathbf{\Omega}_s$, where $s=|\mathbf{x}'|$. In the nonrelativistic source limit, $\mathbf{j}=\rho_m(s)(\mathbf{\Omega}_s \times \mathbf{x}')$. Let I_s be the moment of inertia about the rotation axis and $\mathbf{n}=\mathbf{r}/r$ the exterior radial unit vector. For $r>R_s$, spherical integration gives

$$I_s = \frac{8\pi}{3} \int_0^{R_s} \rho_m(s) s^4 ds, \quad \int \frac{\rho_m(s) \mathbf{x}'}{|\mathbf{r}-\mathbf{x}'|} d^3x' = \frac{I_s}{2r^2} \mathbf{n}. \quad (7)$$

Spherical symmetry makes the angular integration especially simple. Expanding the inverse separation into angular harmonics, the vector $\mathbf{x}'$ retains only the first directional harmonic. Define $\mathbf{n}'=\mathbf{x}'/s$, let $d\Omega'$ be the source solid-angle element, and let δ_{ij} be the identity tensor. The surviving average is $\int n'_i n'_j d\Omega' = 4\pi \delta_{ij}/3$. All other angular contributions integrate to zero. The remaining radial integral is precisely the one defining I_s .

Using $\mathbf{J}=I_s \mathbf{\Omega}_s$ in Eq. (6) yields

$$\mathbf{w}(\mathbf{r}) = \frac{2G_0}{c_0^2} \frac{\mathbf{J} \times \mathbf{r}}{r^3} = \frac{\hbar c_0^3}{16\pi A_0} \frac{\mathbf{J} \times \mathbf{r}}{r^3}. \quad (8)$$

The momentum equation supplies the factor four, and the spherical integral supplies the factor one half. Their product gives the factor two in Eq. (8). The response circulates around the angular-momentum axis and reverses when the source spin reverses.

Choose a fixed positive z direction with unit vector $\hat{\mathbf{z}}$. Write $\mathbf{J}=J\hat{\mathbf{z}}$, where J is signed, and let Ω_s be the signed source angular rate. Define the exterior dragging rate $\omega(r)$ through $\mathbf{w}=\omega(r)\hat{\mathbf{z}} \times \mathbf{r}$. For a uniform sphere of mass M ,

$$\omega(r) = \frac{2G_0 J}{c_0^2 r^3}, \quad J = \frac{2}{5} M R_s^2 \Omega_s, \quad \omega(r) = \frac{4G_0 M R_s^2}{5c_0^2 r^3} \Omega_s. \quad (9)$$

The source rate Ω_s and dragging rate ω are different quantities. The former determines the material circulation; the latter describes the exterior response. At a fixed polar angle, $\mathbf{w}$ falls as r^{-2} , while ω falls as r^{-3} . For a relativistic compact source, its total conserved angular momentum replaces the Newtonian inertia formula. The leading distant response still has Eq. (8), with J defined by that asymptotic conserved quantity. The uniform sphere demonstrates the normalization without assigning a solid-body model to a black hole.

3.2 Continuing through the strong static field

The next step is to determine whether strong static gravity changes the radial dragging law. Define the gravitational length $r_g = G_0 M / c_0^2$. In the Schwarzschild exterior, r now denotes the areal radius, defined by symmetry-sphere area $4\pi r^2$. It agrees asymptotically with the weak-field radial coordinate used above. Let ρ be the isotropic radius, which assigns the same spatial conversion factor in every direction, and set $u = r_g / (2\rho)$. The static clock and ruler factors are [1, 3]

$$f = \frac{1-u}{1+u}, \quad b = (1+u)^{-2}, \quad r = \rho(1+u)^2. \quad (10)$$

These factors satisfy $f/b = 1 - u^2$. They agree at leading weak-field order but differ in the exact exterior. A coordinate length is divided by b to obtain proper length: when the material rulers are shorter, more of them fit into the same comparison-coordinate interval. This explains the reciprocal spatial factor while preserving b as the material length ratio.

Let θ and ϕ be polar and azimuthal angles, define $F = 1 - 2r_g/r$, and let ds^2 denote the spacetime interval. At first order in J , the stationary axial extension is

$$ds^2 = -F c_0^2 dt^2 + F^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta [d\phi - \omega(r) dt]^2 + O(J^2). \quad (11)$$

The squared bracket is retained only through first order; $O(J^2)$ denotes omitted second and higher spin orders. The static clock factor is $f = \sqrt{F}$, and rotation introduces an angular shift [6]. Let γ_{ij} be the spatial metric on a constant-time slice and γ^{ij} its inverse. The lapse f converts comparison time into proper time for observers normal to those slices. In this representation such observers remain at fixed r, θ while following $d\phi/dt = \omega$; remaining at fixed radius generally requires support.

The vacuum momentum constraint determines the radial dependence of that shift. Appendix A gives its evaluation, including the cancellation of the Schwarzschild lapse. The result is

$$\frac{d}{dr} \left(r^4 \frac{d\omega}{dr} \right) = 0. \quad (12)$$

Integration gives a constant angular rate plus a term proportional to r^{-3} . The constant is zero in coordinates nonrotating at infinity. Matching the remaining term to the source angular momentum fixes

$$\omega(r) = \frac{2G_0 J}{c_0^2 r^3} = \frac{\hbar c_0^3 J}{16\pi A_0 r^3}. \quad (13)$$

The first-order result therefore holds without expanding in the static compactness r_g/r . Strong static gravity is retained, while rotation is perturbative at fixed exterior radius $r > 2r_g$. This expansion is not a uniform approximation arbitrarily close to the spin-dependent Kerr horizon. The exact solution used in Section 5 supplies the finite-spin exterior. Equations (8) and (13) together establish the source strength and its inward continuation on the stated branch.

4 Receiver energy motion and phase

We now have the source response $\mathbf{w}$, or equivalently its angular rate ω . A rate by itself does not yet explain what a receiving body does. That explanation requires its Hamiltonian: the energy function whose momentum derivative gives its velocity. This section follows the same rotational term through three descriptions of the receiver. It first contributes a signed energy, then changes the relation between angular momentum and motion, and finally shifts the phase frequencies of oppositely directed waves.

4.1 From the source response to receiver energy

The response becomes observable through the energy of receiving matter. Let H denote the receiver Hamiltonian, expressed in the reference comparison time and energy standards, and let δH_{rot} be its leading rotational contribution. In the weak-field Cartesian variables, let $\mathbf{j}_{\text{rec}}$ denote the receiver's total momentum density. The common conserved-stress coupling gives

$$\delta H_{\text{rot}} = \int \mathbf{w}(\mathbf{x}) \cdot \mathbf{j}_{\text{rec}}(\mathbf{x}) d^3x. \quad (14)$$

Equation (14) adds the momentum-dependent energy over the receiver. Each small element contributes the contraction of its momentum density with the source response at that element. This is useful for an extended receiver. For a sufficiently small receiver, the source response is nearly constant across it and can be taken outside the integral. The remaining integral is the receiver's total canonical momentum.

Take a small, nonspinning test receiver, so that the source response varies negligibly across its dimensions. Its self-gravity and the source's recoil are neglected. Let m_0 be its reference rest mass, $\boldsymbol{\pi}$ its canonical spatial momentum, and $\mathbf{L} = \mathbf{r} \times \boldsymbol{\pi}$ its orbital angular momentum. Canonical momentum is the variable paired with position in Hamilton's equations; the interaction can change its relation to velocity. Define L_z as the component of $\mathbf{L}$ along the fixed z axis. Equation (14) becomes

$$\delta H_{\text{rot}} = \mathbf{w} \cdot \boldsymbol{\pi} = \frac{2G_0}{c_0^2 r^3} \mathbf{J} \cdot \mathbf{L} = \omega L_z. \quad (15)$$

The middle equality follows from the scalar triple product: $(\mathbf{J} \times \mathbf{r}) \cdot \boldsymbol{\pi} = \mathbf{J} \cdot (\mathbf{r} \times \boldsymbol{\pi})$. With the source angular momentum along the chosen z axis, this becomes ωL_z . The source and receiver therefore enter in separate, identifiable ways: ω is fixed by the source, while L_z describes the receiver's orbital momentum. Reversing either one reverses this contribution; reversing both leaves it unchanged.

This is a comparison at specified position and canonical momentum. It does not, by itself, compare two complete circular orbits, whose radii and momenta must also satisfy the equations of motion. Its immediate accomplishment is to identify the single additional term that must be carried into the receiver dynamics.

4.2 Why zero angular momentum permits angular motion

The energy depends on the receiver's momentum as well as its location. For opposite orbital momenta at the same position, this contribution has opposite signs. In the weak-field, slow-receiver limit, the leading terms are

$$H = m_0 c_0^2 + \frac{\boldsymbol{\pi}^2}{2m_0} - \frac{G_0 M m_0}{r} + \frac{2G_0}{c_0^2 r^3} \mathbf{J} \cdot \mathbf{L}. \quad (16)$$

The first three terms are the reference rest energy, kinetic energy, and Newtonian gravitational energy. The final term is the leading contribution linear in source spin. Spin-independent relativistic corrections are outside this weak-field, slow-receiver expression. Let $\mathbf{v} = d\mathbf{r}/dt$ be the coordinate velocity. Hamilton's rule says to differentiate the energy with respect to canonical momentum while holding position fixed. Applying it to Eq. (16) gives

$$\mathbf{v} = \frac{\partial H}{\partial \boldsymbol{\pi}} = \frac{\boldsymbol{\pi}}{m_0} + \mathbf{w}, \quad L_z = m_0 r^2 \sin^2 \theta (\dot{\phi} - \omega). \quad (17)$$

A dot denotes differentiation with respect to reference time t . The second relation is the azimuthal component of the first, at the same retained order. It says that canonical angular momentum measures angular motion relative to the local shift. Consequently, $L_z = 0$ removes the angular motion relative to that shifted local frame, while the angular rate relative to distant nonrotating coordinates remains ω . Zero canonical angular momentum is not the same condition as zero coordinate angular velocity.

To retain the strong static field, return to the isotropic spatial variables of Eq. (10). In the following expression, $\boldsymbol{\pi}^2$ is the Euclidean squared norm of canonical momentum in those comparison coordinates, and ω is evaluated at the corresponding areal radius $r(\rho)$. The receiver Hamiltonian is

$$H = \omega L_z + f \sqrt{m_0^2 c_0^4 + b^2 c_0^2 \boldsymbol{\pi}^2} + O(J^2). \quad (18)$$

In spherical isotropic coordinates, the Euclidean momentum norm in Eq. (18) is $\pi^2 = \pi_\rho^2 + \pi_\theta^2 / \rho^2 + L_z^2 / (\rho^2 \sin^2 \theta)$. The radial and polar momenta may be nonzero. The square root depends on L_z only through its square, so its angular-momentum derivative vanishes at $L_z=0$. The linear term ωL_z still has derivative ω , giving

$$\dot{\phi}|_{L_z=0} = \left. \frac{\partial H}{\partial L_z} \right|_{L_z=0} = \omega(r) \quad \text{to first order in spin.} \quad (19)$$

The result is easy to miss if one looks only at energy values: ωL_z itself is zero at $L_z=0$, but its slope with respect to L_z is not. Motion is determined by that slope. This is why a vanishing rotational energy term at one momentum value can still produce an angular contribution to the coordinate velocity.

Because the source is stationary and axisymmetric, H has no explicit dependence on t or ϕ . The receiver's energy H and canonical angular momentum L_z are conserved. Its angular rate can nevertheless change as it moves through a spatially varying ω . Frame dragging is therefore a conservative change in the relation between momentum and motion. A rotating coordinate choice can remove a constant angular offset, while the source-dependent variation remains in comparisons with the nonrotating standard at infinity.

Figure 1 distinguishes two bodies that share this instantaneous angular rate. A slice-normal observer remains at fixed radius and polar angle and generally needs support. A freely falling receiver with $L_z=0$ can have radial or polar motion as well; its path need not be circular. Along that path it samples the local value of ω , so its angular rate can change while its conserved angular momentum stays zero.

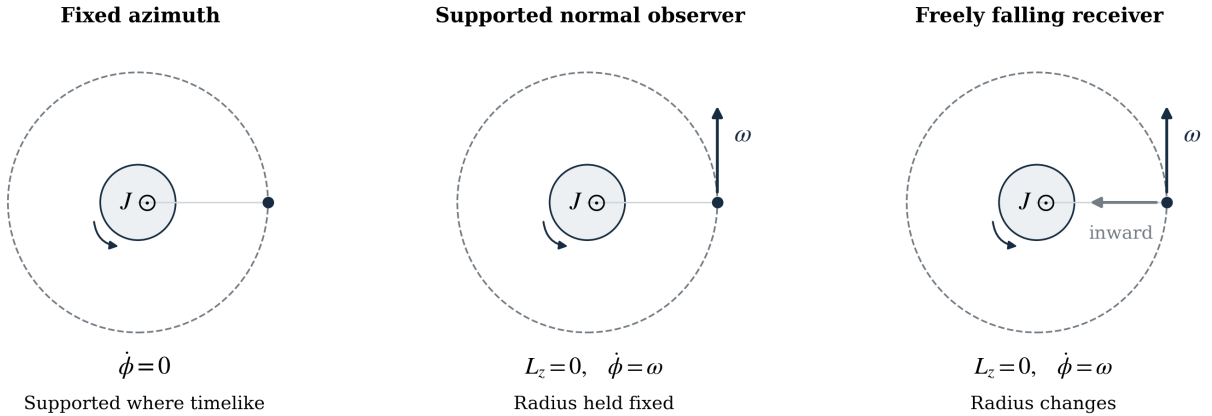


Figure 1. Angular motion is compared with axes nonrotating at infinity. The panels show instantaneous motion in an equatorial schematic, not integrated trajectories. A fixed-azimuth observer has zero coordinate angular rate where such support is possible. The supported slice-normal observer and a freely falling receiver with $L_z=0$ share the local angular rate ω ; the freely falling receiver shown also moves inward. The central dot-in-circle symbol indicates source spin out of the page. Dashed circles mark coordinate locations, not material rings.

4.3 The same coupling in electromagnetic storage

The receiver need not be represented only as a compact material body. Its electromagnetic degrees of freedom carry momentum too. In the weak-field Cartesian Hamiltonian variables used here, that canonical momentum density is $\mathbf{D} \times \mathbf{B}$. Substituting it for the receiver momentum density in the same coupling gives

$$\delta u_{\text{EM}} = \mathbf{w} \cdot (\mathbf{D} \times \mathbf{B}) \quad (20)$$

Equation (20) is added to the quadratic storage density in Eq. (2). The two quadratic terms assign energy to electric and magnetic storage separately; the mixed term depends on their joint orientation and on the source circulation. A field configuration with the opposite momentum reverses $\mathbf{D} \times \mathbf{B}$ and therefore reverses the mixed contribution. This is the energy-level reason that the rotating background distinguishes opposite directions of propagation.

This mixed constitutive response is called magnetoelectric coupling [9]. The expression refers to the stated canonical fields and comparison-coordinate volume. Matter and field momentum are included in the conserved total stress, so this is the field form of the same interaction already used for the receiver. Any internal response absorbed into effective material coefficients must still be counted only once.

4.4 Reading the rotational shift in matter-wave phase

The phase description gives the same result. Let ν be angular frequency in the reference time, let m be a nonzero integer azimuthal index, and let k_i be the spatial wave covector. With i denoting the imaginary unit in the exponential, a matter-wave phase proportional to $e^{-i\nu t + im\phi}$ has $k_\phi = m$ and $L_z = \hbar m$. The signs of m describe opposite orbital phase windings around the source. Compare them at the same position and equal radial and polar wave numbers.

Begin with the phase itself. At a chosen location, introduce a locally rotating azimuth $\tilde{\phi} = \phi - \omega t$, treating the background as constant over the local comparison. Substitution into the phase gives

$$e^{-i\nu t + im\phi} = e^{-i(\nu - m\omega)t + im\tilde{\phi}}, \quad \nu_{\text{local}} = \frac{\nu - m\omega}{f}. \quad (21)$$

The first equality removes the angular phase accumulated by following the local shift. The division by f then converts reference time to the slice-normal observer's proper time. Thus rotation enters the phase through the same signed combination that appears in energy: $H - \omega L_z = \hbar(\nu - m\omega)$. Here ν is angular frequency, measured in radians per unit time; an ordinary frequency in cycles per unit time is obtained by dividing by 2π .

For patterns with the same mass and matched radial and polar wave numbers, let ν_* be the common nonrotating positive frequency for $+m$ and $-m$. At first order in spin the clock and spatial factors retain their nonrotating values. The opposite phase windings contribute opposite signs of $m\omega$, giving reference frequencies $\nu_* + m\omega$ and $\nu_* - m\omega$.

Choosing $m > 0$ gives the signed reference-frequency separation $2m\omega$. The normal observer subtracts each pattern's signed shift, leaving the common value $\nu_{\text{local}} = \nu_*/f$ at this order.

The inverse-response formulation records this phase argument in the language of local propagation. Define K_m as the local inverse matter-wave response. A zero of this inverse response is a pole of the response itself: a frequency at which a freely propagating local wave is allowed. Let Z be a common smooth, real, nonzero normalization near a simple positive-frequency pole. In the conservative local approximation, with background variations over a wavelength neglected, Eq. (18) gives

$$K_m = Z \left[\frac{(\nu - m\omega)^2}{f^2} - c_0^2 \gamma^{ij} k_i k_j - \frac{m_0^2 c_0^4}{\hbar^2} \right]. \quad (22)$$

The terms inside the brackets compare the frequency measured by the normal observer with spatial propagation and rest-mass energy. The inverse spatial metric γ^{ij} converts the wave covector k_i into its proper spatial magnitude. The normalization Z changes the scale of the inverse response without moving its zero. This makes the zero, rather than the absolute size of K_m , the relevant physical quantity.

Let K_s denote the zero-spin inverse response for this paired comparison, and let ν_* be its common positive-frequency zero. A small change δK_m displaces a simple zero by the amount required to cancel that change along the frequency slope. Writing Re for the conservative real part, this first-order rule is

$$\delta \nu_m = - \frac{\delta \text{Re} K_m}{\partial_\nu \text{Re} K_s} \Big|_{\nu=\nu_*}, \quad \delta \nu_{\pm m} = \pm m\omega. \quad (23)$$

Expanding $(\nu - m\omega)^2$ supplies the term $-2\nu m\omega$. Its opposite signs for $+m$ and $-m$ give a particularly useful paired expression:

$$\frac{\text{Re} K_{-m} - \text{Re} K_{+m}}{2m \partial_\nu \text{Re} K_s} \Big|_{\nu=\nu_*} = \omega(r) = \frac{\hbar c_0^3 J}{16\pi A_0 r^3}. \quad (24)$$

At the unperturbed zero, the bracket multiplying Z vanishes. Consequently, differentiating Z contributes nothing there, and the same remaining normalization cancels between the numerator and denominator of Eq. (24). The mass and matched spatial terms cancel from the paired difference as well. The result isolates the source's angular shift using the receiver's local phase response.

This completes the receiver argument. The source response enters energy as ωL_z , angular motion through the momentum derivative of that term, and phase through $\nu - m\omega$. These are mutually consistent descriptions of one conservative coupling. The phase calculation is local; a global bound-state or black-hole oscillation spectrum also requires spatial boundary conditions.

5 The Kerr exterior in clock and ruler variables

5.1 Extending the description to finite spin

The preceding calculation establishes the leading rotational law and its strong-static-field continuation. At finite spin, the nonlinear Einstein-equivalent dynamics admits the Kerr solution [7, 8]. We select a stationary, asymptotically flat, uncharged black-hole exterior with the regularity and horizon conditions of that solution. These conditions distinguish Kerr from the exterior of a generic rotating material body, whose mass and current distribution can carry additional independent multipole moments. The task here is to express the exact black-hole exterior through the same material comparison variables.

The change in task is important. Section 3 derived a first-order rotational coefficient from a spherical source and its exterior constraint. It did not determine every finite-spin multipole of an arbitrary rotating body. Here the specified nonlinear dynamics and black-hole exterior conditions select Kerr. We ask how its exact metric separates into the clock, ruler, and angular-shift variables already used to describe material comparisons.

Define the spin length $a=J/(Mc_0)$ and use Boyer-Lindquist coordinates. Their radial coordinate r reduces to the Schwarzschild areal radius at zero spin, but at finite spin it is a coordinate label rather than the radius of an area $4\pi r^2$. Introduce the functions Σ , Δ , and A by

$$\begin{aligned}\Sigma &= r^2 + a^2 \cos^2 \theta, & \Delta &= r^2 - 2r_g r + a^2, \\ A &= (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta.\end{aligned}\tag{25}$$

The geometric function A has dimensions of length to the fourth power and is distinct from the collective stress stiffness A_0 . The functions Σ and Δ have dimensions of length squared. They collect the radial and angular dependence so that the physical roles of the resulting coefficients can be identified without carrying the full expanded metric through every step.

5.2 Separating clock rate from angular motion

The time and azimuthal terms of a rotating metric are mixed. Completing their square is a way of identifying the angular motion that removes this mixed term locally. With t measured in reference time, the desired form of the interval is

$$ds^2 = -f^2 c_0^2 dt^2 + \gamma_{rr} dr^2 + \gamma_{\theta\theta} d\theta^2 + \gamma_{\phi\phi} (d\phi - \omega dt)^2.\tag{26}$$

To see what is being extracted, write the original time-azimuth terms as $g_{tt}dt^2 + 2g_{t\phi}dt d\phi + g_{\phi\phi}d\phi^2$, where these g components refer to the time coordinate t , not $x^0 = c_0 t$. Expanding the last square in Eq. (26) and matching coefficients gives

$$\gamma_{\phi\phi} = g_{\phi\phi}, \quad \omega = -\frac{g_{t\phi}}{g_{\phi\phi}}, \quad f^2 = \frac{-g_{tt} + g_{t\phi}^2/g_{\phi\phi}}{c_0^2}. \quad (27)$$

The angular rate is chosen to absorb the cross term. The remaining time coefficient is the clock factor for an observer following that angular rate at fixed r and θ . Substituting the Kerr coefficients yields

$$f^2 = \frac{\Sigma \Delta}{A}, \quad \omega = \frac{2r_g \arcc_0}{A}, \quad (28)$$

with spatial coefficients

$$\gamma_{rr} = \frac{\Sigma}{\Delta}, \quad \gamma_{\theta\theta} = \Sigma, \quad \gamma_{\phi\phi} = \frac{A \sin^2 \theta}{\Sigma}. \quad (29)$$

This rearrangement accomplishes a physical separation: ω specifies how the normal observer turns relative to distant coordinates, f specifies that observer's clock rate, and γ_{ij} specifies spatial lengths on the selected time slice. A slice-normal observer is perpendicular, in the spacetime metric, to the constant-time spatial slices. Its angular momentum is zero because it follows the shift rather than moving azimuthally relative to it. Such an observer is commonly called a zero-angular-momentum observer, or ZAMO.

For this observer, $dr = d\theta = 0$ and $d\phi = \omega dt$. All spatial terms in Eq. (26) then vanish along its worldline, leaving $d\tau = f dt$, where τ is its proper time. Keeping r and θ fixed generally requires support. A freely falling body can share the angular rate without sharing the supported worldline, as Figure 1 illustrates. Table 1 collects the distinctions that matter when interpreting f .

Table 1. Observer motions and the clock comparison

Observer	Motion in the coordinates	Meaning of the clock comparison
Fixed azimuth	Fixed r , θ , and ϕ ; requires support.	Clock rate depends on both f and ω ; possible only outside the ergosphere.
Slice-normal observer (ZAMO)	Fixed r and θ ; angular rate ω ; generally supported.	Zero angular momentum; proper-time rate $d\tau/dt = f$.

Observer	Motion in the coordinates	Meaning of the clock comparison
Freely falling receiver with zero angular momentum	Angular rate ω ; radial or polar motion may occur.	Shares the angular rate, but its motion adds a further clock comparison.

The ergosphere is the region outside the horizon where an observer cannot remain at fixed spatial coordinates with a timelike worldline. In Eq. (26), holding $d\phi=0$ leaves a positive $\gamma_{\phi\phi}\omega^2 dt^2$ contribution competing with the negative time term. The slice-normal observer instead follows $d\phi=\omega dt$ and continues to provide a timelike local comparison outside the horizon [8]. A lapse read from this comparison must therefore not be assigned indiscriminately to a fixed-azimuth clock.

Let π_i be the canonical spatial momentum covector, with $\pi_\phi=L_z$. Define E_{local} and ν_{local} as the receiver energy and wave frequency measured by a normal observer. The exact Hamiltonian on the branch with positive local energy, together with the phase conversion, is

$$H=\omega L_z+f\sqrt{m_0^2 c_0^4+c_0^2\gamma^{ij}\pi_i\pi_j}, \quad \nu_{\text{local}}=\frac{\nu-m\omega}{f}. \quad (30)$$

The positive square root is $E_{\text{local}}=(H-\omega L_z)/f$. The formula first removes the angular-shift contribution from the comparison energy and then converts the remainder to the normal observer's local standard. For a receiver at rest relative to that observer, the spatial momentum is zero, so $E_{\text{local}}=m_0 c_0^2$ and $H=f m_0 c_0^2$. The factor f relates this rest-energy comparison to the distant standard; it is not applied a second time to locally measured rest energy.

Stationarity and axial symmetry conserve H and L_z along a free trajectory, but E_{local} can vary as the receiver moves between normal observers at different positions. In the ergosphere, suitable canonical angular momentum opposite to the source spin can make the conserved comparison energy H negative even though the square root, and hence the locally measured energy, remains positive. The sign distinction belongs to the choice of reference time and observer; the Hamiltonian retains the positive-local-energy branch.

The storage interpretation remains tied to the specified corresponding-state family. Applying $p=f^2$ identifies its supporting-power ratio with the squared clock factor of the normal-observer comparison. Other observer motions have their own proper-time rates and phase conversions, as Eq. (30) shows. This keeps the material-state factor distinct from the motion of the observer used to measure it.

5.3 What changes when the spin reverses

Reversing the source spin reverses ω but preserves f^2 at fixed mass, Boyer-Lindquist radius, and polar angle. Expanding the exact lapse gives

$$f^2 = 1 - \frac{2r_g}{r} + \frac{2r_g a^2}{r^3} \left(\cos^2 \theta + \frac{2r_g}{r} \sin^2 \theta \right) + O(a^4). \quad (31)$$

The distinction between odd and even powers of spin explains why the effects appear in different stages. The angular shift is odd in a : reversing the source's rotation reverses its direction. The squared clock factor and the spatial ruler factors are even: they do not distinguish the two senses of rotation at the same specified coordinate location. The first rotational change in angular motion is therefore linear in spin, while the additional scalar clock correction begins at quadratic order.

The positive quadratic correction in Eq. (31) compares locations at fixed M , Boyer-Lindquist r , and θ . It is a statement about that defined comparison, rather than a general measure of a stronger or weaker gravitational interaction. A comparison at fixed circumferential radius holds a different physical quantity fixed and must be specified separately. Its practical use here is to explain why the first-order phase calculation could change ω while retaining the static clock and spatial factors.

5.4 Why the ruler factors become directional

The remaining task is to convert spatial coordinate intervals into proper lengths. In the static isotropic description, one factor b was sufficient for all three directions. Kerr's finite-spin spatial coefficients distinguish motion around the axis from motion in a plane through the axis. A radial coordinate change makes that distinction transparent by putting the radial and polar directions on the same scale.

For a subextremal black hole, introduce the quasi-isotropic radius ρ by

$$r = \rho + r_g + \frac{r_g^2 - a^2}{4\rho}, \quad \rho = \frac{r - r_g + \sqrt{\Delta}}{2}. \quad (32)$$

The exterior restriction is $|a| < r_g$ and $r > r_+$, with $r_+ = r_g + \sqrt{r_g^2 - a^2}$. Differentiating the coordinate relation gives $dr/d\rho = \sqrt{\Delta}/\rho$. Substitution into the radial metric term produces $(\Sigma/\Delta)dr^2 = (\Sigma/\rho^2)d\rho^2$. The polar term is $\Sigma d\theta^2 = (\Sigma/\rho^2)\rho^2 d\theta^2$, so both terms have the same coefficient when measured against Euclidean spherical-coordinate intervals.

Call their shared material ruler factor b_{mer} , where meridional refers to a constant-azimuth section through the rotation axis. Call the factor for the direction around that axis b_ϕ . Matching the azimuthal term separately gives

$$\begin{aligned} b_{\text{mer}} &= \frac{\rho}{\sqrt{\Sigma}}, & b_\phi &= \frac{\rho\sqrt{\Sigma}}{\sqrt{A}}, \\ dl^2 &= \frac{d\rho^2 + \rho^2 d\theta^2}{b_{\text{mer}}^2} + \frac{\rho^2 \sin^2 \theta d\phi^2}{b_\phi^2}. \end{aligned} \quad (33)$$

A ruler factor is a material length ratio, while the reciprocal of that factor converts a comparison-coordinate interval to proper length. The three individual comparisons contained in Eq. (33) are

$$dl_\rho = \frac{d\rho}{b_{\text{mer}}}, \quad dl_\theta = \frac{\rho d\theta}{b_{\text{mer}}}, \quad dl_\phi = \frac{\rho \sin \theta d\phi}{b_\phi}. \quad (34)$$

Here the expressions denote positive small lengths in each selected direction. The factors are evaluated locally. A smaller material ruler factor means more local ruler units span the same comparison-coordinate interval, hence the division by b . Figure 2 shows the directions and the intervals being compared. It depicts the coordinate construction, not a physical surface embedded in ordinary space.

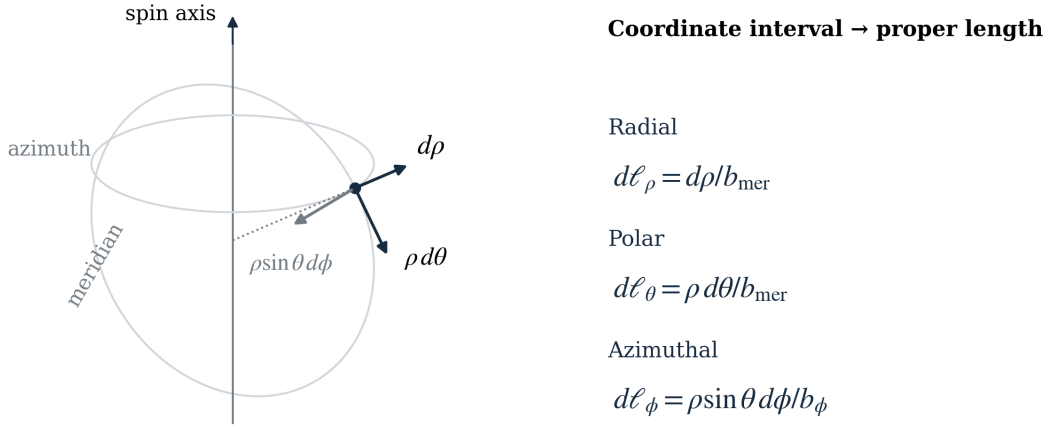


Figure 2. Directional ruler comparisons in quasi-isotropic coordinates. The meridian lies in a plane containing the spin axis; azimuth runs around that axis. The radial and polar intervals share b_{mer} , while the azimuthal interval uses b_ϕ . The circular guide and local arrows identify coordinate directions only; their shapes and lengths do not represent a Kerr embedding or measured distortion.

At zero spin, both factors reduce to $\rho/r = (1 + r_g/(2\rho))^{-2}$, recovering the static isotropic b in Eq. (10). At finite spin they generally differ away from the axis. The coordinate change has not introduced another gravitational response; it has exposed the two spatial conversions already present in the exact Kerr metric.

5.5 From directional rulers to electrical storage

The clock and ruler factors can now be inserted into the material storage law. This step connects the metric comparison back to the electric and magnetic energy variables introduced in Section 2. Define the ruler matrix $\mathbf{R}=\text{diag}(b_{\text{mer}}, b_{\text{mer}}, b_\phi)$ in a local orthonormal basis of the Euclidean comparison coordinates, ordered as radial, polar, and azimuthal. Its determinant $V_R=b_{\text{mer}}^2 b_\phi$ is the material volume ratio. Let bold $\boldsymbol{\epsilon}$ and $\boldsymbol{\mu}$ denote the electric and magnetic storage tensors; superscript T denotes transpose. The directional law of [1] is

$$\frac{\boldsymbol{\epsilon}}{\epsilon_0} = \frac{\boldsymbol{\mu}}{\mu_0} = \frac{\mathbf{R}\mathbf{R}^T}{fV_R}. \quad (35)$$

The matrix expression is compact, but its individual entries show what it accomplishes. The radial and polar numerators are b_{mer}^2 , whereas the azimuthal numerator is b_ϕ^2 . Use the label mer for either radial or polar storage entry. Dividing by the common clock-volume factor gives

$$\begin{aligned} \frac{\epsilon_{\text{mer}}}{\epsilon_0} &= \frac{\mu_{\text{mer}}}{\mu_0} = \frac{1}{fb_\phi}, \\ \frac{\epsilon_\phi}{\epsilon_0} &= \frac{\mu_\phi}{\mu_0} = \frac{b_\phi}{fb_{\text{mer}}^2}. \end{aligned} \quad (36)$$

The appearance of the other directional factor in each result follows from the volume in the denominator: storage energy is compared per coordinate volume, not only along one line. When $b_{\text{mer}}=b_\phi=b$, every entry becomes $1/(fb)$, recovering Eq. (1). The same directional scaling of electric and magnetic storage retains their matched relation.

These tensors set the quadratic terms $1/2 \mathbf{D} \cdot \boldsymbol{\epsilon}^{-1} \mathbf{D} + 1/2 \mathbf{B} \cdot \boldsymbol{\mu}^{-1} \mathbf{B}$. Rotation also supplies the mixed term $\mathbf{w}_{\text{rot}} \cdot (\mathbf{D} \times \mathbf{B})$, with $\mathbf{w}_{\text{rot}} = \omega \rho \sin \theta \mathbf{e}_\phi$ in this Euclidean comparison basis and with the corresponding comparison-volume field variables. Here $\mathbf{e}_\phi$ is the azimuthal unit vector. The factor $\rho \sin \theta$ converts angular rate to the shift's vector component in this basis; the earlier weak-field expression used its own radial comparison coordinate. The same angular shift ω organizes both descriptions.

The exact electromagnetic Hamiltonian therefore has two identifiable ingredients: the clock and directional rulers determine the quadratic storage coefficients, and the angular shift determines the signed momentum coupling. This is the finite-spin counterpart of Eq. (20). The construction completes the return from the exact Kerr interval to the same energy-storage language with which the source-to-receiver calculation began.

5.6 Reading the numerical comparison

For a numerical illustration, define the signed dimensionless spin $\chi = Jc_0/(G_0M^2) = a/r_g$. Table 2 holds the Boyer-Lindquist coordinate $r=4r_g$ and polar angle $\theta=\pi/2$ fixed. All entries are dimensionless. The column f^2 is the squared lapse, also the supporting-power ratio for the specified corresponding-state family; the clock factor itself is its square root. To recover an angular rate from the third column, multiply by c_0/r_g .

Table 2. Kerr comparison factors at $r=4r_g$ in the equatorial plane

χ	f^2	$\omega r_g/c_0$	b_{mer}	b_ϕ
0	0.500000	0	0.728553	0.728553
0.1	0.500156	0.003122	0.728774	0.728433
0.5	0.503817	0.015267	0.734035	0.725582
0.9	0.511763	0.026140	0.746021	0.719213
-0.9	0.511763	-0.026140	0.746021	0.719213

The zero-spin row supplies the baseline: $f^2=0.5$, the angular shift vanishes, and both ruler factors coincide. As positive spin increases, the dragging rate becomes positive and the two ruler factors separate. The last two rows provide the cleanest sign test: changing $\chi=0.9$ to -0.9 reverses only the angular shift. The clock and ruler comparisons are unchanged because they are even in spin.

At $\chi=0.1$, the first-order dragging value is $\omega r_g/c_0=0.003125$, compared with the exact value 0.003122073, a relative difference of about 0.094%. The agreement illustrates the small-spin limit. At larger spin the exact formulas retain the directional ruler split and the scalar correction together.

The numerical verification reported for the underlying calculation evaluates the spherical integral by quadrature for uniform and radially varying densities, tests the receiver frequency splitting, and reconstructs the exact Kerr interval at 256 exterior points with both spin signs. It also checks the quasi-isotropic spatial transformation, the zero-spin limit, and the relativistic energy-momentum relation generated by the Hamiltonian. The reported spherical-integral agreement is better than 10^{-10} in relative error, and the Kerr identities agree to better than 10^{-12} in absolute error in dimensionless units. These are consistency checks of the equations and their implementation within the stated framework; the table illustrates their physical interpretation.

6 Conclusions

The energy-storage description of static gravity extends to rotation by retaining the momentum carried by matter and fields. Supporting exchange establishes the material state, and its relaxed total-energy response supplies the stress functional. The WTDiff dynamics then couples circulating source momentum to the receiver with the same strength that governs the static interaction.

The source integral fixes the factor of two in $\omega=2G_0J/(c_0^2r^3)=\hbar c_0^3J/(16\pi A_0r^3)$. The vacuum momentum constraint then preserves that radial law at first order in spin while retaining the full static compactness. Together, these steps identify the strength of the response and establish its strong-static-field continuation without introducing a separate rotational gravitational coupling.

For the receiver, the essential addition is ωL_z . Its momentum derivative explains why zero canonical angular momentum permits angular motion relative to distant coordinates. The same term appears in wave phase as $v\cdot m\omega$ and gives opposite reference-frequency shifts in the local comparison of paired azimuthal patterns. A freely falling zero-angular-momentum receiver can share the normal observer's angular rate while also moving radially or polarly; equality of angular rates does not make their worldlines identical.

For the selected finite-spin black-hole exterior, completing the Kerr time-azimuth square identifies the clock factor and angular shift. The quasi-isotropic spatial transformation identifies the radial-meridional and azimuthal ruler factors. Those factors then determine the electric and magnetic storage tensors, while the shift supplies their mixed momentum coupling. Spin reversal changes the direction of dragging and preserves the clock and ruler factors, and the zero-spin limit returns continuously to the static description.

These results complete the source-to-receiver rotational calculation within the theory's stated Einstein-equivalent framework. Rotating matter changes how nearby matter carries energy, momentum, and phase, and the Kerr geometry records those comparisons. Matter tells matter how to scale. Geometry is just bookkeeping.

Appendix A The stationary angular momentum constraint

The purpose of this calculation is to identify which static-field factors survive in the equation for the angular shift. The spatial derivative below is metric-compatible; it includes the connection terms needed when comparing components at neighboring positions. Keeping those terms is essential because the radial coordinate is not a Cartesian distance.

This appendix supplies the intermediate calculation behind Eq. (12). Use the first-order interval of Eq. (11) and the coordinate $x^0 = c_0 t$. Let $\gamma = \det \gamma_{ij}$, let $\nabla_i^{(3)}$ denote the covariant derivative compatible with the spatial metric, and let β^i be the coordinate shift per unit x^0 , with only $\beta^\phi = -\omega/c_0$ nonzero. Lowering its index uses $\beta_i = \gamma_{ij} \beta^j$. Define the extrinsic curvature K_{ij} , which records how successive spatial slices fit together, by the convention

$$K_{ij} = \frac{\nabla_i^{(3)} \beta_j + \nabla_j^{(3)} \beta_i}{2\sqrt{F}}. \quad (\text{A1})$$

Write a prime for d/dr . The trace $\gamma^{ij} K_{ij}$ vanishes. The only independent nonzero component and the corresponding volume-weighted raised component are

$$K_{r\phi} = -\frac{r^2 \sin^2 \theta}{2c_0 \sqrt{F}} \omega', \quad \sqrt{\gamma} K_\phi^r = -\frac{r^4 \sin^3 \theta}{2c_0} \omega'. \quad (\text{A2})$$

The vacuum constraint is $\nabla_j^{(3)} K_\phi^j = 0$. For the axisymmetric spatial metric, the connection contributions from the two off-diagonal components cancel, leaving $\partial_r(\sqrt{\gamma} K_\phi^r) = 0$. The lapse has canceled in this product, so the remaining radial equation is $(r^4 \omega')' = 0$. This gives Eq. (12) with its full static radial dependence retained.

The cancellation explains the scope of the result. Strong static curvature is present in the spatial metric and in the intermediate extrinsic curvature, but its lapse factors cancel in the volume-weighted momentum constraint. The surviving equation fixes the same r^{-3} angular-shift law. The argument remains first order in spin; it does not discard the static compactness or establish a finite-spin solution by itself.

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Author contributions. Todd J. Desiato conceived the OQG program and the material-scaling interpretation, directed the theoretical development, reviewed the calculations and literature, and prepared the manuscript.

Data availability. This theoretical study generated no new experimental or observational datasets. The analytical derivations, tabulated Kerr comparison factors, and numerical consistency checks are contained in the article.

Code availability. The verification script, `verify_rotational_storage.py`, and its numerical results are available from the corresponding author. They reproduce the spherical source integral, receiver frequency splitting, and Kerr consistency checks.

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Use of generative artificial intelligence. The author used OpenAI ChatGPT and Codex for literature discovery, mathematical cross-checking, numerical evaluation, organization, drafting, and editing. The author is responsible for reviewing the scientific arguments, equations, references, and final text and for the content of the manuscript.

Supplementary information. The analytical derivations and supporting calculations are included in the article. No separate supplementary files accompany this article.

References

- [1] Desiato, T.J.: Operational Quantum Gravity: From Electromagnetic Power Balance to Gravitational Response. Author manuscript, V2.3 (2026).
- [2] Desiato, T.J.: Operational Quantum Gravity from a Standard Model Stress Hessian to Einstein-Equivalent WTDiff Dynamics. Author manuscript, V0.3 (2026).
- [3] Desiato, T.J.: Matter Wave Scaling and Recoverable Energy Storage in Operational Quantum Gravity. Author manuscript, V1.5 (2026).
- [4] Álvarez, E., Blas, D., Garriga, J., Verdaguer, E.: Transverse Fierz-Pauli symmetry. Nucl. Phys. B **756**, 148–170 (2006). <https://doi.org/10.1016/j.nuclphysb.2006.08.003>
- [5] Barceló, C., Carballo-Rubio, R., Garay, L.J.: Unimodular gravity and general relativity from graviton self-interactions. Phys. Rev. D **89**, 124019 (2014). <https://doi.org/10.1103/PhysRevD.89.124019>
- [6] Hartle, J.B.: Slowly rotating relativistic stars. I. Equations of structure. Astrophys. J. **150**, 1005–1029 (1967). <https://doi.org/10.1086/149400>
- [7] Kerr, R.P.: Gravitational field of a spinning mass as an example of algebraically special metrics. Phys. Rev. Lett. **11**, 237–238 (1963). <https://doi.org/10.1103/PhysRevLett.11.237>
- [8] Visser, M.: The Kerr spacetime: A brief introduction. In: Wiltshire, D.L., Visser, M., Scott, S.M. (eds.) *The Kerr Spacetime: Rotating Black Holes in General Relativity*, pp. 3–37. Cambridge University Press, Cambridge (2009).
- [9] Gibbons, G.W., Werner, M.C.: The gravitational magnetoelectric effect. Universe **5**, 88 (2019). <https://doi.org/10.3390/universe5040088>